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Upper limits on electric dipole moments of τ-lepton and W-boson

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Abstract

We discuss new upper limits on the electric dipole moments (EDM) of the τ-lepton and W-boson, which follow from the precision measurements of the electron and neutron EDM.

1. Introduction

Strict upper limits on the electric dipole moments of common elementary particles, electron and proton, were derived from spectroscopic, almost table-top experiments [1–3]. As to the neutron EDM, the best upper limit on it was obtained as a result of reactor experiments lasting many years [4] (they say that the searches for the neutron EDM killed more theories than any other experiment in the history of physics). And at last, the result for the muon EDM follows from the measurements at the dedicated muon storage ring [5]. These results are summarized in Table 1.

As to the electric dipole moment of the τ-lepton, upper limits on it have been obtained up to now from the analysis of high-energy experiments.

The approach pursued here is based on the precision results [1,4]. We establish upper limit on the EDM of the τ-lepton through the analysis of its possible contribution to the electron EDM. This is a clean theoretical problem. For the W-boson this result is of rather qualitative nature. Additional upper limit on the EDM of the W-boson, also qualitative one, is derived by the analysis of its possible contribution to the neutron EDM.

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Table 1

	e	p	n	μ
d/e, cm	$(0.7 \pm 0.7) \times 10^{-27}$ [1]	$< 0.8 \times 10^{-24}$ [2,3]	$< 0.29 \times 10^{-25}$ [4]	$(0.37 \pm 0.34) \times 10^{-18}$ [5]

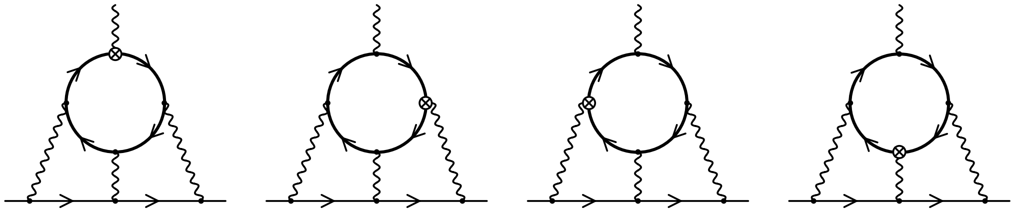

Fig. 1.

2. Dipole moment of τ-lepton and electron EDM

We start with the analysis of the contribution of the τ EDM d_τ to the electron dipole moment d_e. This contribution is described by the diagrams of the type presented in Figs. 1a, b, c, d. Here the loop is formed by the τ line, and the lower solid line is the electron one. The upper wavy line corresponds to the external electric field. The crossed vertices refer to the electromagnetic interaction of the τ EDM

$$L_\tau^{\mathrm{edm}} = -\frac{1}{2} d_\tau \bar{\tau} \gamma_5 \sigma_{\mu\nu} \tau F_{\mu\nu} = i \frac{1}{2} d_\tau \bar{\tau} \sigma_{\mu\nu} \tau \tilde{F}_{\mu\nu}, \qquad \tilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}. \tag{1}$$

Of course, all six permutations of the electromagnetic vertices on the electron line should be considered. The contributions of diagrams 1b and 1c are equal.

This problem is similar to that of the contribution from the light-by-light scattering via muon loop to the electron magnetic moment [6]. The general structure of the resulting contribution to the electron EDM is rather obvious (to the leading order in m_e/m_τ):

$$\Delta d_e = a \frac{m_e}{m_\tau} \left(\frac{\alpha}{\pi} \right)^3 d_\tau, \tag{2}$$

where a is some numerical factor (hopefully, on the order of unity). The factor m_e originates from the necessary helicity-flip on the electron line; then $1/m_\tau$ is dictated by dimensional arguments.

Diagram 1a corresponds to the matrix element $\langle e | \bar{\tau} \sigma^{\mu\nu} \tau | e \rangle = C \bar{u} \sigma^{\mu\nu} u$. We use dimensional regularization with $d = 4 - 2\varepsilon$ dimensions and the method of regions (see the textbook [7]). Only the region where all three loops are hard (loop momenta $\sim m_\tau$) contributes to the leading power term (2); therefore, there are no logarithms $\ln(m_\tau/m_e)$ (contributions of regions with 1 or 2 hard loops are suppressed by an extra factor $(m_e/m_\tau)^2$). In this hard region, the problem reduces to 3-loop vacuum integrals with a single mass m_τ belonging to the simpler topology B_M [8]. We perform the calculation in arbitrary covariant gauge, and use the REDUCE package RECURSOR [8] to reduce scalar integrals to two master integrals. Gauge-dependent terms cancel, and we get

$$C = \frac{m_e}{m_\tau} \frac{e^6 m_\tau^{-6\varepsilon}}{(4\pi)^{3d/2}} \Gamma^3(\varepsilon) \frac{8}{d(d-1)(d-5)} \times \left[-2 \frac{2d^2 - 21d + 61}{d-5} + \frac{d^4 - 9d^3 + 8d^2 + 84d - 126}{2d-9} R \right], \tag{3}$$

where

$$R = \frac{\Gamma(1-\varepsilon)\Gamma^2(1+2\varepsilon)\Gamma(1+3\varepsilon)}{\Gamma^2(1+\varepsilon)\Gamma(1+4\varepsilon)} = 1 + 8\zeta(3)\varepsilon^3 + \cdots, \tag{4}$$

and ζ is the Riemann ζ-function. All divergences cancel, and we arrive at the finite contribution to a:

$$a_1 = \frac{3}{2}\zeta(3) - \frac{19}{12}. \tag{5}$$

In order to calculate the contribution of Figs. 1b, c, d, we expand the corresponding initial expressions in the external photon momentum q up to the linear term. The EDM vertex contains $\varepsilon^{\mu\nu\alpha\beta}$; we put this factor aside, and calculate tensor diagrams with four indices. After summing all diagrams, the result is finite; now we can set $\varepsilon \to 0$, and multiply by $\varepsilon^{\mu\nu\alpha\beta}$ (cf. [9]). The result has the structure of a tree diagram with the electron EDM vertex $\varepsilon^{\mu\nu\alpha\beta} q_\nu \sigma_{\alpha\beta}$. The gauge-dependent terms in it cancel (exactly in d), as well as the divergences. This contribution to a is

$$a_2 = \frac{9}{4}\zeta(3) - 1. \tag{6}$$

As an additional check of our programs, we have reproduced the leading power term in the contribution to the electron magnetic moment originating from the light-by-light scattering via the muon loop (formula (4) in [6]).

The final result for the numerical coefficient is

$$a = a_1 + a_2 = \frac{15}{4}\zeta(3) - \frac{31}{12} = 1.924. \tag{7}$$

With this value of a, the discussed contribution to the electron EDM is

$$\Delta d_e = 6.9 \times 10^{-12} d_\tau. \tag{8}$$

Combining this result with the experimental one [1] (see Table 1) for the electron EDM, we arrive at

$$d_\tau/e = (1 \pm 1) \times 10^{-16}\ \text{cm}. \tag{9}$$

In fact, the results (5) and (6) refer to somewhat different regions of incoming momenta. For (6) all the three momenta are hard, on the order of magnitude about m_τ, but for (5) only two of them belong to this region, and the third one, that of the outer photon, is soft, of vanishing momentum. Still, one may expect that the effective EDM interaction is formed at momenta much higher than m_τ, so that this difference is not of much importance. Besides, the contribution of diagram 1a is anyway numerically small. Thus, result (5) is valid at least for all momenta about $m_\tau \sim 1$–2 GeV.

The upper limits on the τ EDM derived from the accelerator experiments [10–13] belong to the interval of 10^{-16}–10^{-17} e cm, so that our result (9) formally does not improve them. However, all those accelerator data refer to much larger typical momenta of the photon, from 10 to 200 GeV.

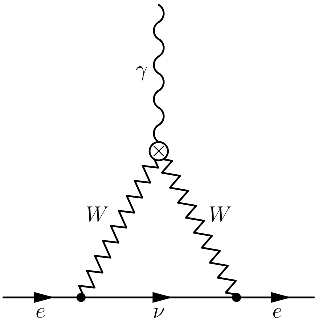

Fig. 2.

3. Electric dipole moment of W-boson

One more contribution to the electron and neutron dipole moments can be given by the EDM d_W of W-boson. This effect was pointed out and investigated long ago [14,15].

It is convenient to start the discussion with the d_W contribution to the electron EDM. Though here we cannot add anything new to the previous works, the problem is considered here as an introduction to the next, more serious problem of the d_W contribution to the neutron EDM. The effect is described by diagram presented in Fig. 2.

Here as well the crossed vertex refers to the electromagnetic interaction of the W-boson EDM (it is obvious from the diagram that here it is the dipole moment of W^-). In this case the EDM interaction is described by the Lagrangian

$$L_W^{\text{edm}} = 2m_W i d_W \tilde{F}_{\alpha\beta} W_\alpha^\dagger W_\beta. \tag{10}$$

The corresponding matrix element is

$$M = \frac{\pi\alpha}{\sin^2\theta_w} d_W m_W \int \frac{d^4k}{(2\pi)^4} \bar{u}_e(p)\gamma_\mu(1+\gamma_5)\frac{\hat{k}}{k^2}\gamma_\nu(1+\gamma_5)u_e(p)\frac{1}{[(k-p)^2 - m_W^2]^2} \times \left\{\tilde{F}_{\mu\nu} - \frac{1}{m_W^2}\big[(k-p)_\mu(k-p)_\alpha\tilde{F}_{\alpha\nu} + \tilde{F}_{\mu\alpha}(k-p)_\alpha(k-p)_\nu\big]\right\}. \tag{11}$$

With straightforward, though rather tedious calculations (somewhat simplified by employing the density matrix of polarized fermion), one arrives at the following result for the contribution of W-boson EDM to electron dipole moment:

$$\Delta d_e = \frac{\alpha}{8\pi\sin^2\theta_w}\frac{m_e}{m_W}\ln\frac{\Lambda^2}{m_W^2} d_W. \tag{12}$$

Here Λ is the cut-off parameter for the logarithmically divergent integral over virtual momenta in the loop. Putting (perhaps, quite conservatively) $\ln(\Lambda^2/m_W^2) \simeq 1$, one obtains with the experimental upper limit on the electron EDM [1] (see Table 1), the following bound on the dipole moment of W-boson:

$$d_W/e \lesssim 2\times 10^{-19}\ \text{cm}. \tag{13}$$

In the case of the W-boson contribution to the neutron EDM, our line of reasoning somewhat differs from that of [14,15]. We note first of all that the electron mass m_e does not enter explicitly matrix element (11). It arises in the result (12) only as the mass of an external fermion, via the Dirac equation $\hat{p}u = m_e u$. Therefore, there are all the reasons to expect that the contribution of d_W to the neutron EDM will be proportional to the neutron mass m_n, i.e. enhanced as compared to (12) by three orders of magnitude. In this case, the forward scattering amplitude of the virtual W-boson can be written in a general form as follows[1]:

$$\bar{u}_n(p)\gamma_\mu(1+\gamma_5)\big[\hat{k}g(k^2) + \hat{p}h(k^2)\big]\gamma_\nu(1+\gamma_5)u_n(p); \tag{14}$$

here k is the total momentum of intermediate hadronic states. Of course, the invariant functions g and h depend in fact not only on k^2, but on (kp) as well. However, in our case $k^2 \sim m_W^2 \gg (kp) \sim m_n m_W$, so that the dependence on (kp) can be safely ignored. By the analogous reason, in the usual case of the deep inelastic neutrino scattering, the structure with $\hat{p}$ in the corresponding amplitude is also omitted. In the present case, however, we should keep in amplitude (14) $\hat{p}$, in addition to the common $\hat{k}$, since after integrating over d^4k both structures give comparable contributions to the result.

At last, the usual dimensional and scaling arguments dictate that asymptotically, for $k^2 \sim m_W^2$, both functions g and h behave as follows:

$$g\big(k^2\big) = \frac{g_0}{k^2}, \qquad h\big(k^2\big) = \frac{h_0}{k^2}.$$

In particular, one can neglect the gluon corrections in these functions. Without any additional parameters, it is natural to assume that $g_0, h_0 \sim 1$.

Now, the same calculations as those in the case of electron EDM, result in the following expression for the discussed contribution to the neutron dipole moment:

$$\Delta d_n = \frac{\alpha}{8\pi \sin^2\theta_w}\frac{m_n}{m_W}\left[g_0 \ln\frac{\Lambda^2}{m_W^2} + h_0\left(\ln\frac{\Lambda^2}{m_W^2} + 1\right)\right]d_W. \tag{15}$$

For numerical estimate we put

$$g_0 \ln\frac{\Lambda^2}{m_W^2} + h_0\left(\ln\frac{\Lambda^2}{m_W^2} + 1\right) \sim 1,$$

so that

$$\Delta d_n \sim \frac{\alpha}{8\pi \sin^2\theta_w}\frac{m_n}{m_W}d_W \approx \frac{\alpha}{2\pi}\frac{m_n}{m_W}d_W.$$

Then, with the result of [4] for the neutron EDM (see Table 1), we arrive at the following quite strict upper limit on the W-boson dipole moment:

$$d_W/e \lesssim 2 \times 10^{-21}\ \text{cm}. \tag{16}$$

Acknowledgements

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[1] Compare with the corresponding structure $\bar{u}_e(p)\gamma_\mu(1+\gamma_5)(\hat{k}/k^2)\gamma_\nu(1+\gamma_5)u_e(p)$ in formula (11) for electron.

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