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# Study of the process $e^{+} e^{-} \rightarrow \pi^{0} \pi^{0} \gamma$ in c.m. energy range $600-970 \mathrm{MeV}$ at CMD-2 

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#### Abstract

The cross section of the process $e^{+} e^{-} \rightarrow \pi^{0} \pi^{0} \gamma$ has been measured in the c.m. energy range $600-970 \mathrm{MeV}$ with the CMD-2 detector. The following branching ratios have been determined: $\mathcal{B}\left(\rho^{0} \rightarrow \pi^{0} \pi^{0} \gamma\right)=\left(5.2_{-1.3}^{+1.5} \pm 0.6\right) \times 10^{-5}$ and $\mathcal{B}\left(\omega \rightarrow \pi^{0} \pi^{0} \gamma\right)=\left(6.4_{-2.0}^{+2.4} \pm 0.8\right) \times 10^{-5}$. Evidence for the $\rho^{0} \rightarrow f_{0}(600) \gamma$ decay has been obtained: $\mathcal{B}\left(\rho^{0} \rightarrow f_{0}(600) \gamma\right)=$ $\left(6.0_{-2.7}^{+3.3} \pm 0.9\right) \times 10^{-5}$. From a search for the process $e^{+} e^{-} \rightarrow \eta \pi^{0} \gamma$ the following upper limit has been obtained: $\mathcal{B}\left(\omega \rightarrow \eta \pi^{0} \gamma\right)<3.3 \times 10^{-5}$ at $90 \%$ CL. © 2003 Published by Elsevier B.V. Open access under CC BY license.


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## 1. Introduction

Radiative transitions of vector mesons into two pseudoscalar mesons have been attracting attention
since long ago as a possible test of various low energy theoretical models and a source of information on controversial scalar states [1-13]. After the reliable observation of the $f_{0}(980)$ and $a_{0}(980)$ states in the $\phi(1020)$ meson decays by SND [14] and CMD-2 [15], recently confirmed by KLOE [16], the interest moved to the $\rho$ and $\omega$ meson decays.

Information on the $\rho(\omega)$ decays to the $\pi \pi \gamma\left(\eta \pi^{0} \gamma\right)$ final states is rather scarce. Because of the large background from the initial state radiation in the process $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-}$, a search for $\rho(\omega)$ decays into the $\pi^{+} \pi^{-} \gamma$ final state is difficult. Among many such experiments [17] only one succeeded in the observation of the decay $\rho^{0} \rightarrow \pi^{+} \pi^{-} \gamma$ [18]. A long search for the $\omega \rightarrow \pi^{0} \pi^{0} \gamma$ decay (see [17] and references therein) finally proved successful for the GAMS Collaboration, which observed it in $\pi^{-} p$ collisions with the branching fraction of $(7.4 \pm 2.5) \times$ $10^{-5}$ [19]. Recently a high statistics study of the $\rho(\omega)$ energy range has been performed by the SND Collaboration [20,21]. They observe $\rho$ and $\omega$ decays into $\pi^{0} \pi^{0} \gamma$, both with a branching ratio higher than that predicted by vector dominance. While for the $\rho$ meson the excess can be explained by the $\rho^{0} \rightarrow$ $f_{0}(600) \gamma$ decay, first evidence for which is reported by SND, the reasons for the higher probability of the corresponding $\omega$ decay are not yet clear.

In our recent paper [22] we described a study of the process $e^{+} e^{-} \rightarrow \pi^{0} \pi^{0} \gamma$ in the c.m. energy range $920-1380 \mathrm{MeV}$, i.e., above the threshold of $\omega \pi^{0}$ production, using the CMD-2 detector at the VEPP-2M $e^{+} e^{-}$collider. In this Letter we report on the measurement of the cross section of the process $e^{+} e^{-} \rightarrow$ $\pi^{0} \pi^{0} \gamma$ in the c.m. energy range $600-970 \mathrm{MeV}$ with CMD-2. Also described is the first search for the process $e^{+} e^{-} \rightarrow \eta \pi^{0} \gamma$ in this energy range.

## 2. Experiment

The general purpose detector CMD-2 has been described in detail elsewhere [23]. Its tracking system consists of a cylindrical drift chamber (DC) and double-layer multiwire proportional Z-chamber, both also used for the trigger, and both inside a thin $\left(0.38 X_{0}\right)$ superconducting solenoid with a field of 1 T . The barrel CsI calorimeter with a thickness of $8.1 X_{0}$ placed outside the solenoid has energy resolution for
photons of about $9 \%$ in the energy range from 100 to 700 MeV . The angular resolution is of the order of 0.02 radians. The end-cap BGO calorimeter with a thickness of $13.4 X_{0}$ placed inside the solenoid has energy and angular resolution varying from $9 \%$ to $4 \%$ and from 0.03 to 0.02 radians, respectively, for the photon energy in the range 100 to 700 MeV . The barrel and end-cap calorimeter systems cover a solid angle of $0.92 \times 4 \pi$ radians.

This analysis is based on a data sample corresponding to integrated luminosity of $7.7 \mathrm{pb}^{-1}$ collected in 1998-2000 in the energy range $600-970 \mathrm{MeV}$. The step of the c.m. energy scan varied from 0.5 MeV near the $\omega$ peak to 5 MeV far from the resonance. The beam energy spread is about $4 \times 10^{-4}$ of the total energy. The luminosity is measured using events of Bhabha scattering at large angles [24].

We use Monte Carlo simulation (MC) to model the response of the detector and determine the efficiency. Due to the beam background additional ("fake") clusters can appear in the calorimeter. The corresponding probability as well as photon energy and angular spectra are obtained directly from the data using the process $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0}$. Then these photons are mixed with the detector response during simulation.

## 3. Data analysis

At the initial stage, we select events which have no tracks in the DC, five photons, the total energy deposition $1.7<E_{\text {tot }} / E_{\text {beam }}<2.2$, the total momentum $P_{\text {tot }} / E_{\text {beam }}<0.3$ and at least three photons detected in the CsI calorimeter. The minimum photon energy is 30 MeV for the CsI and 40 MeV for the BGO calorimeter. 350 events have been found with these cuts. Then a kinematic fit requiring energymomentum conservation is performed with an additional reconstruction of two $\pi^{0}$ mesons. We require good reconstruction quality $\left(\chi^{2}<5\right)$ and the ratio of the reconstructed to measured energy to be $0.75<$ $\omega_{i} / E_{i}<1.8$ for each photon. After this stage 219 events remain in the whole energy range.

The dominant background comes from the processes
$e^{+} e^{-} \rightarrow \eta \gamma, \quad \eta \rightarrow 3 \pi^{0}$,



Fig. 1. The $E_{\text {tot }}$ (left) and $\chi^{2}$ distributions. The points with error bars represent experimental events, hatched histograms show the MC simulation for the background processes and open histograms are a sum of the signal and background MC. The arrows indicate the cuts imposed.
$e^{+} e^{-} \rightarrow \pi^{0} \gamma$,
$e^{+} e^{-} \rightarrow 3 \gamma, 4 \gamma$.
Events from the process (1) can imitate signal events if two soft photons are lost. The processes (2), (3) can meet the selection criteria with one or two additional ("fake") photons coming from shower splitting, "noisy" electronic channels in the calorimeter and beam background.

To determine the background contribution, the following procedure is used. All the above processes are simulated using the Monte Carlo. To decrease the statistical uncertainty, ten times more events than expected from the background cross section and luminosity are used at each energy. Then the same selection criteria as for the data are applied. The obtained number of selected events is divided by ten and subtracted from the experimental one at each energy. In total, about 29 background events are expected after this procedure.

Fig. 1 shows the $E_{\text {tot }}$ and $\chi^{2}$ distribution for the data, signal and background MC. We use the $\chi^{2}$ distributions to estimate the accuracy of the background estimation. The experimental distribution was fitted by a sum of MC and background. The ratio $N_{\mathrm{bg}}^{\exp } / N_{\mathrm{bg}}^{\mathrm{MC}}=$ $1.2 \pm 0.2$ was obtained. We conclude that the background level is estimated well and its systematic error
does not exceed $40 \%$. This results in a $6 \%$ systematic uncertainty in the cross section.

### 3.1. Approximation of the cross sections

At each energy point the cross section of the process $\sigma$ is calculated from the observed number of events and background MC expectation using the following formula:
$\sigma(\sqrt{s})=\frac{N_{\exp }-N_{\mathrm{bg}}}{L \varepsilon(1+\delta)}$,
where $N_{\exp }$ is the number of observed events, $N_{\mathrm{bg}}$ is the expected number of background events from $\mathrm{MC}, L$ is the integrated luminosity, $\varepsilon$ is the detection efficiency and $(1+\delta)$ is the radiative correction at the corresponding energy.

To calculate the detection efficiency, we use Monte Carlo simulation taking into account the neutral trigger (NT) efficiency. NT is based on the information from the CsI calorimeter and its efficiency depends on the number of clusters and total energy deposition. The NT efficiency is estimated using events of the process $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0}$. We require the charged trigger signal and three or more clusters in the CsI calorimeter, and study the NT efficiency as a function of the energy deposition in CsI. The NT efficiency



Fig. 2. The cross section of the process $e^{+} e^{-} \rightarrow \pi^{0} \pi^{0} \gamma$. The points with error bars represent the experimental data, while the solid and dashed curves correspond to the results of the fit I and III, respectively.
varies from $85 \%$ at c.m. energy of 600 MeV to about $95 \%$ at 980 MeV .

The radiative corrections are calculated according to [25]. The dependence of the detection efficiency on the energy of the emitted photon is determined from simulation.

The obtained Born cross section of the process $e^{+} e^{-} \rightarrow \pi^{0} \pi^{0} \gamma$ is shown in Fig. 2 while Table 1 lists detailed information on the analysis of this reaction. Since no events were selected in the energy range 600 to 690 MeV , our results are presented as upper limits at $90 \%$ C.L. The Feldman-Cousins procedure [26] was used to calculate errors (upper limits) at each energy. This cross section (the "dressed" one from the column VII) was used in the approximation of the energy dependence with resonances.

Meanwhile, for applications to various dispersion integrals like that for the leading order hadronic contribution to the muon anomalous magnetic moment, the "bare" cross section should be used. Following the procedure in Ref. [27], the latter is obtained from the "dressed" one by multiplying it by the vacuum polarization correction $|1-\Pi(s)|^{2}$, where $\Pi(s)$ is the photon polarization operator calculated taking into account the effects of both leptonic and hadronic vacuum polarization. The values of the correction and the "bare" cross section $\hat{\sigma}$ are presented in two last columns of Table 1.

The maximum likelihood method is applied to fit the experimental data obtained from the relation (4).

We parameterize the amplitude of the process by a sum of the $\rho$ and $\omega$ contributions. The former contains the $\rho \rightarrow \omega \pi^{0}$ transition plus one more mechanism beyond the vector dominance model which is chosen to be the $\rho^{0} \rightarrow f_{0}(600) \gamma$ one. Because of the small width of the $\omega$ meson, the $\rho \rightarrow \omega \pi^{0}$ amplitude is rapidly falling below the $\omega \pi^{0}$ threshold. This allows to distinguish $\rho^{0} \rightarrow \omega \pi^{0}$ and $\rho^{0} \rightarrow f_{0}(600) \gamma$ mechanisms by studying the energy dependence of the cross section. On the contrary, the small width of the $\omega$ meson prevents from distinguishing various mechanisms possibly existing in the $\omega \rightarrow \pi^{0} \pi^{0} \gamma$ decay by its cross section. For this reason we parameterize the $\omega$ meson amplitude by the $\omega \rightarrow \rho \pi$ transition only.

The Born cross section of the process is written as

$$
\begin{equation*}
\sigma_{\pi^{0} \pi^{0} \gamma}(s)=\int\left|A_{\pi^{0} \pi^{0} \gamma}(s)\right|^{2} d \Phi \tag{5}
\end{equation*}
$$

where $d \Phi$ is the final state phase space and

$$
\begin{align*}
A_{\pi^{0} \pi^{0} \gamma}= & A_{\rho \rightarrow \omega \pi^{0}}\left(\frac{m_{\rho}^{2}}{D_{\rho}}+\alpha \frac{m_{\rho^{\prime}}^{2}}{D_{\rho^{\prime}}}\right) \\
& +A_{\rho \rightarrow f_{0}(600) \gamma} \frac{m_{\rho}^{2}}{D_{\rho}} \\
& +A_{\omega \rightarrow \pi^{0} \pi^{0} \gamma} \frac{m_{\omega}^{2}}{D_{\omega}} . \tag{6}
\end{align*}
$$

Here the first term describes the amplitude of the $e^{+} e^{-} \rightarrow \rho, \rho^{\prime} \rightarrow \omega \pi^{0}$ transition, while the second and third ones are the $e^{+} e^{-} \rightarrow \rho \rightarrow f_{0}(600) \gamma$ and

Table 1
The c.m. energy, integrated luminosity, detection efficiency, number of observed events, background expectation, radiative correction, Born cross section $\sigma$, vacuum polarization correction and "bare" cross section $\hat{\sigma}$ of the process $e^{+} e^{-} \rightarrow \pi^{0} \pi^{0} \gamma$

| $\sqrt{s}(\mathrm{MeV})$ | $L\left(\mathrm{nb}^{-1}\right)$ | $\varepsilon(\%)$ | $N_{\text {exp }}$ | $N_{\text {bg }}$ | $1+\delta$ | $\sigma(\mathrm{pb})$ | $\|1-\Pi(s)\|^{2}$ | $\hat{\sigma}(\mathrm{pb})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 600 | 56.1 | 10.9 | 0 | 0.2 | 0.892 | $<411$ | 0.993 | < 408 |
| 630 | 115.1 | 11.9 | 0 | 0.1 | 0.888 | < 192 | 0.995 | < 191 |
| 660 | 235.5 | 12.8 | 0 | 0.4 | 0.884 | $<80$ | 0.997 | $<80$ |
| 690 | 196.2 | 13.5 | 0 | 0.2 | 0.880 | $<96$ | 0.999 | $<96$ |
| 720 | 419.7 | 14.1 | 1 | 0.4 | 0.879 | $<76$ | 0.999 | $<76$ |
| 750 | 210.5 | 14.7 | 3 | 0.1 | 0.884 | $106_{-68}^{+84}$ | 0.994 | $105_{-68}^{+83}$ |
| 760 | 206.5 | 14.9 | 2 | 0.3 | 0.885 | $62_{-38}^{+82}$ | 0.991 | $62_{-38}^{+81}$ |
| 764 | 39.7 | 14.9 | 1 | 0.0 | 0.883 | $191{ }_{-120}^{+334}$ | 0.990 | $189{ }_{-119}^{+331}$ |
| 770 | 109.2 | 15.0 | 1 | 0.3 | 0.872 | <284 | 0.991 | <282 |
| 774 | 195.3 | 15.1 | 4 | 0.9 | 0.852 | $123{ }_{-66}^{+110}$ | 0.994 | $122_{-66}^{+109}$ |
| 778 | 199.1 | 15.2 | 6 | 1.3 | 0.817 | $190_{-88}^{+132}$ | 0.994 | $189{ }_{-87}^{+131}$ |
| 780 | 194.7 | 15.2 | 11 | 1.1 | 0.801 | $417{ }_{-92}^{+161}$ | 0.983 | $410_{-90}^{+158}$ |
| 781 | 255.7 | 15.2 | 5 | 1.5 | 0.798 | $112_{-41}^{+90}$ | 0.971 | $109{ }_{-40}^{+87}$ |
| 782 | 631.0 | 15.3 | 30 | 5.1 | 0.803 | $322_{-59}^{+62}$ | 0.958 | $309{ }_{-57}^{+60}$ |
| 783 | 275.7 | 15.3 | 15 | 2.6 | 0.815 | $361{ }_{-107}^{+126}$ | 0.946 | $342_{-101}^{+119}$ |
| 784 | 337.2 | 15.3 | 16 | 3.3 | 0.835 | $295{ }_{-85}^{+111}$ | 0.937 | $276{ }_{-80}^{+104}$ |
| 785 | 198.8 | 15.3 | 9 | 1.6 | 0.859 | $283{ }_{-102}^{+145}$ | 0.932 | $264{ }_{-95}^{+135}$ |
| 786 | 190.8 | 15.3 | 10 | 1.4 | 0.885 | $332{ }_{-124}^{+147}$ | 0.932 | $309_{-116}^{+137}$ |
| 790 | 149.4 | 15.4 | 4 | 0.6 | 0.966 | $153-75$ | 0.939 | $144_{-70}^{+117}$ |
| 794 | 178.7 | 15.5 | 4 | 0.4 | 1.000 | $130_{-60}^{+100}$ | 0.944 | $123_{-57}^{+94}$ |
| 800 | 261.7 | 15.6 | 2 | 0.7 | 1.010 | $32_{-24}^{+55}$ | 0.948 | $30_{-23}^{+52}$ |
| 810 | 247.3 | 15.8 | 3 | 1.0 | 1.006 | $51_{-40}^{+59}$ | 0.950 | $48_{-38}^{+56}$ |
| 820 | 295.7 | 15.9 | 3 | 0.7 | 1.001 | $49_{-34}^{+49}$ | 0.951 | $46_{-32}^{+46}$ |
| 840 | 602.8 | 16.3 | 13 | 0.8 | 0.999 | $124_{-38}^{+44}$ | 0.953 | $118_{-36}^{+42}$ |
| 880 | 375.4 | 17.1 | 4 | 0.6 | 0.984 | $54_{-26}^{+37}$ | 0.958 | $51_{-25}^{+35}$ |
| 920 | 458.2 | 18.1 | 6 | 1.1 | 0.901 | $65_{-29}^{+44}$ | 0.964 | $63_{-28}^{+42}$ |
| 940 | 327.8 | 18.7 | 12 | 0.5 | 0.854 | $219_{-60}^{+82}$ | 0.966 | $212+58$ |
| 950 | 226.1 | 19.1 | 14 | 0.4 | 0.855 | $369_{-100}^{+117}$ | 0.968 | $357{ }_{-97}^{+113}$ |
| 958 | 250.0 | 19.3 | 17 | 0.3 | 0.859 | $402_{-101}^{+116}$ | 0.969 | $390_{-98}^{+112}$ |
| 970 | 249.8 | 19.8 | 23 | 0.9 | 0.867 | $516_{-101}^{+116}$ | 0.972 | $502_{-98}^{+112}$ |

$e^{+} e^{-} \rightarrow \omega \rightarrow \rho \pi^{0}$ amplitudes. $m_{V}$ is the mass and $D_{V}$ is the propagator of the vector meson $V$ given by $D_{V}(s)=s-m_{V}^{2}+i \sqrt{s} \Gamma_{V}(s), \Gamma_{V}(s)$ is
the corresponding energy dependent width. The real parameter $\alpha=g_{\rho^{\prime} \omega \pi} / g_{\rho \omega \pi}$ is the ratio of the coupling constants for the $\rho$ and $\rho^{\prime}$ mesons. The $A_{\rho \rightarrow \omega \pi^{0}}$

Table 2
The fit results in various models

| Fit parameters | Fit I | Fit II | Fit III |
| :--- | :--- | :--- | :--- |
| $\mathcal{B}\left(\omega \rightarrow \pi^{0} \pi^{0} \gamma\right), 10^{-5}$ | $6.4_{-2.0}^{+2.4} \pm 0.8$ | $6.2_{-2.0}^{+2.4} \pm 0.7$ | $11.8_{-1.9}^{+2.1} \pm 1.4$ |
| $g_{\rho \omega \pi}, \mathrm{GeV}^{-1}$ | $\equiv 16.7$ | $16.2 \pm 1.4$ | $18.6 \pm 1.1$ |
| $\mathcal{B}\left(\rho \rightarrow f_{0}(600) \gamma \rightarrow \pi^{0} \pi^{0} \gamma\right), 10^{-5}$ | $2.0_{-0.9}^{+1.1} \pm 0.3$ | $2.3_{-1.2}^{+1.4} \pm 0.3$ | $\equiv 0$ |
| $\mathcal{B}\left(\rho \rightarrow \pi^{0} \pi^{0} \gamma\right), 10^{-5}$ | $5.2_{-1.3}^{+1.5} \pm 0.6$ | $5.4_{-1.4}^{+1.6} \pm 0.6$ | $2.2 \pm 0.3 \pm 0.3$ |
| $\chi^{2} /$ n.d.f. | $19.2 / 28$ | $19.0 / 27$ | $26.7 / 28$ |

amplitude, proportional to the coupling constant $g_{\rho \omega \pi}$ of the $\rho \rightarrow \omega \pi$ transition, is written as in our previous analysis above 1 GeV [22].

The coupling constant $g_{\rho \omega \pi}$ and the following branching ratios are used as fit parameters:

$$
\begin{align*}
& \mathcal{B}\left(\rho^{0} \rightarrow f_{0}(600) \gamma \rightarrow \pi^{0} \pi^{0} \gamma\right) \\
& \quad=\frac{1}{\sigma_{\rho}} \int\left|A_{\rho \rightarrow f_{0}(600) \gamma}\left(m_{\rho}\right)\right|^{2} d \Phi \\
& \mathcal{B}\left(\omega \rightarrow \pi^{0} \pi^{0} \gamma\right)=\frac{1}{\sigma_{\omega}} \int\left|A_{\omega \rightarrow \pi^{0} \pi^{0} \gamma}\left(m_{\omega}\right)\right|^{2} d \Phi \tag{7}
\end{align*}
$$

Then the total branching fraction of the $\rho^{0} \rightarrow \pi^{0} \pi^{0} \gamma$ decay is calculated from the following formula:

$$
\begin{align*}
& \mathcal{B}\left(\rho \rightarrow \pi^{0} \pi^{0} \gamma\right) \\
& \quad=\frac{1}{\sigma_{\rho}} \int\left|A_{\rho \rightarrow \omega \pi^{0}}\left(m_{\rho}\right)+A_{\rho \rightarrow f_{0}(600) \gamma}\left(m_{\rho}\right)\right|^{2} d \Phi . \tag{8}
\end{align*}
$$

In (7) and (8) $\sigma_{V}$ is the cross section at the resonance peak without taking into account other contributions:

$$
\begin{align*}
\sigma_{V} & =\sigma_{e^{+} e^{-} \rightarrow V \rightarrow \pi^{0} \pi^{0} \gamma}\left(m_{V}^{2}\right) \\
& =\frac{12 \pi \mathcal{B}\left(V \rightarrow e^{+} e^{-}\right) \mathcal{B}\left(V \rightarrow \pi^{0} \pi^{0} \gamma\right)}{m_{V}^{2}} \tag{9}
\end{align*}
$$

$\mathcal{B}\left(V \rightarrow e^{+} e^{-}\right)$and $\mathcal{B}\left(V \rightarrow \pi^{0} \pi^{0} \gamma\right)$ are the corresponding branching ratios.

### 3.2. Results of the fits

In all the following fits the $\rho, \omega$ and $\rho^{\prime}$ meson masses and widths are fixed at the world average values [17]. The parameter $\alpha=g_{\rho^{\prime} \omega \pi} / g_{\rho \omega \pi}$ was also fixed at the value obtained in our analysis above 1 GeV [22]. Since the $f_{0}(600)$ mass and width are badly known [17], we use a wide range of these
parameters: $M_{f_{0}(600)}=400-800 \mathrm{MeV}, \Gamma_{f_{0}(600)}=$ $300-600 \mathrm{MeV}$. For the $\rho$ and $\rho^{\prime}$ resonances the energy dependence of the total width was described similarly to Ref. [22] while for the $\omega$ meson the total width was assumed to be energy independent. We perform three main fits: with $g_{\rho \omega \pi}$ equal to the value $(16.7 \pm 0.4 \pm$ $0.6) \mathrm{GeV}^{-1}$ obtained in our analysis above 1 GeV [22] (fit I), with free $g_{\rho \omega \pi}$ (fit II) and without a contribution from the $\rho^{0} \rightarrow f_{0}(600) \gamma$ decay (fit III). The results of the fits are shown in Table 2 and in Fig. 2 by the curves.

The value of the coupling constant $g_{\rho \omega \pi}=16.2 \pm$ 1.4 obtained in the second fit is in good agreement with the one from our measurement of the $e^{+} e^{-} \rightarrow$ $\omega \pi^{0}$ cross section above 1 GeV [22], where a comparison with other measurements and theoretical evaluations can be found.

The fits taking into account the $\rho^{0} \rightarrow f_{0}(600) \gamma$ decay mode (fits I and II) better describe the data. The branching fraction $\mathcal{B}\left(\rho^{0} \rightarrow f_{0}(600) \gamma\right)$ differs from zero by two standard deviations. However, the fit III also has good $\chi^{2} /$ n.d.f. $=26.7 / 28$. There are additional reasons to choose a fit with the $\rho \rightarrow$ $f_{0}(600) \gamma$ decay.

- The branching fraction of the $\omega \rightarrow \pi^{0} \pi^{0} \gamma$ decay obtained in the fit III, $\mathcal{B}\left(\omega \rightarrow \pi^{0} \pi^{0} \gamma\right)=$ $\left(11.8_{-1.9}^{+2.1} \pm 1.4\right) \times 10^{-5}$ is above the GAMS result $(7.4 \pm 2.5) \times 10^{-5}$ [19] by two standard deviations. The latter result obtained in $\pi^{-} p$ collisions has no $\rho$ contribution.
- The value of the coupling constant $g_{\rho \omega \pi}=18.6 \pm$ 1.1 obtained in the fit III is above that from our previous analysis [22] by almost two standard deviations.
- The recent analysis of the process $e^{+} e^{-} \rightarrow$ $\pi^{0} \pi^{0} \gamma$ by SND [21] also showed evidence for the $\rho^{0} \rightarrow f_{0}(600) \gamma$ decay.

Thus, we choose the first model as our final result. The $\rho^{0} \rightarrow f_{0}(600) \gamma$ branching fraction is calculated from the results listed in Table 2 taking into account that $\mathcal{B}\left(f_{0}(600) \rightarrow \pi^{0} \pi^{0}\right)=1 / 3$.

### 3.3. Invariant mass spectrum

Fig. 3 shows the $\pi^{0} \pi^{0}$ invariant mass distribution for $\pi^{0} \pi^{0} \gamma$ events from data in the $\omega$ meson energy range ( $770-800 \mathrm{MeV}$ ). The experimental distribution agrees well with the $\omega \rightarrow \rho \pi^{0}$ decay model; however, a contribution from the $\omega \rightarrow f_{0}(600) \gamma$ decay cannot be excluded. The existing statistics is not enough to distinguish between these contributions, therefore we quote only the total branching fraction of the $\omega \rightarrow$ $\pi^{0} \pi^{0} \gamma$ decay.

### 3.4. Search for the decay $\omega \rightarrow \eta \pi^{0} \gamma$

For a search of events of the process $e^{+} e^{-} \rightarrow \eta \pi^{0} \gamma$ we first apply the same criteria as for the preliminary selection of $e^{+} e^{-} \rightarrow \pi^{0} \pi^{0} \gamma$ events. Next, a kinematic fit requiring energy-momentum conservation is performed with the additional reconstruction of one soft $\pi^{0}$ meson and a good reconstruction quality, $\chi^{2}<6$, is


Fig. 3. The $\pi^{0} \pi^{0}$ invariant mass distribution in the $\omega$ meson energy range. The points with error bars represent the experimental data, the solid histogram shows the MC simulation of the $\omega \rightarrow \rho \pi^{0}$ decay, the dashed one corresponds to the $\omega \rightarrow f_{0}(600) \gamma$ decay. The hatched histogram is the estimated background contribution.
required. To reject the dominant background from the process $e^{+} e^{-} \rightarrow \pi^{0} \pi^{0} \gamma$, we perform an additional kinematic fit with the $\pi^{0} \pi^{0} \gamma$ hypothesis and reject events that are consistent with it, $\chi_{\pi^{0} \pi^{0} \gamma}^{2}<6$. Then we look for a possible $\eta$ signal in the invariant mass of two hard photons of the remaining three, $M_{\gamma \gamma}$. The $M_{\gamma \gamma}$ distribution is approximated with a Gaussian for the signal and a polynomial function for the background. The Gaussian mean value and width are fixed from the MC simulation of the signal events. The background shape is obtained using the $\pi^{0} \pi^{0} \gamma$ MC. In all energy ranges the resulting $\eta \pi^{0} \gamma$ signal is consistent with zero. Fig. 4 shows the $M_{\gamma \gamma}$ distribution for events from the $\omega$ resonance region: $381 \mathrm{MeV}<E_{\text {beam }}<$ 401 MeV . The $90 \%$ CL upper limit for the number of $\eta \pi^{0} \gamma$ events is obtained: $N_{\eta \pi^{0} \gamma}<2.4$. Using the detection efficiency of $1.3 \%$, we set the following upper limit for the $e^{+} e^{-} \rightarrow \eta \pi^{0} \gamma$ cross section:
$\sigma\left(e^{+} e^{-} \rightarrow \eta \pi^{0} \gamma\right)<57 \mathrm{pb}$,
and for the branching fraction of the $\omega$ meson:
$\mathcal{B}\left(\omega \rightarrow \eta \pi^{0} \gamma\right)<3.3 \times 10^{-5}$.


Fig. 4. The $M_{\gamma \gamma}$ distribution for the $\eta \pi^{0} \gamma$ candidates. The histogram represents experimental events and the solid curve shows the fit result. The dashed curve corresponds to the $\eta \pi^{0} \gamma \mathrm{MC}$.

Table 3
Main sources of systematic errors

| Source | Contribution (\%) |
| :--- | :---: |
| Selection criteria | 8 |
| Background subtraction | 6 |
| Model uncertainty | 5 |
| Luminosity | 2 |
| Trigger efficiency | 2 |
| Radiative corrections | 1 |
| Total | 12 |

### 3.5. Systematic errors

The main sources of systematic uncertainties in the cross section determination are listed in Table 3. The systematic error due to selection criteria is obtained by varying the photon energy threshold, total energy deposition, total momentum, and $\chi^{2}$. The model uncertainty corresponds to different detection efficiencies for the $\omega \pi^{0}, \rho \pi^{0}$ and $f_{0}(600) \gamma$ intermediate states. It also includes dependence on the $f_{0}(600)$ mass and width. The uncertainty in the determination of the integrated luminosity comes from the selection criteria of Bhabha events, radiative corrections and calibrations of DC and CsI. The error of the neutral trigger efficiency was estimated by trying various fitting functions for the energy dependence and variations of the cluster threshold. The uncertainty of the radiative corrections comes from the dependence on the emitted photon energy and the accuracy of the theoretical formulae. The resulting systematic uncertainty of the cross section in Table 1 as well as of the branching fractions in Table 2 is $12 \%$.

## 4. Discussion

The obtained values of the branching fractions, $\mathcal{B}\left(\rho^{0} \rightarrow \pi^{0} \pi^{0} \gamma\right)=\left(5.2_{-1.3}^{+1.5} \pm 0.6\right) \times 10^{-5}$ and $\mathcal{B}\left(\omega \rightarrow \pi^{0} \pi^{0} \gamma\right)=\left(6.4_{-2.0}^{+2.4} \pm 0.8\right) \times 10^{-5}$, are in good agreement with the previous measurements by GAMS [19] and SND [21]. Both values are significantly higher than those predicted by the vector dominance model with the standard value of the coupling constant: $\sim 1 \times 10^{-5}$ and $\sim 3 \times 10^{-5}$ [4], respectively. An attempt to explain the obtained branching ratios results in a high value of $g_{\rho \omega \pi}$ contradicting the other

Table 4
Predictions for branching fractions of $\rho, \omega \rightarrow \pi^{0} \pi^{0} \gamma, \eta \pi^{0} \gamma$ decays

| Mode | Branching fraction |
| :--- | :---: |
| $\rho^{0} \rightarrow \pi^{0} \pi^{0} \gamma$ | $(1.1-4.7) \times 10^{-5}$ |
| $\omega \rightarrow \pi^{0} \pi^{0} \gamma$ | $(1.4-8.2) \times 10^{-5}$ |
| $\rho^{0} \rightarrow \eta \pi^{0} \gamma$ | $2 \times 10^{-10}-4 \times 10^{-6}$ |
| $\omega \rightarrow \eta \pi^{0} \gamma$ | $8.3 \times 10^{-8}-6.3 \times 10^{-6}$ |

observations like, e.g., the experimental values of the $\omega \rightarrow \pi^{+} \pi^{-} \pi^{0}$ and $\omega \rightarrow \pi^{0} \gamma$ widths.

Theoretical papers on the subject [1-13] offer a broad choice of effects influencing the discussed decays. In addition to the $\omega \rho \pi$ transition they include: $\rho-\omega$ mixing, pion and kaon loops, various scalar $\left(f_{0}(600), f_{0}(980), a_{0}(980)\right)$ and tensor $\left(f_{2}(1270)\right.$ and $a_{2}(1320)$ ) intermediate mesons decaying into $\pi \pi$ ( $\eta \pi$ ).

Predictions of these models differ rather strongly from each other, reflecting various approaches applied by their authors [10]. While most of the recent papers agree that the observed value of the branching fraction for the $\rho^{0} \rightarrow \pi^{0} \pi^{0} \gamma$ decay can be ascribed to the intermediate $f_{0}(600)$ state, the situation with the corresponding $\omega$ decay remains controversial. The corresponding ranges of the predicted values of branching fractions are summarized in Table 4. Note that from the upper limits for the non- $\left(\omega \pi \pi^{0} \pi^{0} \gamma\right)$ cross section obtained by us at higher energy in Ref. [22] a significant contribution from the $f_{0}(980) \gamma$ or $f_{2}(1270) \gamma$ mechanisms seems not very likely.

Much higher data samples of the $\rho$ and $\omega$ decays expected in experiments at the upgraded collider VEPP-2000 in Novosibirsk [28] will help to significantly improve our understanding of their radiative decays.

From the obtained results on the cross section of the radiative processes $e^{+} e^{-} \rightarrow \pi^{0} \pi^{0} \gamma, \eta \pi^{0} \gamma$ one can estimate a possible contribution of the previously unstudied radiative processes to the leading order hadronic correction to the muon anomalous magnetic moment. To this end we first calculate the contribution of the process $e^{+} e^{-} \rightarrow \pi^{0} \pi^{0} \gamma$ using the "bare" cross section, $\hat{\sigma}$, from Table 1 in the energy range below 920 MeV . The result contains a piece coming from the $\omega \rightarrow \pi^{0} \pi^{0} \gamma$ decay already taken into account in Ref. [29] in the whole $\omega$ meson contribu-
tion, $a_{\mu}^{\omega}=(37.96 \pm 1.07) \times 10^{-10}$. This $\omega$ meson contribution is subtracted from the value above using the branching ratio $\mathcal{B}\left(\omega \rightarrow \pi^{0} \pi^{0} \gamma\right)$ with the result $(6.08 \pm 0.82) \times 10^{-12}$. A possible contribution from the process $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \gamma$ is twice that of $e^{+} e^{-} \rightarrow \pi^{0} \pi^{0} \gamma$, so that
$a_{\mu}^{\mathrm{LO}, \pi \pi \gamma}(600-920 \mathrm{MeV})=(18.2 \pm 2.5) \times 10^{-12}$.
Adding the contribution from the $\eta \pi \gamma$ final state, we finally obtain
$a_{\mu}^{\mathrm{LO}, \mathrm{rad}}(600-920 \mathrm{MeV})<0.24 \times 10^{-10}$ at $90 \% \mathrm{CL}$.
Adding the upper limit from the energy range 9202000 MeV obtained previously [22], we obtain
$a_{\mu}^{\mathrm{LO}, \mathrm{rad}}(600-2000 \mathrm{MeV})<0.7 \times 10^{-10}$ at $90 \% \mathrm{CL}$
or about $10 \%$ of the current uncertainty of $a_{\mu}^{\mathrm{LO}, \text { had }}$ [29,30]. The obtained upper limit does not contradict the lower estimate of Ref. [31], in which the value $(0.10 \pm 0.13) \times 10^{-10}$ has been obtained using the QCD sum rules, but is by far smaller than their upper estimate of $(6.95 \pm 5.60) \times 10^{-10}$ based on the PDG value for the $\gamma \gamma$ width of the $f_{0}(600)$ meson.

## 5. Conclusions

The following results are obtained in this Letter.

- Using a data sample corresponding to integrated luminosity of $7.7 \mathrm{pb}^{-1}$, the cross section of the process $e^{+} e^{-} \rightarrow \pi^{0} \pi^{0} \gamma$ has been measured in the c.m. energy range $600-970 \mathrm{MeV}$. The values of the cross section are consistent with those obtained by the SND detector [21] and have similar accuracy. The following branching ratios have been determined: $\mathcal{B}\left(\rho \rightarrow \pi^{0} \pi^{0} \gamma\right)=$ $\left(5.2_{-1.3}^{+1.5} \pm 0.6\right) \times 10^{-5}$ and $\mathcal{B}\left(\omega \rightarrow \pi^{0} \pi^{0} \gamma\right)=$ $\left(6.4_{-2.0}^{+2.4} \pm 0.8\right) \times 10^{-5}$.
- We confirm evidence for the $\rho \rightarrow f_{0}(600) \gamma$ decay with the branching fraction $\mathcal{B}\left(\rho \rightarrow f_{0}(600) \gamma\right)=$ $\left(6.0_{-2.7}^{+3.3} \pm 0.9\right) \times 10^{-5}$ reported by the SND Collaboration [21].
- A first search for the process $e^{+} e^{-} \rightarrow \eta \pi^{0} \gamma$ was performed allowing to set the $90 \% \mathrm{CL}$ upper limits: $\sigma\left(e^{+} e^{-} \rightarrow \eta \pi^{0} \gamma\right)<57 \mathrm{pb}$ in the c.m. energy range $685-920 \mathrm{MeV}$ and $\mathcal{B}\left(\omega \rightarrow \eta \pi^{0} \gamma\right)<$ $3.3 \times 10^{-5}$.
- A possible contribution of the studied radiative processes to the muon anomalous magnetic moment was estimated to be negligible.


## Acknowledgements

The authors are grateful to the staff of VEPP2 M for the excellent performance of the collider, and to all engineers and technicians who participated in the design, commissioning and operation of CMD-2. This work is supported in part by grants DOE DEFG0291ER40646, NSF PHY-9722600, NSF PHY0100468, INTAS 96-0624, RFBR-98-02-1117851 and RFBR-03-02-16843.

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