



# Neutrino phenomenology and scalar Dark Matter with $A_4$ flavor symmetry in Inverse and type II seesaw

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## Abstract

We present a TeV scale seesaw mechanism for exploring the dark matter and neutrino phenomenology in the light of recent neutrino and cosmology data. A different realization of the Inverse seesaw (ISS) mechanism with  $A_4$  flavor symmetry is being implemented as a leading contribution to the light neutrino mass matrix which usually gives rise to vanishing reactor mixing angle  $\theta_{13}$ . Using a non-diagonal form of Dirac neutrino mass matrix and  $3\sigma$  values of mass square differences we parameterize the neutrino mass matrix in terms of Dirac Yukawa coupling “ $y$ ”. We then use type II seesaw as a perturbation which turns out to be active to have a non-vanishing reactor mixing angle without much disturbing the other neutrino oscillation parameters. Then we constrain a common parameter space satisfying the non-zero  $\theta_{13}$ , Yukawa coupling and the relic abundance of dark matter. Contributions of neutrinoless double beta decay are also included for standard as well as non-standard interaction. This study may have relevance in future neutrino and Dark Matter experiments.

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## 1. Introduction

The link between neutrino oscillation and modern cosmology needs an elucidation since both of them infer physics beyond Standard Model (BSM). Several theories have been deciphered to bridge between these two separate sectors of particle physics and cosmology [1]. There is

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now a plethora of evidences for the existence of dark matter (DM) that constructs approximately a quarter of the energy density of the universe [2–5]. Despite a number of recent studies of simplified DM models their nature remains rather elusive. The most successful Standard Model (SM) of particle physics too has not been able to furnish any signature of DM candidates and their properties. This is one of the pressing problems in both high energy physics and cosmology. This may surmise new physics beyond the standard model in near future. Therefore, searching for a concrete realization to provide a hint towards physics BSM will be of utmost interest. It will be more fascinating if the discovery of neutrino oscillation and the existence of DM can be framed within the same particle physics model.

Presence of DM in the universe has been well established by astrophysics and cosmology experiments, although the exact particle nature of DM is yet unknown. According to the Planck 2013 data [5], 26.8% of the energy density of the present universe is composed of DM. The present abundance of DM or relic density is represented as

$$\Omega_{DM}h^2 = 0.1187 \pm 0.0017, \quad (1)$$

where  $\Omega$  is the density parameter and  $h = \text{Hubble parameter}/100$  is a parameter of order unity [6].

Authors in [7] proposed a ten-point test that new particles have to pass in order to be considered as good DM candidates. The existence of dark matter is universally accepted, its nature remains elusive. It is usually assumed to be a single particle, but it may also be more than one. In specific models, it is often considered to be a fermion, scalar or vector [8]. Among the requirements the potential DM candidate must meet, the stability is protected by invoking some parity symmetry like  $Z_2$  which is supposed to appear as a residual of a discrete flavor symmetry. There have been extensive studies in this field adopting various flavor symmetry groups [9–11]. We have plenty of examples where different kinds of DM were extensively studied with their stability in several ways. Recently connection between neutrino and the DM, using various flavor symmetries is drawing more attention in particle physics and cosmology. Here also we present a picture to construct a bridge between these two different sectors of particle physics adopting the  $A_4$  based ISS realization. The most peculiar signatures of the ISS scenario are the additional decay channels of the Higgs boson into a heavy and ordinary neutrino, which confirms the SM particles to be a gateway to the scalar DM. In order for the SM particles being a portal to the dark sector, there must be at least two particles, one fermion and one boson in the dark sector. Here in our model Higgs boson, is considered as a DM candidate, couples with SM neutrino through a right handed neutrino. Two neutral components of this Higgs which is a triplet under  $A_4$  are responsible in making correlation with neutrino mass and Dark Matter. The stability of the DM is explained by a remnant  $Z_2$  symmetry. This  $Z_2$  symmetry also prevents the interaction of other particle contents of the model with the DM. Apart from the stability issue one more important test it must pass is to satisfy the observed relic density given by equation (1). For getting the correct relic abundance we require to take the DM mass from 50 GeV onwards. The Yukawa, which is responsible in making correlation between neutrino mass and DM coupling also needs to be fixed in such a way that the potential DM candidate gives rise to correct relic abundance.

Several seesaw mechanisms have shown a promising role in explaining neutrino mass and mixing. The Inverse Seesaw (ISS) has been found to be an entirely different realization, which beautifully offers an explanation for having a tiny neutrino mass at the cost of proposing the RH neutrino masses at the TeV scale which may be probed at the LHC experiments. The essence of the ISS lies in the fact that the double suppression by the mass scale associated with  $M$  makes it possible to have such a scale much below than that involved in the canonical seesaw mechanism. Which in turn renders us with SM neutrinos at sub-eV scale obtained with  $m_D$  at electroweak

scale,  $M$  at TeV scale and  $\mu$  at keV scale as explained in [12]. This RH neutrino mass at TeV scale helps us to get the required mediator mass in order to obtain the appropriate relic abundance of relics. In addition to the ISS we are working with the Type II seesaw mechanism which turns out to be instrumental to have the non-vanishing reactor mixing angle. Both the inverse and type II seesaw are realized adopting the  $A_4$  flavor symmetry. Then we have also studied the effective mass prediction to neutrinoless double beta decay for standard and non-standard contributions due to light neutrino exchanges.

We organize the paper as follows. In Section 2 we present our model. Section 3 provides the stability issue of DM. Non-zero reactor angle is explained in the Section 4. Section 5 has been presented with the analysis on Neutrinoless double beta decay. Section 6 offers the observation of the Relic abundance of DM in the context of the proposed model. In Section 7 we have shown the numerical analysis. Finally, in Section 8 we end up with our conclusion.

## 2. Neutrino mass model with various seesaw scenarios

### 2.1. Inverse seesaw mechanism

In our work we focus on the simplest ISS mechanism which is able to open up a new window to get the right handed neutrino mass at a scale much below the one that involved in the canonical seesaw [12–19].

The fulfillment of the ISS scheme requires the extension of the SM fermion sector by the addition of three RH neutrinos  $N$  and three extra SM singlet neutral fermions  $S_{iL}$  to the active neutrinos  $\nu_{iL}$ , with  $i = 1, 2, 3$ . It is worth stating that, the implementation of the ISS allows us to make use of extra symmetries in order to provide the neutrinos the following bilinear terms,

$$\mathcal{L} = -\bar{\nu}_L m_D N - \bar{S}_L M N - \frac{1}{2} \bar{S}_L \mu S_L^C + H.C. \quad (2)$$

The above Lagrangian implies a  $9 \times 9$  leptonic mass matrix,

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix}. \quad (3)$$

In spite of its many phenomenological successes the ISS has a drawback that the right-handed mass term in the  $M_{\nu 22}$  entry of  $M_\nu$  is allowed by symmetries. This is a typical problem of inverse seesaw models. But it is prevented here by using  $Z_3$  symmetry. After block diagonalization of the equation (3) we get the lightest neutrino mass eigenvalue as

$$m_\nu^I = m_D (M^T)^{-1} \mu M^{-1} m_D^T, \quad (4)$$

which is considered as leading order contribution to the neutrino mass. Unlike the canonical seesaw mechanism that got its position in GUT, the ISS still lacks a special framework where the six new neutrinos could find their places in the elemental particle content and normally can get a mass term.

Non-Abelian discrete flavor symmetries have played an important role in particle physics. In particular the symmetry group  $A_4$  has been immensely found of utmost operation [20–24]. In this work we have analyzed the model presented by the authors in [9], extended with additional flavons with inverse and type II seesaw. The flavor symmetry group  $A_4$  is the group of permutation of four objects, isomorphic to the symmetry group of a tetrahedron.  $A_4$  has four

Table 1

Fields and their transformation properties under  $SU(2)_L$ , the  $A_4$  flavor symmetry,  $Z_2, Z_3$  flavor symmetry.

	$L_e$	$L_\mu$	$L_\tau$	$l_e^c$	$l_\mu^c$	$l_\tau^c$	$N$	$N_4$	$h$	$\eta$	$S_4$	$S$	$\phi_R$	$\phi_s$	$\zeta$	$\xi$	$\Delta$
$SU(2)_L$	2	2	2	1	1	1	1	1	2	2	1	1	1	1	1	1	3
$A_4$	1	1'	1''	1	1''	1'	3	1	1	3	1	3	1	1	1'	1''	1
$Z_2$	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	1	1	1	1
$Z_3$	$\omega$	$\omega$	$\omega$	$\omega^2$	$\omega^2$	$\omega^2$	$\omega^2$	$\omega^2$	1	1	$\omega$	$\omega$	1	$\omega$	1	1	$\omega$

irreducible representations, among which there are three singlets and one triplet. The group has got two generators. We summarize the  $A_4$  based ISS model by assigning the matter fields as shown in Table 1. Four right handed neutrinos are introduced, three of which  $N = (N_1, N_2, N_3)$  are supposed to transform as a triplet of  $A_4$  and the rest as a singlet  $N_4$ . We assign the SM type Higgs  $\eta$  to the  $A_4$  triplet, which is considered as a DM candidate in the present analysis. We have four additional SM fermion singlets among which ‘ $S$ ’ is transforming as  $A_4$  triplet and  $S_4$  as  $A_4$  singlet. To get a desired neutrino mass matrix structure we are extending the Higgs sector by introducing six more Higgs fields, boosted by two additional symmetries  $Z_2$  and  $Z_3$  whose quantum numbers are given in Table 1. The triplet multiplication rules of  $A_4$  that has been used in the present analysis are given below (for more details see [25,26]).

The representations are given as follows

$$a, b \sim 1, \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \sim 3.$$

$$(ab)_1 = a_1b_1 + a_2b_2 + a_3b_3$$

$$(ab)_{1'} = a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3$$

$$(ab)_{1''} = a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3$$

$$(ab)_{3_1} = a_2b_3 + a_3b_1 + a_1b_2$$

$$(ab)_{3_2} = a_3b_2 + a_1b_3 + a_2b_1$$

### 2.2. Type II seesaw with triplet Higgs

For the type II seesaw mechanism to be implemented the SM is extended by the inclusion of an additional  $SU(2)_L$  triplet scalar field  $\Delta$  having  $U(1)_Y$  charge twice that of lepton doublets with its  $2 \times 2$  matrix representation as

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & \Delta^+/\sqrt{2} \end{pmatrix}. \tag{5}$$

The Vacuum expectation value of the SM Higgs  $\langle \phi_0 \rangle = v/\sqrt{2}$ , the trilinear mass term  $\mu_{\phi\Delta}$  generate an induced VEV for Higgs triplet as  $\Delta^0 = v_\Delta/\sqrt{2}$ , where  $v_\Delta \simeq \mu_{\phi\Delta} v^2/\sqrt{2}M_\Delta^2$  [27]. The type II seesaw contribution to light neutrino mass is given by

$$m_{LL}^{II} = f_\nu v_\Delta, \tag{6}$$

where the analytic formula for induced VEV for neutral components of the Higgs scalar triplet, derived from the minimization of the scalar potential [27], is

$$v_\Delta \equiv \langle \Delta^0 \rangle = \frac{\mu_{\phi\Delta} v^2}{\sqrt{2} M_\Delta^2} \tag{7}$$

In the low scale type II seesaw mechanism operative at the TeV scale, barring the naturalness issue, one can consider a very small value of the trilinear mass parameter to be

$$\mu_{\phi\Delta} \simeq 10^{-8} \text{ GeV.}$$

The sub-eV scale light neutrino mass with type II seesaw mechanism constrains the corresponding Majorana Yukawa coupling as

$$f_\nu^2 < 1.4 \times 10^{-5} \left( \frac{M_\Delta}{1 \text{ TeV}} \right)$$

Within the reasonable value of  $f_\nu \simeq 10^{-2}$ , the triplet Higgs scalar VEV is  $v_\Delta \simeq 10^{-7}$  GeV which is in agreement with oscillation data. It is worth to note here that the tiny trilinear mass parameter  $\mu_{\phi\Delta}$  controls the neutrino overall mass scale, but does not play any role in the couplings with the fermions. The structure of the matrix  $m_{LL}^I$ , with  $w = f_\nu v_\Delta$  is explained in Section 4.

### 3. Stabilizing the Dark Matter

A simple way to establish the stability of the DM is by invoking some parity symmetry like  $Z_2$ . Here is an attempt to search for a theory which is responsible for explaining neutrino phenomenology and Dark Matter (DM) stability as well. In this ISS realization the symmetry  $A_4 \times Z_2 \times Z_3$  spontaneously breaks to  $Z_2$  accommodating a stable DM candidate. The  $A_4 \times Z_2 \times Z_3$  symmetry here only allows the coupling of the  $\eta$  with the singlet RH neutrinos rather than with charged fermions or quarks. It is worth noting that the alignment  $\langle \eta \rangle \sim (1, 0, 0)$  breaks spontaneously  $A_4 \times Z_2 \times Z_3$  to  $Z_2$  since  $(1, 0, 0)$  remains manifestly invariant under one of the generators of the group  $A_4$ .

The stability of the DM candidate is guaranteed by this remnant symmetry. The  $Z_2$  residual symmetry is defined by

$$N_2 \rightarrow -N_2, S_2 \rightarrow -S_2, \eta_2 \rightarrow -\eta_2$$

$$N_3 \rightarrow -N_3, S_3 \rightarrow -S_3, \eta_3 \rightarrow -\eta_3$$

The leading order Yukawa Lagrangian for the neutrino sector is given by the following equation.

$$L_\nu^I = y_1^\nu L_e(N\eta)_1 + y_2^\nu L_\mu(N\eta)_{1'} + y_3^\nu L_\tau(N\eta)_{1'} + y_4^\nu L_e N_4 h + y_s(SS)\phi_s + y'_s S_4 S_4 \phi_s + y_R(NS)\phi_R + y'_R N_4 S_4 \phi_R. \tag{8}$$

The following flavon alignments help us to get a desired neutrino mass matrix.

$\langle \Phi_R \rangle = v_R, \langle \Phi_s \rangle = v_s, \langle h \rangle = v_h, \langle \eta \rangle = v_\eta(1, 0, 0)$ . It is clear from the equation (9), (10) that,  $m_D$  is connected to  $v_\eta$  and  $v_h$ , and that  $M$  is determined by the VEV  $v_R$ . In this way, the order of magnitude involved in the equation (4) is such that,  $m_\nu \propto \frac{(v_\eta + v_h)^2}{v_R^2} \mu$ . Here  $v_\eta$  and  $v_h$  are of the order of electroweak breaking,  $v_R$  is of the order of TeV scale. Therefore, to get  $m_\nu$  in sub-eV,  $\mu$  which is coming from the VEV of  $\Phi_s$  should be of the order of keV. The two components of  $\eta$  are not generating the VEV [9], considered potential DM candidate. Decomposition of the following terms present in the equation (8) has been shown as follows

$$y_s(SS)\phi_s = y_s(S_1S_1 + S_2S_2 + S_3S_3)\phi_s,$$

$$y_R(NS)\phi_R = y_R(N_1S_1 + N_2S_2 + N_3S_3)\phi_R.$$

The chosen flavon alignments and the  $A_4$  product rules allow us to have the Yukawa coupling matrices as follows

$$m_D = \begin{pmatrix} y_1^v \langle \eta \rangle & 0 & 0 & y_4^v \langle h \rangle \\ y_2^v \langle \eta \rangle & 0 & 0 & 0 \\ y_3^v \langle \eta \rangle & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} x_1 a & 0 & 0 & y_1 b \\ x_2 a & 0 & 0 & 0 \\ x_3 a & 0 & 0 & 0 \end{pmatrix}, \tag{9}$$

$$M = \begin{pmatrix} y_R \langle \phi_R \rangle & 0 & 0 & 0 \\ 0 & y_R \langle \phi_R \rangle & 0 & 0 \\ 0 & 0 & y_R \langle \phi_R \rangle & 0 \\ 0 & 0 & 0 & y'_R \langle \phi_R \rangle \end{pmatrix} = \begin{pmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_1 & 0 & 0 \\ 0 & 0 & M_1 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix}, \tag{10}$$

$$\mu_s = \begin{pmatrix} y_s \langle \phi_s \rangle & 0 & 0 & 0 \\ 0 & y_s \langle \phi_s \rangle & 0 & 0 \\ 0 & 0 & y_s \langle \phi_s \rangle & 0 \\ 0 & 0 & 0 & y'_s \langle \phi_s \rangle \end{pmatrix} = \begin{pmatrix} \mu_1 & 0 & 0 & 0 \\ 0 & \mu_1 & 0 & 0 \\ 0 & 0 & \mu_1 & 0 \\ 0 & 0 & 0 & \mu_2 \end{pmatrix}. \tag{11}$$

The above three matrices lead to the following light neutrino mass matrix under ISS framework

$$m_\nu = \begin{pmatrix} \frac{y_1^2 b^2 \mu_2}{M_2^2} + \frac{a^2 x_1^2 \mu_1}{M_1^2} & \frac{a^2 x_1 x_2 \mu_1}{M_1^2} & \frac{a^2 x_1 x_3 \mu_1}{M_1^2} \\ \frac{a^2 x_1 x_2 \mu_1}{M_1^2} & \frac{a^2 x_2^2 \mu_1}{M_1^2} & \frac{a^2 x_2 x_3 \mu_1}{M_1^2} \\ \frac{a^2 x_1 x_3 \mu_1}{M_1^2} & \frac{a^2 x_2 x_3 \mu_1}{M_1^2} & \frac{a^2 x_3^2 \mu_1}{M_1^2} \end{pmatrix}. \tag{12}$$

The assigned  $A_4$  charge of this Higgs triplet  $\eta$  restricts the interaction of  $\eta$  with the charged leptons. Now the Lagrangian for the charged lepton mass is given by

$$\mathcal{L}_l^I = y_e L_e l_e^c h + y_\mu L_\mu l_\mu^c h + y_\tau L_\tau l_\tau^c h \tag{13}$$

Following is the mass matrix for charged leptons.

$$m_l = \begin{pmatrix} y_e \langle h \rangle & 0 & 0 \\ 0 & y_\mu \langle h \rangle & 0 \\ 0 & 0 & y_\tau \langle h \rangle \end{pmatrix} \tag{14}$$

#### 4. The reactor mixing angle

It is needless to say that there is a menagerie of theories, put forward in establishing the  $\theta_{13}$  as having a nonzero value. Here also we are trying to present such a picture by including a perturbation called type II perturbation to the above Lagrangian given by equation (8) which is realized within the type II seesaw mechanism [27–32]. The type II seesaw term is followed by this term

$$\mathcal{L}^{II} = f_\nu (L_e L_\tau + L_\mu L_\mu + L_\tau L_e) \frac{\zeta \Delta}{\Lambda} + f_\nu (L_e L_\mu + L_\mu L_e + L_\tau L_\tau) \frac{\xi \Delta}{\Lambda} \tag{15}$$

where  $\Lambda$  is the cutoff scale. With the type II perturbation the Lagrangian takes the following form

$$\begin{aligned} \mathcal{L} = & y_e L_e l_e^c h + y_\mu L_\mu l_\mu^c h + y_\tau L_\tau l_\tau^c h + y_1^\nu L_e (N\eta)_1 + y_2^\nu L_\mu (N\eta)_{1'} + y_3^\nu L_\tau (N\eta)_{1''} \\ & + y_4^\nu L_e N_4 h + y_s (SS)\phi_s + y_s' S_4 S_4 \phi_s + y_R (NS)\phi_R + y_R' N_4 S_4 \phi_R \quad (16) \\ & + f_\nu \frac{(L_e L_\tau + L_\mu L_\mu + L_\tau L_e)\zeta \Delta}{\Lambda} + f_\nu \frac{(L_e L_\mu + L_\mu L_e + L_\tau L_\tau)\xi \Delta}{\Lambda}. \end{aligned}$$

The last two terms represent the perturbation to the leading order terms in the above Lagrangian giving rise to non-zero  $\theta_{13}$ .

Here we have implemented the  $A_4$  group to explain the structure of the neutrino mass matrix (17) originating from the type II seesaw mechanism. The  $SU(2)_L$  triplet Higgs field  $\Delta_L$  is supposed to transform as a  $A_4$  singlet. Two more flavon fields  $\zeta$  and  $\xi$  have been introduced which are assumed to transform as  $A_4$  singlets as summarized in Table 1. The flavon alignments which help in constructing the  $m_{LL}^{II}$  matrix are as follows  $\langle \Delta \rangle \sim v_\Delta$ ,  $\langle \zeta \rangle \sim v_\zeta$ ,  $\langle \xi \rangle \sim v_\xi$ .  $\zeta$  and  $\xi$  are assumed to take the VEV in the same scale  $v_\zeta = v_\xi = \Lambda$ . With these flavon alignments the structure of mass matrix  $m_{LL}^{II}$  will take the form

$$m_{LL}^{II} = \begin{pmatrix} 0 & -w & w \\ -w & w & 0 \\ w & 0 & -w \end{pmatrix}. \quad (17)$$

### 5. Neutrinoless Double Beta Decay

The time period for Neutrinoless Double Beta Decay rate is directly proportional to the square of the effective neutrino mass  $m_\nu^{ee}$ . Which implies that in determining the time period for Neutrinoless Double Beta Decay, the effective mass plays a non-trivial role in the standard three generation picture. (See Fig. 1.) The effective neutrino mass can be given by

$$|m_\nu^{ee}| = |U_{ei}^2 m_i|. \quad (18)$$

The Unitary matrix is the PMNS matrix which is the neutrino mixing matrix in the basis where the charged lepton mass matrix is diagonal [33,34]. In addition to this, following non-standard contributions become transparent in the present model.

- Two separate contributions due to light and heavy neutrino exchanges to  $0\nu\beta\beta$  come into play. And this event is established by writing the flavor eigenstates as a linear combination of light and heavy mass eigenstates. The only contribution that becomes effective in the ISS regime comes from the contribution due to light neutrino exchanges.

$$v_\alpha = N_{\alpha i} v_i + U_{\alpha j} \xi_j, \quad (19)$$

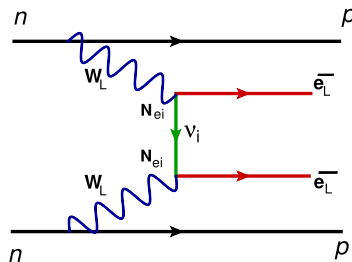


Fig. 1. Feynman diagram contributing to neutrinoless double beta decay due to light neutrino exchanges.

where,  $N_{\alpha i}$  and  $U_{\alpha j}$  are the mixing matrices for light and heavy neutrino respectively. The effective mass takes different values depending on the framework (quasi degenerate or normal/inverted hierarchies), the neutrino mass states are in. Now considering the light neutrino contribution (the only contribution for ISS in this model), the key formula for determining the effective neutrino mass is given by

$$m_{\nu,LL}^{ee} \simeq U_{e1}^2 m_1 + U_{e2}^2 e^{2i\alpha} m_2 + U_{e3}^2 e^{2i\beta} m_3. \tag{20}$$

- The triplet Higgs contribution from the type II seesaw. The contribution from the triplet Higgs is of the order of  $10^{-13} m_i$  which is much suppressed as compared to the dominant contributions [33].

Of special importance is the fact that, the chosen value of Yukawa coupling giving rise to the observed relic abundance of our DM candidate, constrains the lightest neutrino mass significantly in the presented forum. The fine tuned Yukawa couplings (0.994–1) are noticed to play an important role in achieving the lightest neutrino mass and inturn to get the effective neutrino mass prediction within the GERDA bound (0.5 eV). The type II perturbation strength is found to play some role in giving  $m_{lightest}$  within the PLANK bound (0.065 eV for IH). The introduced model also evinces the role of leptonic mixing matrix elements and the lightest neutrino mass as the effective neutrino mass is dependent upon them.

### 6. Relic density of Dark Matter

The relic abundance of a DM particle  $\chi$  is given by the Boltzmann equation [35–38]

$$\frac{dn_\chi}{dt} + 3Hn_\chi = - \langle \sigma v \rangle (n_\chi^2 - (n_\chi^{eqb})^2), \tag{21}$$

where  $n_\chi$  is the number density of the DM particle  $\chi$  and  $n_\chi^{eqb}$  is the number density when  $\chi$  was in thermal equilibrium.  $H$  is the Hubble rate and  $\langle \sigma v \rangle$  is the thermally averaged annihilation cross-section of the DM particle  $\chi$ . Numerical solution of the Boltzmann equation is given by [36]

$$\Omega_\chi h^2 \approx \frac{1.04 \times 10^9 x_F}{M_{pl} \sqrt{g_*} (a + 3b/x_F)}, \tag{22}$$

where  $x_F = \frac{m_\chi}{T_F}$ ,  $T_F$  is the freeze-out temperature,  $g_*$  is the number of relativistic degrees of freedom at the time of freeze-out. DM particles with electroweak scale mass and couplings freeze out at temperatures in the range  $x_F \approx 20-30$ . This in turn simplifies to [39]

$$\Omega_\chi h^2 \approx \frac{3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\langle \sigma v \rangle}. \tag{23}$$

For complex scalar DM, the annihilation rate is given by equation (24). The relic abundance is related to the cross section of the DM–DM interaction. The terms in equation (8) evince the interaction 2. While finding the allowed parameter space satisfying the correct relic abundance and neutrino oscillation parameters we vary the Relic mass and the Majorana fermion mass (the right handed neutrino) both of which are involved in the cross section formula as shown in [40] reads as

$$(\sigma v)_{\text{complex scalar}}^{\chi\chi^\dagger} = \frac{v^2 y^4 m_\chi^2}{48\pi (m_\chi^2 + m_\psi^2)^2}. \tag{24}$$



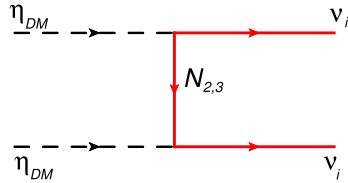


Fig. 2. Feynman diagram showing the scattering of  $\eta_2$  and  $\eta_3$ .

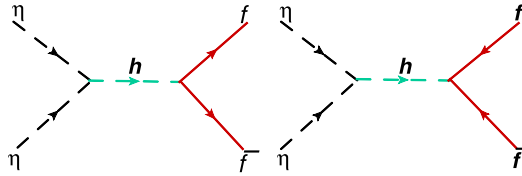


Fig. 3. Self annihilation of  $\eta_2$  and  $\eta_3$  into SM fermions (conventions are followed from [42]).

With  $v$  = relative velocity of the two relic particles and is typically  $0.3c$  at the freeze out temperature,  $\chi$  is the relic particle (DM),  $y$  is the Yukawa coupling,  $m_\chi$  is the mass of the relic,  $m_\psi$  is the mass of the right handed neutrino.

The dark matter relic abundance may get affected by some kind of annihilation processes which might have taken place between the two neutral scalars depending on their mass difference  $\Delta m = m_{\eta_2} - m_{\eta_3}$ . If the mass splitting is of the order of freeze-out temperature,  $T_f$  the coannihilation between the two neutral scalars plays a significant role in finding the relic abundance of dark matter. But if  $\Delta m$  is larger than the freeze-out temperature, then the immediate heavier neutral scalar affects the dark matter relic density notably. The self annihilation between dark matter and next to lightest neutral component of scalar triplet  $\eta$  contribute to the annihilation cross section of dark matter. Many authors in [35,37,41] explored this kind of self annihilation effects on dark matter relic abundance. To calculate the effective annihilation cross section we are following the analysis of [35]. The various annihilation channels and interactions can be given by Fig. 3.

For low mass scheme ( $m_{DM} < M_W$ ), the self annihilation of either  $\eta_2$  or  $\eta_3$  into SM particles takes place via SM Higgs boson as shown in Fig. 3. The according annihilation cross section [37,41] is followed by equation (25).

$$\sigma_{xx} = \frac{|Y_f|^2 |\lambda_x|^2}{16\pi s} \frac{(s - 4m_f^2)^{3/2}}{\sqrt{s - 4m_x^2} ((s - m_h^2)^2 + m_h^2 \Gamma_h^2)}, \tag{25}$$

where  $x \rightarrow \eta_{2,3}$ ,  $\lambda_x$  is the coupling of  $x$  with SM Higgs boson  $h$  and  $Y_f$  is the Yukawa coupling of fermions, which has been estimated to be 0.32 albeit the full possible range of values is  $\lambda_f = 0.26-0.63$  [6].  $\Gamma_h = 4.15$  MeV is the SM Higgs decay width,  $m_h$  is 126 GeV.  $s$  is the thermally averaged center of mass squared energy given by

$$s = 4m^2 + m^2 v^2, \tag{26}$$

where  $v$  is the relative velocity and  $m$  is the mass of the relic. In order to yield the correct relic abundance we need to constrain the Yukawa coupling along with the relic mass and the mediator mass. Similar to the works done in [43,44] here also we consider the neutral component of the scalar triplet as the DM candidate. We choose the relic mass as lighter than the W boson

Table 2  
Neutrino oscillation data for normal mass ordering.

Oscillation parameters	bfp	$3\sigma$ CI
$\Delta m_{21}^2$ [ $10^{-5}$ eV <sup>2</sup> ]	7.5	(7.02, 8.07)
$\Delta m_{31}^2$ [ $10^{-3}$ eV <sup>2</sup> ]	2.457	(2.317, 2.607)
$\sin^2 \theta_{12}$	0.304	(0.270, 0.344)
$\sin^2 \theta_{13}$	0.0218	(0.0186, 0.0250)
$\sin^2 \theta_{23}$	–	0.381–0.643

Table 3  
Neutrino oscillation data for inverted mass ordering.

Oscillation parameters	bfp	$3\sigma$ CI
$\Delta m_{21}^2$ [ $10^{-5}$ eV <sup>2</sup> ]	7.5	(7.02, 8.07)
$\Delta m_{23}^2$ [ $10^{-3}$ eV <sup>2</sup> ]	–2.449	–2.590, –2.307
$\sin^2 \theta_{12}$	0.304	0.270, 0.34
$\sin^2 \theta_{13}$	0.0219	0.0188, 0.0251
$\sin^2 \theta_{23}$	–	0.388, 0.644

mass  $m_{DM} \leq M_W$ . And interestingly for the relic we stick to a comparatively low mass region, which is around 50 GeV. The mediator mass here in our case, i.e., the Majorana neutrino mass is required to vary from 153 GeV to 154 GeV to obtain the observed relic density. This type of findings have been extensively studied in the literature [40,45]. For a light DM with a mass below 10 GeV, the LHC searches have a better awareness for complex scalar DM cases. Moreover, the LHC has a better reach than direct detection experiments with DM masses up to around 500 GeV for the complex scalar DM case.

## 7. Numerical analysis

The latest global fit [46] value with their best fit point (bfp) for  $3\sigma$  range of neutrino oscillation parameters used to study neutrino phenomenology are given in Table 2 and Table 3.

Cosmological constraint says that,

$$m_1 + m_2 + m_3 \leq 0.23 \text{ eV.}$$

The Yukawa coupling governing the interaction is present in the established mathematical expression which computes the scattering cross section of this interaction in turn the relic abundance of the potential DM. As a proper choice of Yukawa coupling, the mediator mass along with the complex scalar mass allows us to achieve the observed relic abundance we need to put constraints on them. In our work we first fix the above mentioned parameters to get the relic abundance which is reported by PLANCK 2013 data. Fixing the relic mass around 50 GeV and varying the mediator mass from 153 to 154 GeV we get the idea of Yukawa coupling yielding the correct relic abundance. Since the required relic abundance for the potential DM candidate desires a mediator mass at a much lower scale (around 153 GeV) the ISS realization helps us to keep the RH neutrino (which is here, the mediator particle governing the t-channel scattering as shown in 2) mass at a scale much below than that one involved in the canonical seesaw. The

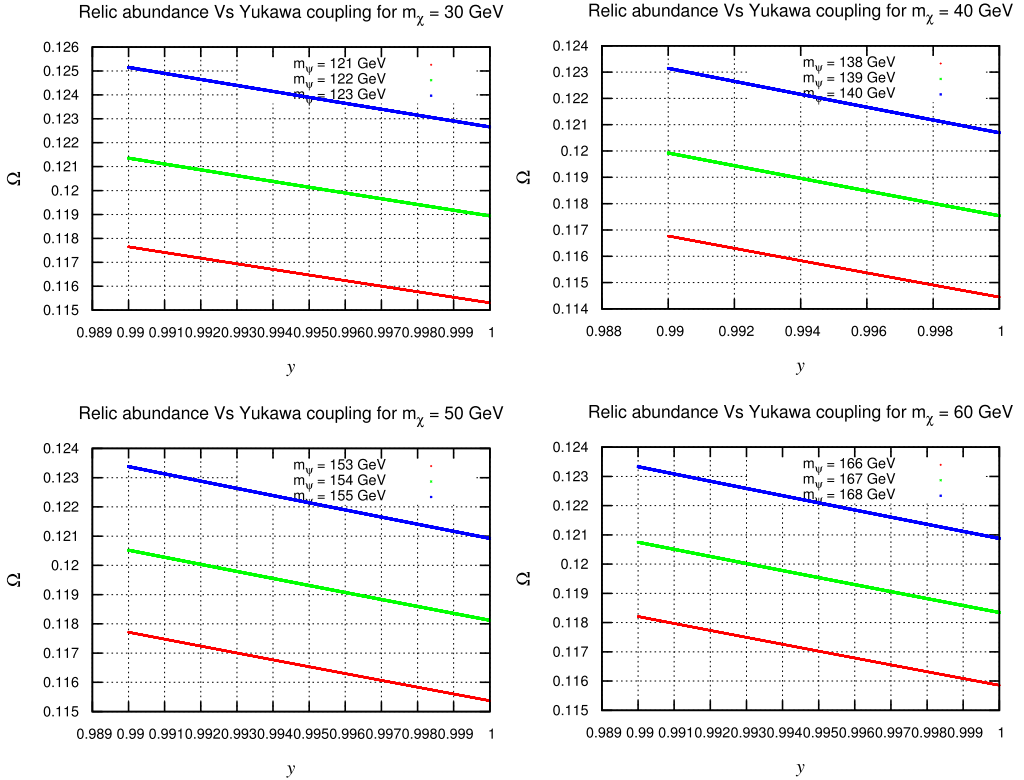


Fig. 4. Variation of relic abundance with Yukawa coupling.

Yukawa coupling needs to fall between 0.99 and 1 to have a better reach of the relic abundance as shown in Fig. 4.

We redefine the parameters of the matrix shown by the equation (12) in terms of  $p$ ,  $q$  and  $r$ . Where,  $p = \frac{ax_1\sqrt{\mu_1}}{M_1}$ ,  $q = \frac{ax_2\sqrt{\mu_1}}{M_1}$  and  $r = \frac{ax_3\sqrt{\mu_1}}{M_1}$ . From the requirement of bringing the light neutrino mass matrix into TBM form we equate the 11-element of  $m_\nu$  to  $2q^2 - pq$  [9]. This is done in accordance with adjusting the Yukawa couplings and the associated VEVs. Along with this redefinition we also make  $q = r$  by  $x_2 = x_3$  for numerical analysis. This form of light neutrino mass matrix has an inverse hierarchical neutrino mass spectrum and a zero eigenvalue with  $m_3 = 0$ . For numerical analysis we take another couple of definitions for the Yukawa couplings  $x_1 = x$  and  $x_2 = x_3 = y$ . We have kept  $x = 1$  and varied  $y$  for computing the oscillation parameters and  $m_\nu^{ee}$ , however there are no significant changes observed by keeping  $y$  fixed and varying  $x$ . Each value of  $y$  gives rise to various sets of the neutrino mass matrix parameters  $p$ ,  $q$ . We parameterize the light neutrino mass matrix obtained from the ISS realization with the help of recent neutrino oscillation data given in Table 2 and Table 3. Along with the redefined parameters of the light neutrino mass matrix and using equations (9), (10), (11) the new light neutrino mass matrix is found to be of TBM type given by equation (27)

$$m_\nu = \begin{pmatrix} 2q^2 - pq & pq & pq \\ pq & q^2 & q^2 \\ pq & q^2 & q^2 \end{pmatrix}. \tag{27}$$

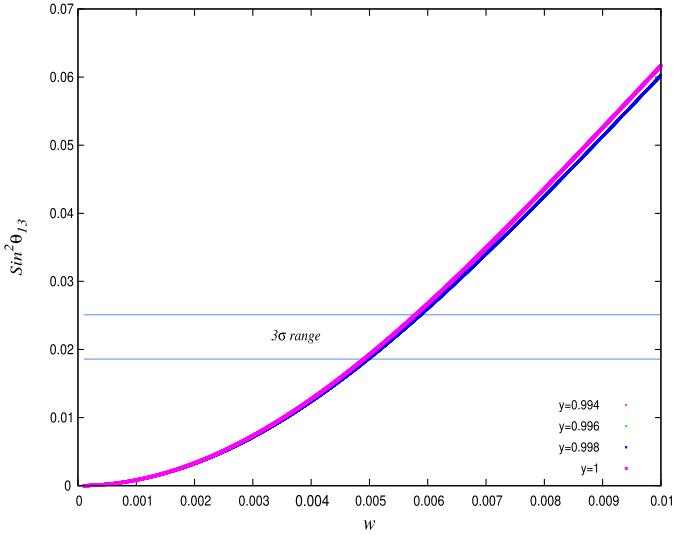


Fig. 5. Generation of non-zero  $\sin^2\theta_{13}$  varying the type II strength for best fit values.

We have analyzed the model only for IH case as the light neutrino mass matrix structure only allows us to have the inverted hierarchy mass pattern. After diagonalizing the complete mass matrix the mass eigenvalues are found to be  $m_1 = -2(pq - q^2)$ ,  $m_2 = q(p + 2q)$  and  $m_3 = 0$ . Then we parameterize the mass matrix keeping  $x = 1$  while at the same time varying  $y$  between a range around 0.994–1. Choosing each set of  $p, q$  values which have been found different for different “ $y$ ” values, we get several light neutrino mass matrices. The same Yukawa coupling  $y$  is being varied in the dark matter sector too for showing its contribution to obtain the correct relic abundance. With the discovery of non-zero reactor mixing angle, it is a customary to reflect the concept theoretically. In our work we try to provide a platform which reproduces the same. For that purpose, we include type II perturbation [27] to the leading order neutrino mass matrix as explained in Section 4. This perturbation brings out non-zero  $\theta_{13}$  in  $3\sigma$  range along with  $m_3 \neq 0$  leaving the light neutrino masses with IH nature only. The numerical value of the perturbation term  $w = f_\nu v_\Delta$  critically depends upon the Majorana coupling  $f_\nu$ , trilinear mass parameter  $\mu\phi\Delta$ , and  $M$ . Accordingly, we vary the type II seesaw strength from  $10^{-6}$  to 0.01 to produce non-zero  $\theta_{13}$ . It is observed from Fig. 5 that, the type II seesaw strength of  $10^{-3}$  eV is generating the non-zero  $\theta_{13}$  in the  $3\sigma$  range in all cases.

The perturbation matrix takes the following structure

$$m_\nu^{II} = \begin{pmatrix} 0 & -w & w \\ -w & w & 0 \\ w & 0 & -w \end{pmatrix}.$$

After adding the perturbation we get the neutrino mass matrix as follows

$$m_\nu = m_\nu^I + m_\nu^{II}.$$

Now the elements of these diagonalized matrices are associated with the parameters of the model and the type II perturbation term. The set of  $p, q$  values obtained for each  $y$  value and chosen for analysis are listed in Table 4, Table 5, Table 6. In addition  $p, q$  corresponds to some

Table 4

Values of  $p, q$  obtained by solving for IH case with best fit central value of  $3\sigma$  deviations.

Parameters	$y = 0.994$	$y = 0.996$	$y = 0.998$	$y = 1$
$p$	0.366138	0.366146	0.366154	0.357719
$q$	0.0899502	0.089768	0.0895865	0.091516

Table 5

Values of  $p, q$  obtained by solving for IH case with an upper bound of  $3\sigma$  deviations.

Parameters	$y = 0.994$	$y = 0.996$	$y = 0.998$	$y = 1$
$p$	0.371351	0.371359	0.371367	0.362663
$q$	0.0911924	0.0910077	0.0908236	0.0928181

Table 6

Values of  $p, q$  obtained by solving for IH case with a lower bound of  $3\sigma$  deviations.

Parameters	$y = 0.994$	$y = 0.996$	$y = 0.998$	$y = 1$
$p$	0.360693	0.3607	0.360708	0.352452
$q$	0.088626	0.0884465	0.0882677	0.0901551

Table 7

Comparison of relic abundance  $\Omega$  with various choices of Yukawa couplings, DM mass, RH neutrino mass.

$m_\chi$	$m_\psi$	$y$	$\Omega$
30 GeV	(121–122) GeV	(0.99–1)	✓
40 GeV	(139) GeV	(0.99–1)	✓
50 GeV	(153–154) GeV	(0.99–1)	✓
60 GeV	(166–167) GeV	(0.99–1)	✓

Table 8

Summary of results obtained from various allowed mass schemes.

$3\sigma$ ranges	$\theta_{13}$	$\theta_{12}$	$\theta_{23}$	$\Delta m_{21}^2$	$\Delta m_{23}^2$	$\Sigma \text{ mod } m_i$
bfp	✓	✓	✓	✓	✓	✓
Lower bound	✓	✓	✓	✓	✓	✓
Upper bound	✓	✓	✓	✓	×	✓

complex sets of solution too. Taking them under consideration, no significant changes in the numerical analysis have been observed.

A comparison among the various sets of results obtained in the DM phenomenology part has been made in Table 7 and neutrino phenomenology has been shown in Table 8.

The light neutrino mass matrix (27) is having only two unknown parameters, solution for which demand two equations. Two mass squared differences which we get from neutrino oscillation data, lead to those two parameters. Then using the solutions for  $p$  and  $q$  the light neutrino mass matrix is obtained. Then we fix the mass eigenvalues from that light neutrino mass matrix.

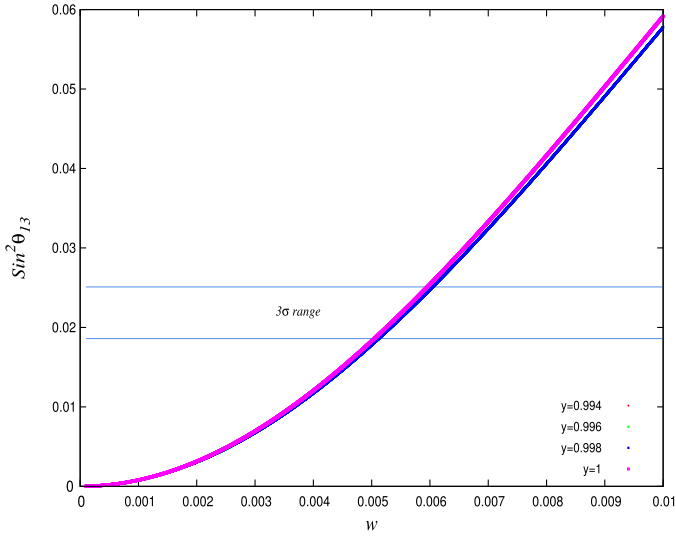


Fig. 6. Generation of non-zero  $\sin^2\theta_{13}$ , varying the type II strength using upper bound of  $3\sigma$  deviations.

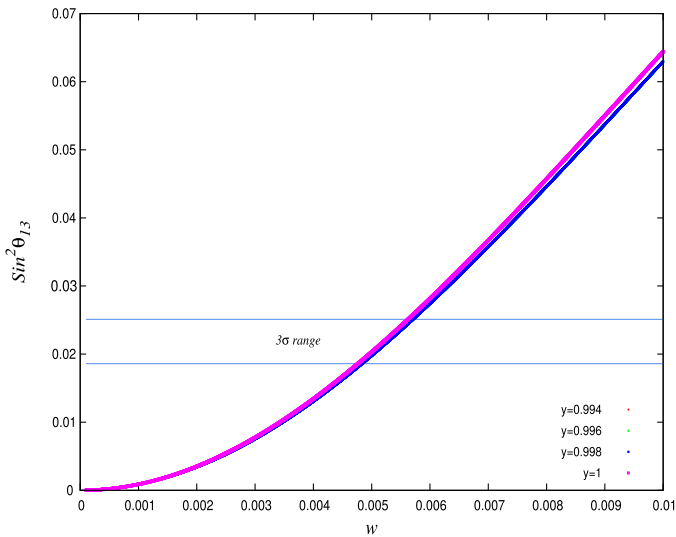


Fig. 7. Generation of non-zero  $\sin^2\theta_{13}$ , varying the type II strength using lower bound of  $3\sigma$  deviations.

Using the best fit central values from the oscillation data, we numerically fit the leading order neutrino mass matrix. A thorough analysis has been carried out to check whether the oscillation parameters are near to reach or not by taking the upper and lower bound of  $3\sigma$  deviation as well. Here we try to exhibit an unexplored parameter space satisfying both the DM relic abundance and neutrino phenomenology.

The scattering cross section of the decay channel described by Fig. 3 to various SM fermions have been calculated. They are found to have an order of  $10^{-60} \text{ cm}^2/10^{-42} \text{ GeV}^{-2}$  which is much smaller than the cross section which has been achieved for the t-channel contribution (of

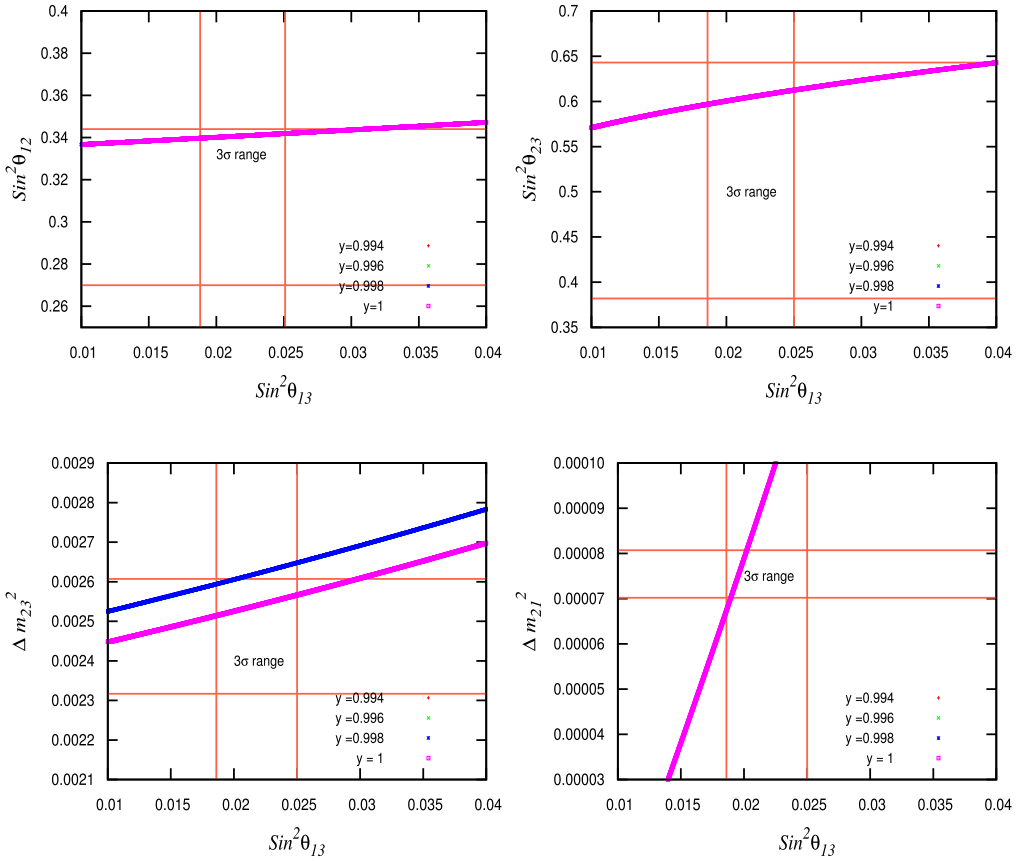


Fig. 8. Variation of  $\sin^2 \theta_{12}$ ,  $\sin^2 \theta_{23}$ ,  $\Delta m_{23}^2$  and  $\Delta m_{21}^2$  with  $\sin^2 \theta_{13}$  with best fit value.

the order of  $10^{-44}$  cm<sup>2</sup>). They will have little contribution (can be neglected therefore) to the relic abundance of the potential DM candidate. We have already noticed that for obtaining the observed  $\Omega$  we need to fix the Yukawa coupling. Fixing the Yukawa coupling as varying from 0.99 to 1, varying  $m_{DM}$  from 30 to 60 GeV and varying  $M_R$  from 120 to 167 GeV, we study the order of relic abundance. We fit the values of oscillation parameters using recent cosmological constraints for inverted mass ordering. We compute all the oscillation parameters also by varying the type II seesaw strength. Variation of type II seesaw strength with the non-vanishing  $\theta_{13}$ , has been shown in Fig. 5, Fig. 6, Fig. 7. The production of other oscillation parameters, e.g. the two mixing angles and two mass squared splitting as a function of nonzero  $\theta_{13}$  has been shown in Fig. 8, Fig. 9 and Fig. 10 for different values of Yukawa coupling. The sum of absolute masses has also been calculated to see whether it satisfies the Planck upper bound or not. Seeing that, the sum of absolute neutrino masses can give some clue on neutrinoless double beta decay, a little study has been performed to check whether the presented model is able to contribute to the  $0\nu\beta\beta$  physics. In Fig. 11 we plot for the contribution of the effective mass to  $0\nu\beta\beta$  decay due to light neutrino exchanges for standard contribution showing the variation of effective mass with the type II seesaw strength. Fig. 12 displays the variation of  $m_\nu^{ee}$  with the lightest neutrino mass, in our model  $m_3$ . In Fig. 13 we present the variation of effective mass with  $m_{lightest}$  and type II

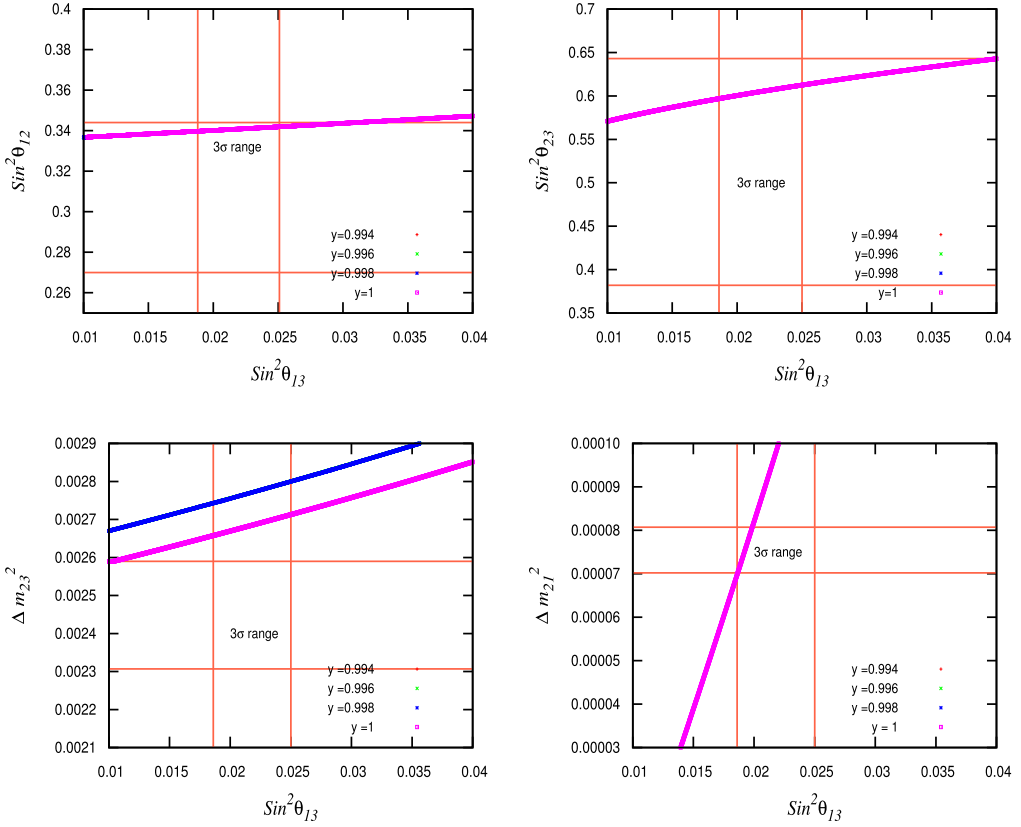


Fig. 9. Variation of  $\sin^2 \theta_{12}$ ,  $\sin^2 \theta_{23}$ ,  $\Delta m_{23}^2$  and  $\Delta m_{21}^2$  with  $\sin^2 \theta_{13}$  with upper bound of  $3\sigma$  deviation.

seesaw strength taking the upper and lower bound of  $3\sigma$  deviation. Since the presented model only presents a hierarchy of inverted kind the lowest mass range has been selected which is resulted from the perturbation. The variation in  $m_{\nu}^{ee}$  for non-standard contribution with different  $y$  values have been checked and found to be in agreement with the experimental bounds. The effective mass for non-standard contribution has been obtained around 0.0489 almost for all the values of Yukawa couplings chosen for the analysis. It is worth noting that the variation in Yukawa coupling leaves trivial impacts on  $m_{\nu}^{ee}$  for non-standard contribution. For showing the variation of  $m_{\nu}^{ee}$  with  $m_3$ , we choose those values of  $m_3$  obtained as a result of adding the type II seesaw strength.

The following observations have been made from the results and analysis.

- The relic abundance has been found to match the value shown by PLANCK 2013 data, for a choice of Yukawa coupling ranging from 0.99 to 1 provided the Relic mass is fixed at 50 GeV keeping the mediator mass at a range from 153 to 154 GeV. A detailed analysis of the choice of Yukawa coupling, the Relic mass ( $m_{\chi}$ ) and the mediator mass ( $m_{\psi}$ ) for this particular model has been presented in Table 7.
- The oscillation parameters are near to reach only when the Yukawa coupling is varied from 0.994 to 1 and as a further increase/decrease of the Yukawa coupling does not yield good



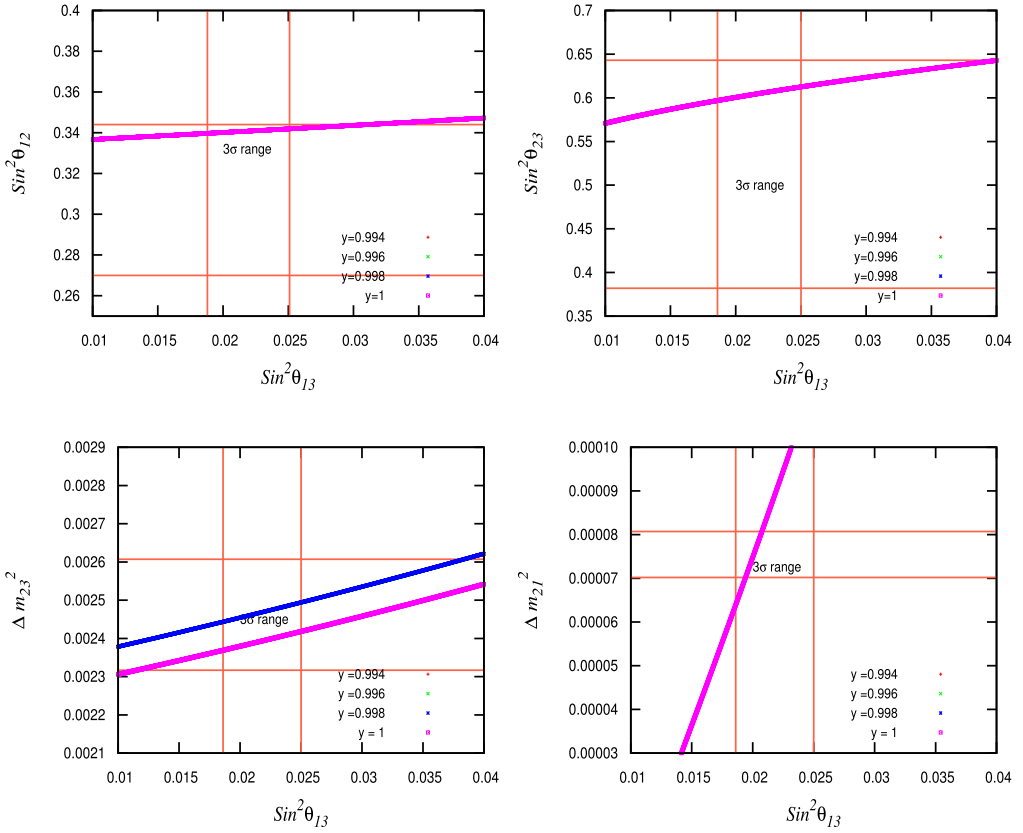


Fig. 10. Variation of  $\sin^2 \theta_{12}$ ,  $\sin^2 \theta_{23}$ ,  $\Delta m_{23}^2$  and  $\Delta m_{21}^2$  with  $\sin^2 \theta_{13}$  with lower bound of  $3\sigma$  deviation.

neutrino phenomenology we have considered those corresponding values of relic abundance obtained for Yukawa coupling ranging from 0.994 to 1.

- It has been noticed that the proposed model evidences correct neutrino phenomenology using the best fit and lower  $3\sigma$  bound in case of inverted hierarchy mass pattern only. All the oscillation parameters have been seen to come inside the frame while taking the best fit and lower  $3\sigma$  bound.
- The non-zero value of  $\theta_{13}$  has been found to be consistent with the variation of type II seesaw strength.
- Both the standard and new physics contribution to  $0\nu\beta\beta$  decay in the allowed hierarchy is obtained in the vicinity of experimental results [GERDA].

### 8. Conclusion

An  $A_4$  based IH neutrino mass model originating from both Inverse and type II seesaw have been studied. Here ISS is implemented as a leading order contribution to the light neutrino mass matrix yielding zero reactor mixing and  $m_3 = 0$ . Then the type II seesaw has been used in order to produce non-zero reactor mixing angle, which later on produces  $m_3 \neq 0$  keeping the hierarchy as inverted only. We have studied the possibility of having a common parameter space

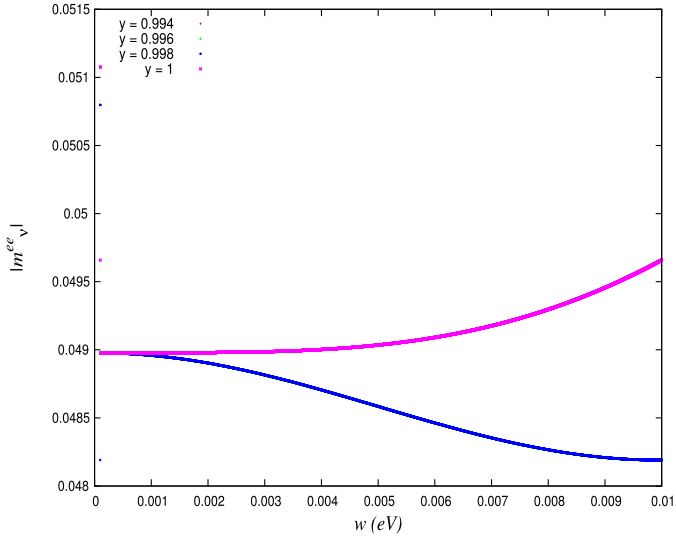


Fig. 11. Variation of effective mass  $m_{\nu}^{ee}$  with type II seesaw strength using bfp.

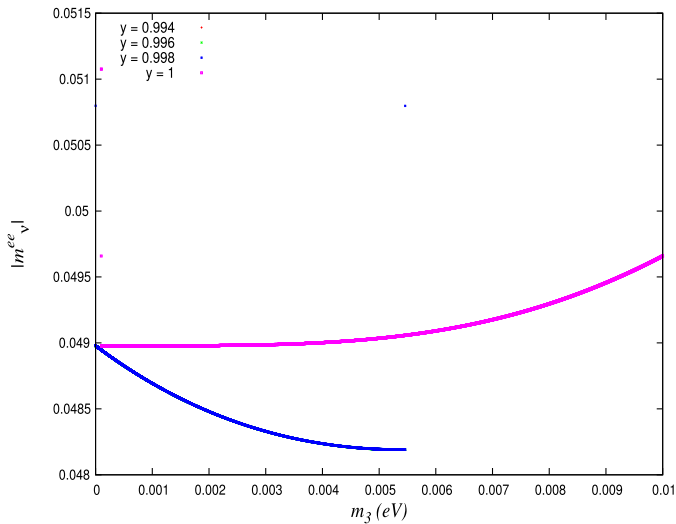


Fig. 12. Variation of effective mass  $m_{\nu}^{ee}$  with the lightest neutrino mass using bfp.

where both the Neutrino oscillation parameters in the  $3\sigma$  range and DM relic abundance have a better reach. With a proper choice of Yukawa coupling ( $y$ ), right handed neutrino (mediator particle) mass ( $m_{\psi}$ ), and complex scalar (potential DM candidate) mass ( $m_{\chi}$ ) the variation in relic abundance as a function of Yukawa coupling has been shown. For a choice of Yukawa coupling between 0.994 and 0.9964,  $m_{DM}$  around 50 GeV, the mediator mass needs to fall around 153 GeV to match the correct relic abundance. The same Yukawa coupling has got a key role in generating the Neutrino oscillation parameters as well. We have studied the prospect of producing non-zero  $\theta_{13}$  by introducing a perturbation to the light neutrino mass matrix using type II seesaw

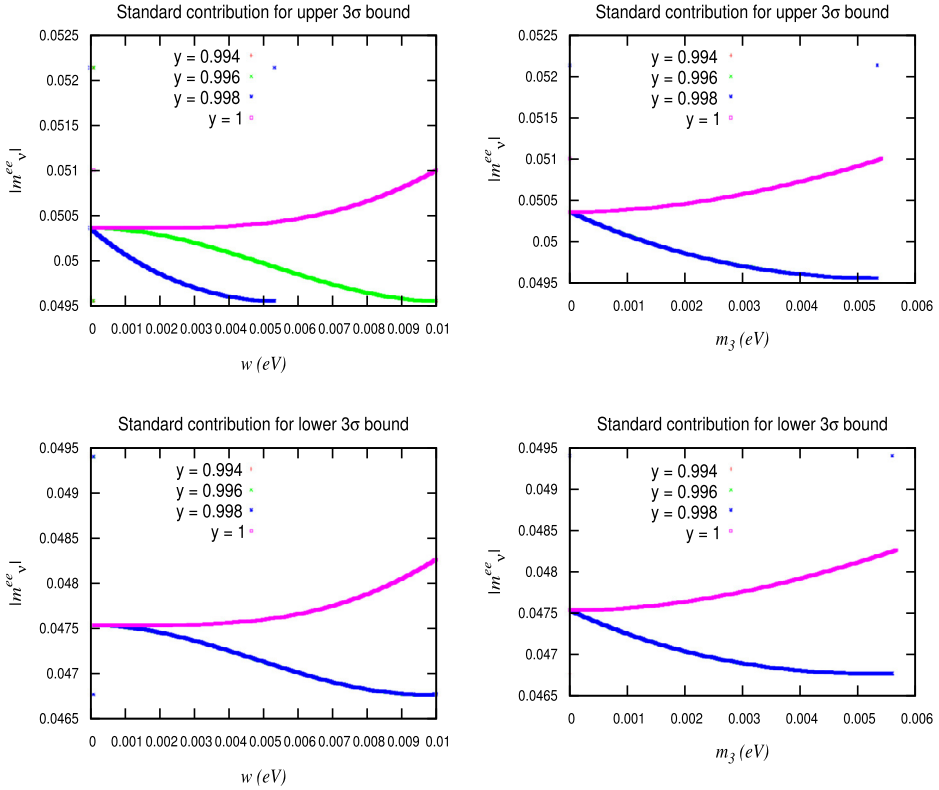


Fig. 13. Variation of effective mass  $m_{\nu}^{ee}$  with type II seesaw strength and the  $m_3$  for upper and lower  $3\sigma$  bounds.

within the  $A_4$  model. We have also determined the strength of the type II seesaw term which is responsible for the generation of non-zero  $\theta_{13}$  in the correct  $3\sigma$  range. We have also checked whether the proposed model can project about neutrinoless double beta decay or not. In context to the presented model we have found a wide range of parameter space where one may have a better reach for both neutrino and dark matter sector as well. This model may have relevance in studying baryon asymmetry of the universe, which we leave for future study.

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