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Two Degree of Freedom Controller Design by AGTM\AGMP Matching Method for Time Delay Systems

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Abstract

This paper proposes the design of a general Two Degree Of Freedom (2-DOF) controller for time delay systems using a novel method which combines model order reduction, approximate model matching concepts as well as optimization techniques. The desired spectrum is embodied in the form of a transfer function which can be constructed from a set of time domain specifications. The problem of finding the parameters of the 2-DOF controller is formulated as that of obtaining the solution of a set of non-homogeneous linear equations. These set of non-homogeneous linear equations is obtained by using Approximate Generalized Time Moments (AGTM) matching concept, where closed loop response at certain frequency points in s-plane is matched with that of the desired model response. Using genetic algorithm optimum selection of frequency points (expansion points) in s-plane are obtained, which results in an optimum solution of controller parameters. This leads to a high degree of matching of the closed loop response with that of the desired model. The proposed method not only ensures the stability of the closed loop system with a 2-DOF controller but also satisfies the required performance criteria. The developed method does not pose any restriction on either the order of the model or on the structure/order of the controller transfer function. Moreover, this method is computationally simple and easy to implement. Simulation results demonstrate the effectiveness of the proposed method.

Keywords: model matching; model order reduction; optimization techniques; time delay; 2-DOF controller

1. Introduction

The Degree Of Freedom (DOF) is the number of closed loop transfer functions that can be adjusted independently [1]. The inadequacies of the 1-DOF controller structure are due to its dependence on the loop transmission to achieve both the filter-type and feedback-type system specifications. It is impossible to achieve both of these specifications simultaneously with a 1-DOF controller. The 1-DOF system is very sensitive to parameter
variation in an important frequency range. The 1-DOF configuration also has an inherent weakness in the matter of rejection of external disturbances or corrupting signals. 2-DOF controller guarantees stability and at the same time matches the desired performance. Moreover, it accesses both reference and output signal at the same cost as that of 1-DOF controller [2-4].

However, the methods involved in tuning parameters of the 2-DOF controller are not well established. In [5] Coefficient Diagram Method (CDM) was used in designing the parameters of the 2-DOF controller for position control of a DC motor. In CDM method denominator and the numerator of the transfer function are considered independently of each other. The method proposed in this paper considers the controller transfer function as a whole. A 2-DOF controller for motor drives with a first order plant model was proposed in [6]. The parameters of the proposed controller were designed using a systematic procedure to match the prescribed motor drive specifications.

By the parametric 2-DOF controller configuration, the multivariable linear quadratic optimal system design was developed in [7], and its application to the flight control of the longitudinal motion of aircraft was also illustrated. When comparing with these methods, main advantage of the proposed method in this paper is that, it is a general method and not specific to any class of systems.

The present work is an extension of, 2-DOF controller design method described in [8]. The 2-DOF control structure was used for the control of integrating the process with time delay [9]. A design method for a combined PID and feedforward controller was proposed in [10] for a first order plus dead time model. Work in [11] describes 2-DOF controller tuning method for integral plus time delay plants. The method developed in the present work does not impose any restriction on the structure or order of the plant. 2-DOF PID controllers for SISO systems were discussed in [12] with mathematical analysis which includes equivalent transformations, giving explanations about the effect of the 2-DOF structure. Two design methods for PID controllers were proposed in [13], based on the 2-DOF direct synthesis approach. But the method does not yield a general form of analytical expressions for PID controller parameters.

This paper blends the model order reduction as well as model matching concepts suitable to result in a good 2-DOF controller design procedure. The importance of integrating the order reduction and controller design procedure has been given in [14]. Work on controller design using model matching dates several decades [15]. In the proposed work, model order reduction does not reduce the order of the plant. All the information related to the plant is retained in the developed algorithm in 2-DOF controller design. Pade approximation is commonly used for model order reduction and was originally introduced by Pade [16]. The main drawback of this method is that one cannot guarantee the stability of the resultant reduced order model in the case of reduced order modeling, or it doesn’t ensure the stability of the closed loop system with controller, in the case of controller design. A partial solution to this problem has been proposed by Pal [17] by defining a set of parameters that are more generalized than the time-moments and Markov parameters, called the Approximate Generalized Time Moments (AGTM) or Approximate Generalized Markov Parameters (AGMP) and used for controller design in [18-20]. To find out the expansion points of AGTM/AGMP method, optimization method called Genetic Algorithm (GA) is used [21].

The effectiveness of the developed method is illustrated on two different models. The rest of this brief is organized as follows. The proposed method for the design of a 2-DOF controller for time delay systems is given in Section II. Section III discusses the application of this method to different examples. Finally, Section IV contains concluding remarks.

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>AGMP</td>
<td>Approximate Generalized Markov Parameters</td>
</tr>
<tr>
<td>AGTM</td>
<td>Approximate Generalized Time Moments</td>
</tr>
<tr>
<td>C_l(s)</td>
<td>Closed Loop Transfer Function</td>
</tr>
<tr>
<td>M(s)</td>
<td>Closed Loop Model Transfer Function</td>
</tr>
<tr>
<td>J</td>
<td>Performance Index</td>
</tr>
<tr>
<td>2-DOF</td>
<td>Two Degree Of Freedom</td>
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</table>
2. Proposed Method

For any open loop plant, \( G(s) \) having time delay \( \tau_{d1} \), with a feedback transfer function \( H(s) \), the closed loop transfer function with a 2-DOF compensator becomes:

\[
C_L(s) = \frac{G(s)}{r(s)} = \frac{q(s)}{k(s)+h(s)G(s)H(s)}
\]

(1)

Where \( G(s) = G_1(s)e^{-\tau_{d1}s} \)

A 2-DOF controller

\[ r(s) \]
\[ + \]
\[ q(s) \]
\[ 1/k(s) \]
\[ 1 \]
\[ G(s) \]
\[ u(s) \]
\[ y(s) \]

Fig. 1. A control system with 2-DOF compensator.

In the approximate model matching method, a general model transfer function having the specified performance should be selected, having no restrictions on its order or time delay. The model transfer function is:

\[
M(s) = M_1(s)e^{-\tau_{d2}s}
\]

(3)

\( M(s) \) is the model transfer function with time delay \( \tau_{d2} \), which embodies the desired performance. The closed loop system with the plant and 2-DOF controller should have the similar response as that of the specified model. For approximate model matching:

\[
C_L(s) = \frac{q(s)G(s)}{k(s)+h(s)G(s)H(s)} = M(s)
\]

(4)

In general, \( q(s) \), \( h(s) \) and \( k(s) \) can be chosen as polynomials of order \( n_1 \), \( n_2 \) and \( n_3 \) respectively, with \( k(s) \) taken as a monic polynomial as given in Eq.(5).

\[
\begin{align*}
q(s) &= q_{n_1}s^{n_1} + q_{n_1-1}s^{n_1-1} + \cdots + q_1s + q_0 \\
h(s) &= h_{n_2}s^{n_2} + h_{n_2-1}s^{n_2-1} + \cdots + h_1s + h_0 \\
k(s) &= s^{n_3} + k_{n_3-1}s^{n_3-1} + \cdots + k_1s + k_0
\end{align*}
\]

(5)

\( q_{n_1}, \ldots, q_1, q_0, h_{n_2}, \ldots, h_1, h_0, k_{n_3-1}, \ldots, k_1, k_0 \) are the unknown coefficients of the three polynomials which are to be determined. The total number of unknown coefficients to be determined is,

\[
n_x = n_1 + n_2 + n_3 + 2
\]

(6)

Eq.(5) can be written in vector form as:

\[
\begin{align*}
q(s) &= U X_q \\
h(s) &= V X_h \\
k(s) &= s^{n_3} + W X_k
\end{align*}
\]

(7)

U, V, and W are row vectors with each element being a function of \( s \) as shown in Eq.(8).
X_q, X_h and X_k are column vectors, whose elements are unknown coefficients of q(s), h(s) and k(s) respectively, as shown in Eq.(9).

\[
X_q = [q_{n_1}, ..., q_1, q_0]^T \\
X_h = [h_{n_2}, ..., h_1, h_0]^T \\
X_k = [k_{n_3-1}, ..., k_1, k_0]^T
\]  

As explained in the section-II, the generalized time moments of the closed loop plant, C_L(s) are matched with those of the desired model, M(s) at the expansion points \(\delta_i\), \(i=1,2,...,n_X\).

\[
M(s)|_{s=\delta_i} = C(s)|_{s=\delta_i}
\]  

The expansion points \(\delta_i\) can be a positive or negative real number or a complex point, chosen from any of the four quadrants of the s-plane. The number of expansion points depends upon the number of unknown parameters of the controller. From Eq.(10), following expression is obtained:

\[
M(\delta_i) = \frac{q(\delta_i)g(\delta_i)}{k(\delta_i)+h(\delta_i)g(\delta_i)h(\delta_i)}
\]

After substituting from Eq.(5) in Eq.(11) with \(s=\delta_i\), \(i=1,2,...,n_X\), Eq.(11) becomes:

\[
G(\delta_i)U(\delta_i)X_q = M(\delta_i)((\delta_i)^{n_3} + W(\delta_i)X_k) + M(\delta_i)G(\delta_i)H(\delta_i)V(\delta_i)X_h
\]  

Rearranging Eq.(12) to form a more condensed matrix equation as:

\[
AX = B
\]  

where

\[
A = \begin{bmatrix}
G(\delta_1)U(\delta_1) & -M(\delta_1)G(\delta_1)H(\delta_1)V(\delta_1) & -M(\delta_1)W(\delta_1) \\
\vdots & \vdots & \vdots \\
G(\delta_{n_X})U(\delta_{n_X}) & -M(\delta_{n_X})G(\delta_{n_X})H(\delta_{n_X})V(\delta_{n_X}) & -M(\delta_{n_X})W(\delta_{n_X})
\end{bmatrix}
\]

\[
X = \begin{bmatrix}
X_q \\
X_h \\
X_k
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
(\delta_1)^{n_3}M(\delta_1) \\
\vdots \\
(\delta_{n_X})^{n_3}M(\delta_{n_X})
\end{bmatrix}
\]

In Eq.(13), \(n_X\) simultaneous linear algebraic equations are obtained which may be solved to get the elements of X_q, X_h and X_k and gives the unknown coefficients of the 2-DOF controller polynomials q(s), h(s) and k(s).

In the controller design scenario, the choice of the expansion points is governed by the stability and performance of the closed loop system. The controller has to be designed such that the closed loop system responses satisfy the desired specifications while guaranteeing closed loop stability as well. In the present work, this problem of choosing the best expansion points has been cast as a constrained optimization problem which is solved by Genetic Algorithm (GA). The optimization problem, in general, can be stated as:

Find \(\delta_i\), \(i=1,2,...,n_X\) so as to minimize:
\[ J = \int_{t_i}^{t_f} (y_m(t) - y(t))^2 \, dt \]  

(15)

subject to the constraints: \( \text{re} \{ \text{pole}(G(s)) \} \leq 0 \).

where \( y_m(t) \) and \( y(t) \) are the responses of the desired model, \( M(s) \) and the closed loop plant with a 2-DOF controller. Where, \( t_i \) and \( t_f \) are initial time and final time respectively.

3. Results

The effectiveness of the proposed method is illustrated on two examples.

3.1 Model I: Closed loop system with a PID controller

The closed loop system consists of, a plant with time delay and a PID controller in [12], is taken as the model for the proposed method. In Fig. 2, \( G(s) \) is the plant having time delay and \( C(s) \) is the PID controller.

![Fig. 2. Closed loop with a PID controller.](image)

The open loop plant \( G(s) \):

\[ G(s) = \frac{1}{s+1} e^{0.2 \cdot s} \]  

(16)

Designed PID controller in [12] with parameters as given: \( K_p = 0.6 \), \( T_I = 0.4 \) and \( T_D = 0.084 \).

With this PID controller, the closed loop system meets the desired specifications in [12]. The objective is to meet the same specifications with a closed loop system having a reduced order 2-DOF controller, using the developed method.

The optimum expansion points are found out by using genetic algorithm. In these cases, population size, elite count, crossover fraction, migration fraction and the number of generations taken are 20, 2, 0.8, 0.2 and 100 respectively. The performance index in Eq.(15) is the area between the desired response and the designed one. The performance index, \( J \), is minimized so as to make the designed response as much closer as that of the specified response.

<table>
<thead>
<tr>
<th>Model</th>
<th>2-DOF controller polynomials</th>
<th>Optimal Expansion points</th>
<th>Performance Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>( q(s) = 0.4235 s + 0.4015 )</td>
<td>(-2.2089) (-0.3199) (-1.4815) (-1.1168) (0.9439)</td>
<td>(0.00025613)</td>
</tr>
<tr>
<td></td>
<td>( h(s) = 6.224 s + 2.504 )</td>
<td>(-1.1168) (0.9439)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( k(s) = s - 0.004149 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Applying the proposed methodology, Performance Index, 2-DOF controller parameters and optimal expansion points obtained are given in Table 1. A minimized performance index obtained, \( J = 0.00025613 \) shows good matching between the desired and designed responses. The closed loop response of PID controller and the proposed 2-DOF controller with the same plant is given in Fig.3. From Fig.3, it is clear that the designed response matches that of the desired response excellently. Convergence curve given in Fig.4 shows the smooth convergence of GA.
3.2 Model II. Closed Loop with a PI controller for PT 326 Thermal Process

Here, the plant to be controlled, \( G(s) \), is PT 326 thermal process in [22].

\[
G(s) = \frac{0.58}{1.57s + 1} e^{-0.56s}
\]  

(17)

The model closed loop transfer function is \( M(s) \), taken from [23], having a different time delay:

\[
M(s) = \frac{5.531s + 3.349}{9.649s^2 + 3.319s + 1.345} e^{-1.84s}
\]

(18)

The desired specification is embodied in the model transfer function \( M(s) \). The aim is to achieve the desired specification with closed loop transfer function having plant \( G(s) \) and a reduced order 2-DOF controller, using the proposed method.

Using the developed method, a 2-DOF model matching controller has designed with \( q(s) \), \( h(s) \) and \( k(s) \) as shown in Table 2. The closed loop response of the proposed 2-DOF controller with the plant \( G(s) \), and the model response is given in Fig.5. The closed loop response of 2-DOF controller matches the model response ie, the desired specification, as closely as possible. It is clear from the convergence curve of GA, given in Fig.6, that GA converges with a value of performance index \( J = 0.6989 \).
Performance index, 2-DOF controller parameters and optimal expansion points obtained using the developed method are given in Table 2.

<table>
<thead>
<tr>
<th>Model</th>
<th>2-DOF controller polynomials</th>
<th>Optimal Expansion points</th>
<th>Performance Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>( q(s) = -3.5908s + 6.9821 )</td>
<td>0.4004, 0.3443, 0.2973, 0.3208, 0.2473</td>
<td>0.6989</td>
</tr>
<tr>
<td></td>
<td>( h(s) = -26.8661s - 12.0111 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( k(s) = s + 8.5801 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Conclusion

The present work proposes an algorithm for the design of a general 2-DOF controller for time delay systems, which makes the response of the closed loop system as close as possible to that of the given model. The desired model embodied in a transfer function can be obtained from the given time domain specifications. The developed method can be easily used for the design of any order controller. There is no restriction in the selection of the order of the controller or model. The proposed method is simple and takes less computational time.
The effectiveness of the proposed method was illustrated by two different models. The work can also be extended to the design of the 2-DOF controller for interval system, fractional order system, etc.

References


