ELSEVIER

Available online at www.sciencedirect.com

SciVerse ScienceDirect

Procedia - Social and Behavioral Sciences 80 (2013) 7 – 24

Procedia
Social and Behavioral Sciences

The 20th International Symposium on Transportation and Traffic Theory

On joint railway and housing development strategy

Xiaosu Ma[a] and Hong K. Lo[b,*]

[a]*Future Urban Mobility IRG, Singapore-MIT Alliance for Research and Technology, Singapore*
[b]*Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Hong Kong*

Abstract

Transit Oriented Development (TOD) with railway service is recognized as a sustainable mode of development for highly dense megacities. In addition to providing safe and efficient transit services, reducing auto dependence and therefore less need for highway expansions, the improved accessibility of TOD influences commuters' residential location choices and the resultant housing value. Traditionally, statistical approaches have been used to estimate the relationship between railway development and housing value for individual sites. To some degree, TOD has also been studied with integrated land-use transport models. While useful, they lack an analytical framework to study the region-wide impacts of TOD on residential location and travel choices and the resultant land value changes. In this study, the joint railway and housing development strategy is modeled based on a combined equilibrium formulation with the bid-rent process. The problem is formulated as a mathematical program with equilibrium constraints, in which the upper level optimizes the objective for the joint development strategy by deciding on the combination of housing supplies and railway service levels. Analytical results are obtained for a single corridor in a multi-modal transport network, which are further illustrated by sensitivity analyses. A numerical example is constructed to demonstrate the approach and compare with other separate development strategies. The results generaly confirm the synergy between railway and housing developments.

Selection and peer-review under responsibility of Delft University of Technology

Keywords: bid-rent; joint railway and housing development

1. Introduction

Transit Oriented Development (TOD) with railway service is recognized as a sustainable mode of development for highly dense megacities. Compared with cities with a high dependency on auto travel, TOD can substantially mitigate traffic congestion and avoid urban sprawl due to the ever increasing urban population. In addition to offering a sustainable mode of transportation, Litman [1] stated that transit can help reducing the amount of land that must be paved for roads and parking facilities, and providing a catalyst for more compact urban redevelopment. However, most railway projects involve huge investments, which are usually one of the major barriers for implementing TOD in developing countries. Therefore, a successful TOD project requires policies to encourage the provision of quality transit services with affordable fares, which are also financially sustainable.

Hong Kong is one of the few exceptions whose railway services are commercially operated, with reputable quality and profitability, often serving as a benchmark for new projects in the region and beyond. The railway company, Mass Transit Railway Corporation (MTRC), builds, funds, owns, and operates the railway, and jointly develops housing above railway stations with a private developer [2]. In this way, the transport infrastructure development is cross-subsidized by housing development profits. The experience of Hong Kong sheds light on an approach that exploits the synergy between railway and housing developments [3]. The railway service improves the accessibility of housing to be developed and hence increases its value, which helps finance the infrastructure investment. Empirical studies show that property values generally increase in areas near quality transit stations [4]. In Bogotá, Colombia, for example, a reduction of five minutes of walking distance to nearby BRT stations increases property values up to 9.3% [5]. Munoz-Raskin [6] showed that middle-income households in Bogotá tend to spend 2.3% to 14.4% more for properties close to BRT stations. In the case of

* Corresponding author. Tel : +852-2358-8389; Fax: +852-2358-1534.
E-mail address: cehklo@ust.hk

Selection and peer-review under responsibility of Delft University of Technology
doi:10.1016/j.sbspro.2013.05.002

Hong Kong, on average, profit from housing development accounts for almost 50% of the operating revenue and contributes to about 90% of the MTRC's total profit [2,3,7]. On the other hand, housing development generates traffic for the railway service and hence increases its fare revenue. Higher fare revenue can afford better services, which in turn yield still higher patronage, thus initiating a positive cycle.

The joint railway and housing development strategy depends very much on the value capture, or the housing value in association with the accessibility improvement introduced. Classically, this type of study is accomplished via hedonic analyses [8]. The seminal work of Rosen [9], which explored the role of housing attributes in the consumer housing market, initiated a vast volume of studies that investigated both the theoretical foundation and empirical analyses of this approach. In particular, hedonic models have been developed to relate housing price to characteristics of the property such as proximity to amenities, accessibility, etc. Recently, Cervero and Murakami [10] discussed how the joint rail and property development model works in Hong Kong. While useful, these statistical approaches are typically developed based on regression analysis, which tend to be location specific without considering the overall system effect. For example, adding a new railway link will change the relative accessibility between locations and may induce region-wide shifts in residence choices, and ultimately, changes in housing value. That is, not only will locations along the new railway link be affected, but also other locations. To some degree, TOD has also been studied with integrated land-use transport models. To this end, it is more indicative if the study is integrated in a broader analytical framework, so that the impacts of transportation improvements at the network level can be incorporated directly.

In this study, an integrated modeling framework is developed to study the impact of joint railway and housing developments on region-wide residential location and travel choices, and the resultant housing value changes. It is based on a combined equilibrium formulation for residents' location and travel choices with the bid-rent process [9,11,12,13]. In Ma and Lo [13], time-dependent transport supply and demand management (TS-DM) strategies by a welfare-oriented transport provider were investigated. By internalizing the effect of land use/value changes in the combined equilibrium formulation, the interactions between highway expansion, road pricing, and land use/value changes can be jointly considered, allowing for analyses such as exposing winners and losers of transport improvements, and possibly cross-subsidization between them and land sale. In this paper, to study the synergy between joint railway and housing development, the combined equilibrium formulation is extended to incorporate the joint optimization of railway and housing developments. In other words, we study the case of a transport provider who also serves as a housing developer, i.e. a single joint railway and property developer. Whereas joint railway and housing development by one developer is not common in Western countries, it is not uncommon in Asia, especially in Hong Kong and mainland China. In recent years, many Asian megacities have started to explore similar approaches.

Specifically, the problem is formulated as a mathematical program with equilibrium constraints (MPEC). The lower level problem constitutes the combined equilibrium formulation encapsulating the bid-rent process, whereas the upper level objective is to maximize the combined profit from the joint railway and housing development by deciding on the combination of housing supplies (e.g. location and type of housing) and the railway service levels (e.g. headway and fare). Section 2 describes the general formulation of the modeling framework. Analytical results are derived for a single corridor in a multi-modal transport network. A monotonic relationship is found between changes in land value and railway service quality, i.e. better railway services always lead to higher land value or housing prices. Sensitivity analyses are performed to illustrate these properties. Section 3 describes the numerical studies, and Section 4 provides some concluding remarks. The results generally show that joint railway and housing development strategy is systematically better than treating railway and housing developments separately. There are benefits to be gained by exploiting the synergy between them. The results also indicate that even if the joint development were to be invested by a single profit-oriented developer, whose strategies would introduce quality transport system improvement on its own initiative, in return for gains in housing revenue. That is to say, the private developer would not compromise the transport system performance.

2. The formulation

2.1 *Residents' location and travel choices*

Residents' choices include both residential location and corresponding travel choices, as modeled by a combined bid-rent and nested multinomial logit framework. The highest level is the residential location choice, then travel modal choice, finally route choice.

2.1.1 *Travel choice*

The generalized travel cost by railway includes travel time, expressed in monetary terms, monetary costs, e.g. fare, and travel comfort index. Since normally a railway network constitutes a limited number of railway lines operated by a single company, and most of the regular railway passengers choose a unique path between OD pairs, without loss of generality, it is assumed that there exists one railway path between each OD pair[1]. As a result, the generalized travel cost on railway between OD pair r and s valuated by travelers in income group k is formulated as:

$$c_{\text{rail}}^{rsk} = (ct\_i^{rs} + ct\_w^{rs}) \cdot vot^k + cp^{rs}, \tag{1}$$

where ct_i^{rs} and ct_w^{rs} are the in-vehicle travel time and waiting time between OD pair r and s, respectively; vot^k is the value of time for income group k; cp^{rs} is the fare between r and s. The in-vehicle travel time between r and s, ct_i^{rs}, is assumed to be fixed once the railway is built. The waiting time, ct_w^{rs}, is assumed to be proportional to the headway, hw^{rs}, expressed as:

$$ct\_w^{rs} = \kappa_{cti} \cdot hw^{rs}, \tag{2}$$

where κ_{cti} is a positive headway/waiting time parameter. In this formulation, the railway service level is measured by the headway, hw^{rs} and the fare, cp^{rs}, between r and s, which are the decision variables of railway design for the developer. Note that in the real world the average headway/waiting time experienced by a traveler depends on the actual headway of each railway line he takes. For simplicity, the headway defined in this study is based on each OD pair, which constitutes a subset of the actual line-based headway combinations.

For the auto mode, the generalized auto travel cost captures travel time and other monetary cost. The auto travel cost for travelers in income group k between r and s through auto path p, notated as $c_{p|\text{auto}}^{rsk}$, is the sum of the path's corresponding link travel times t_a, and toll charges ρ_a,

$$c_{p|\text{auto}}^{rsk} = \sum_a \delta_{a,p}^{rs} (vot^k \cdot t_a + \rho_a), \tag{3}$$

where $\delta_{a,p}^{rs}$ is the path-link incidence indicator, equals 1 if link a is on path p between r and s; 0 otherwise. The BPR link performance function is adopted to calculate link time t_a. The probability of travelers in income group k between r and s using path p is expressed as:

$$\Pr_{p|\text{auto}}^{rsk} = \frac{\exp(-\beta_{\text{route}}{}^k \cdot c_{p|\text{auto}}^{rsk})}{\sum_{p' \in P^{rs}} \exp(-\beta_{\text{route}}{}^k \cdot c_{p'|\text{auto}}^{rsk})}. \tag{4}$$

The expected perceived auto travel cost between OD pair r and s of income group k can be expressed as the following logsum term:

$$c_{\text{auto}}^{rsk} = -\frac{1}{\beta_{\text{route}}{}^k} \cdot \ln\left(\sum_{p' \in P^{rs}} \exp(-\beta_{\text{route}}{}^k \cdot c_{p'|\text{auto}}^{rsk}) \right). \tag{5}$$

Note that (3)-(5) represent a money-based multi-class stochastic traffic assignment formulation by categorizing travelers into groups with different VOTs. Under the further condition of homogeneous VOT, it produces the same unique user equilibrium link flow patterns[2]. Accordingly, the probability that travelers in income group k choose travel mode m among alternatives $m \in \{\text{rail}, \text{auto}\}$ between OD pair r and s is expressed as:

$$\Pr_m^{rsk} = \frac{\exp(-\beta_{\text{m}}{}^k \cdot c_m^{rsk})}{\sum_{m' \in \{\text{rail, auto}\}} \exp(-\beta_{\text{m'}}{}^k \cdot c_{m'}^{rsk})}, \tag{6}$$

where c_m^{rsk} is the systematic part of the expected travel cost of travel mode m between OD pair r and s for travelers in income group k, with a random term ω_m^{rsk} following the Gumbel distribution. $\beta_{\text{m}}{}^k$ is the scale parameter for income group k using travel mode m which is inversely proportional to the standard deviation of ω_m^{rsk}. Also, the travel disutility of a resident in income group k working in workplace s chooses to live in residential location r is expressed as:

$$\mu^{rsk} = -\frac{1}{\beta_{\text{m}}{}^k} \ln \sum_{m' \in \{\text{rail, auto}\}} \exp(-\beta_{\text{m'}}{}^k \cdot c_{m'}^{rsk}). \tag{7}$$

Finally, the corresponding railway flow and auto path flows between OD pair r and s for travelers in income group k, are expressed as:

$$q_{\text{rail}}^{rvsk} = q^{rvsk} \cdot \Pr_{\text{rail}}^{rsk}, \tag{8}$$

---

[1] This is purely for simplicity. Extending this to a network of rail lines can be achieved without any conceptual difficulty, similar to the approach developed for the auto mode shown below.

[2] See more discussions on the equilibrium flow conditions in Ma and Lo [13], Daganzo [14], and Yang and Huang [15].

$$f_{p|\text{auto}}^{rvsk} = q^{rvsk} \cdot \text{Pr}_{\text{auto}}^{rsk} \cdot \text{Pr}_{p|\text{auto}}^{rsk} . \tag{9}$$

where q^{rvsk} is the demand between OD pair r and s for travelers residing in housing type v in income group k, which is determined by the location choices in the following section.

2.1.1.1 *Location choice*

A combined bid-rent and multinomial logit framework is developed to formulate the residential location choice problem. The bid-rent theory considers that residents bid for residential locations based on the principle of utility maximization subject to budget constraints [9]. The term $WP^{sk/rv}$, or Willingness-to-Pay (WP), represents the maximum value that a bidder of income group k at workplace s is willing to pay for a housing unit of type v at residential location r in order to maximize his overall utility [12]. The WP function is expressed as:

$$WP^{sk/rv} = b^{sk} - \mu^{rsk} + \alpha^k \cdot ls^v + wp , \tag{10}$$

where b^{sk} is a utility index, representing the bidder's desired level of utility at equilibrium. μ^{rsk} is the corresponding travel disutility between r and s as perceived by residents in income group k, as defined in (7) . ls^v represents the hedonic attribute of housing type v. In this formulation, it refers to the housing lot size. α^k is the taste parameter of lot size by residents of income group k. wp adjusts the bids to the observed or actual housing rents and can be considered as a calibration parameter. For the stochastic bid-rent process, WP is considered as a random variable consisting of the systematic part as expressed in (10) and the random term $\omega^{sk/rv}$ following the identical and independently distributed (IID) Gumbel distribution, with β as the scale parameter. By adjusting the utility index, b^{sk}, the housing supply and demand equilibrium can be attained with all residents able to find a residence given the assumption that the total number of housing supply equals the total number of housing demand, expressed as [12]:

$$\sum_{r'\in R}\sum_{v'\in V} \Psi^{r'v'} = \sum_{s'\in S}\sum_{k'\in K} D^{s'k'} , \tag{11}$$

where $\Psi^{r'v'}$ is the housing supply of type v' in residential location r', which is endogenously determined by housing developers as described in the following section. $D^{sk(\tau)}$ is the total number of residents of income group k working in s. It is assumed to be exogenously fixed; hence the dimension of the utility index b^{sk} is $S \times K$.

Accordingly, the probability of housing type v at residential location r being occupied by residents of income group k working in workplace s is expressed as:

$$\text{Pr}^{sk/rv} = \frac{\exp(\beta \cdot WP^{sk/rv})}{\sum_{s'k'\in SK} \exp(\beta \cdot WP^{s'k'/rv})} . \tag{12}$$

Based on (12), the OD flows of residents in income group k between residential location r and workplace s is obtained by:

$$q^{rvsk} = \Psi^{rv} \cdot \text{Pr}^{sk/rv} . \tag{13}$$

In addition, according to the discrete choice theory for the logit model [16], the expected maximum willingness-to-pay, which constitutes the eventual housing rent, after factoring in the supply influence [13] is expressed as:

$$\varphi^{rv} = \frac{1}{\beta} \cdot \ln\left(\sum_{s'k'\in SK} \exp\left(\beta \cdot WP^{s'k'/rv}\right)\right) - \frac{1}{\beta} \cdot \ln(\Psi^{rv}) . \tag{14}$$

Accordingly, the consumer surplus of a resident of income group k working in workplace s choosing to reside in housing type v at location r is expressed as:

$$CS^{rvsk} = WP^{sk/rv} - \varphi^{rv} . \tag{15}$$

The consumer surplus measures the willingness-to-pay of income group k for housing type v at residence location r minus its actual rent and can be regarded as an incentive or utility in residential location choice from the perspective of residents, following the principle of utility maximization.

2.1.1.2 *Combined equilibrium formulation*

The combined bid-rent and nested multinomial choice framework formulated in Sections 2.1.1 and 2.1.2 constitutes an equilibrium model. The entire equilibrium problem can be cast into equivalent Nonlinear Complementarity Problem (NCP), given that the total number of housing supply equals the total number of housing demand, and solved accordingly [17]:

$$f_{p|\text{auto}}^{rvsk}(f_{p|\text{auto}}^{rvsk} - \Psi^{rv} \cdot \Pr^{sk/rv} \cdot \Pr_{\text{auto}}^{rsk} \cdot \Pr_{p|\text{auto}}^{rsk}) = 0, \quad \forall r, v, s, p, k , \tag{16}$$

$$f_{p|\text{auto}}^{rvsk} - \Psi^{rv} \cdot \Pr^{sk/rv} \cdot \Pr_{\text{auto}}^{rsk} \cdot \Pr_{p|\text{auto}}^{rsk} \geq 0, \quad \forall r, v, s, p, k , \tag{17}$$

$$q_{\text{rail}}^{rvsk}(q_{\text{rail}}^{rvsk} - \Psi^{rv} \cdot \Pr^{sk/rv} \cdot \Pr_{\text{rail}}^{rsk}), \ \forall r, v, s, k , \tag{18}$$

$$q_{\text{rail}}^{rvsk} - \Psi^{rv} \cdot \Pr^{sk/rv} \cdot \Pr_{\text{rail}}^{rsk} \geq 0, \ \forall r, v, s, k , \tag{19}$$

$$b^{sk}(\sum_{rv} \Psi^{rv} \cdot \Pr^{sk/rv} - D^{sk}) = 0, \quad \forall s, k , \tag{20}$$

$$\sum_{rv} \Psi^{rv} \cdot \Pr^{sk/rv} - D^{sk} \geq 0, \quad \forall s, k , \tag{21}$$

$$f_{p|\text{auto}}^{rvsk} \geq 0, \ \forall r, v, s, p, k , \tag{22}$$

$$q_{\text{rail}}^{rvsk} \geq 0, \ \forall r, v, s, k , \tag{23}$$

$$b^{sk} \geq 0, \ \forall s, k . \tag{24}$$

(16)-(17) and (18)-(19) model residents' travel choices, where the auto path flow, $f_{p|\text{auto}}^{rvsk}$, and railway flow, q_{rail}^{rvsk}, follow exactly the probabilities as defined in (4) and (6), respectively. Similarly, (20)-(21) model the residents' residential location choice at equilibrium. (20)-(21) assure that every resident can eventually be located, i.e. the total housing supply equals the total housing demand.

The above reformulation can be further reformulated as an unconstrained optimization problem, by minimizing the following gap function to zero [18,19].

$$\begin{aligned} \min G(\mathbf{Z}) = & \sum_{rvskp} \vartheta\left(f_{p|\text{auto}}^{rvsk},\ f_{p|\text{auto}}^{rvsk} - \Psi^{rv} \cdot \Pr^{sk/rv} \cdot \Pr_{\text{auto}}^{rsk} \cdot \Pr_{p|\text{auto}}^{rsk}\right) \\ & + \sum_{rvsk} \vartheta\left(q_{\text{rail}}^{rvsk},\ q_{\text{rail}}^{rvsk} - \Psi^{rv} \cdot \Pr^{sk/rv} \cdot \Pr_{\text{rail}}^{rsk}\right) \\ & + \sum_{sk} \vartheta\left(b^{sk},\ \sum_{rv} \Psi^{rv} \cdot \Pr^{sk/rv} - D^{sk}\right) \end{aligned} , \tag{25}$$

where $\vartheta(\cdot)$ is defined as:

$$\vartheta(c,d) = \frac{1}{2}\phi^2(c,d), \tag{26}$$

$$\phi(c,d) = \sqrt{c^2 + d^2} - (c+d). \tag{27}$$

c and d in (26)-(27) are real numbers. By minimizing (25), the auto path flow, $f_{p|\text{auto}}^{rvsk}$, and railway flow, q_{rail}^{rvsk}, can be obtained. Equivalently, by setting the gap function to be zero, i.e. $G(\mathbf{Z}) = 0$, it can be considered as the equilibrium constraint to be satisfied in a mathematical program with equilibrium constraint (MPEC), as discussed in the next section. It can be verified readily that when $G(\mathbf{Z})$ attains the value of zero, the entire NCP (16)-(24) is satisfied [19].

2.2 *Developer's investment decisions*

Section 2.1 describes residents' location and travel choices based on fixed housing and transport supply, which is formulated as a combined equilibrium formulation. On top of that, the developer's joint railway and housing investment decisions are determined, as shown schematically in Fig. 1.

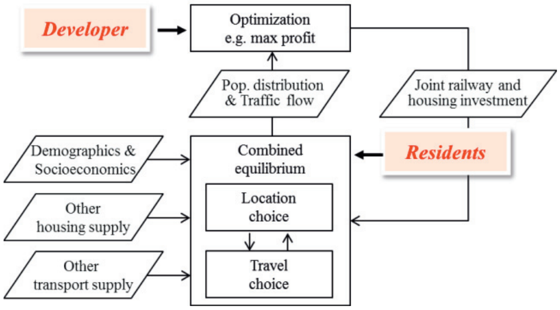

Fig. 1. The joint railway and housing development model

The investment cost on the railway, $B_{\tilde{T}}$, include the initial construction cost on all the railway lines, $B_{\tilde{T}C}$, and the operational cost, $st \cdot b_{\tilde{T}O}$, expressed as:

$$B_{\tilde{T}} = B_{\tilde{T}C} + st \cdot b_{\tilde{T}O}, \tag{28}$$

where st is the number of trains operated on a railway line, $b_{\tilde{T}O}$ is the overall operational cost per train, where the initial purchase price of a train is also converted and included in $b_{\tilde{T}O}$ for simplicity. Note that the highway network is assumed to be given and fixed. The revenue from the railway investment equals the total fare collected, expressed as:

$$R_{\tilde{T}} = \sum_{rs} q_{\text{rail}}^{rs} \cdot cp^{rs}, \tag{29}$$

where q_{rail}^{rs} is the number of railway passengers between OD pair r and s, as defined in (8). On the other hand, the investment cost on the housing supply, B_H, is the sum of investment cost on different housing types and locations, expressed as:

$$B_H = \sum_v \sum_r b_H^v \cdot \Psi^{rv}, \tag{30}$$

where b_H^v is the unit investment cost of housing type v. In this study, it is assumed to be increasing with the unit housing lost size ls^v, as defined in (10). Ψ^{rv} is the number of housing unit of type v to be invested in residential location r. In this formulation, since the total housing demand is fixed and equal to the total housing supply, the total number of housing units Ψ is also fixed. Thereby, the decisions on housing supply are measured by the proportion of type v housing units to be invested in residential location r, i.e. $\Pr^{rv}$. And the number of housing units of type v in residential location r is expressed as:

$$\Psi^{rv} = \Psi \cdot \Pr^{rv}, \tag{31}$$

The total revenue from housing investment, R_H, equals the total housing rent collected from the housing market, expressed as:

$$R_H = \sum_v \sum_r \varphi^{rv} \cdot \Psi^{rv}, \tag{32}$$

where φ^{rv} is the resultant housing rent of type v in residential location r at equilibrium.

In this study, both the railway and housing investment decisions are made by a single profit-oriented developer. The problem is casted as a mathematical program with equilibrium constraints, expressed as:

$$\underset{\Pr^{rv}, hw^{rs}, cp^{rs}}{\text{Maximize}}\ \Pi = R_H + R_{\tilde{T}} - B_H - B_{\tilde{T}}, \tag{33}$$

subject to

$$G(\mathbf{Z}) = 0, \tag{34}$$

Constraints (1)-(15),

$$\sum_r \sum_v \Pr^{rv} = 1, \tag{35}$$

$$B_H + B_{\tilde{T}} \le B, \tag{36}$$

$$\Pr^{rv} \ge 0, \forall r, v, \tag{37}$$

$$\underline{hw} \le hw^{rs} \le \overline{hw}, \forall r, s, \tag{38}$$

$$\underline{cp} \le cp^{rs} \le \overline{cp}, \forall r, s, \tag{39}$$

where $\Pr^{rv}$ is the proportion of type v housing units to be invested in residential location r; hw^{rs} is the operational headway between OD pair r and s; cp^{rs} is the train fare between OD pair r and s. (36) is the budget constraint for the overall investment. $\underline{hw}$ and $\overline{hw}$, $\underline{cp}$ and $\overline{cp}$ are lower and upper bounds for headway and fare, respectively, to reflect technological barriers and planning regulations. Generally, the above MPEC is non-linear and non-convex, but can be solved by commercial non-linear mathematical programming solvers. The global optimality of solutions is not guaranteed, as is typical for MPEC problems. However, under some simplified conditions, some results can be derived analytically, as discussed in in the following section.

2.3 *Impact of joint railway and housing development*

The combined equilibrium formulation in (1)-(15) models residents' location and travel choices given a fixed transport and housing system. It is of interest to analyze the resultant benefit redistribution among different stakeholders due to different railway and housing investment strategies, such as changes in resident consumer surplus, railway and housing developer' producer surplus, as well as the overall social welfare. Despite that the following analytical results are developed for the case of one OD pair or a single corridor, the results are revealing. In Section 2.4, we will conduct sensitivity analyses to further demonstrate these properties. Generally speaking, the analytical results obtained for the simple one OD pair with multiple travel modes are echoed by the numerical studies that implement the model for a network with multiple OD pairs, as described in Section 3.

To simplify notation, superscripts r and s are dropped in the following discussion. The decision variables are denoted as $\Pr^v, hw,$ and cp. The general conditions are defined below:

(H_0): The network comprises one residential location and one workplace, with residents belonging to different income groups ($k = 1,2,...,K$). The housing demand for each income group, H^k, is fixed.

(H_1): There are several housing types v with different lot sizes ls^v to be invested, such that $ls^1 < ls^2 < ... < ls^V$. The supply for each housing type v is given by $\Psi^v = \Psi \cdot \Pr^v$. The total housing demand $H = \sum_k H^k$ equals the total housing supply Ψ.

(H_2): There are two travel modes between the OD pair, i.e. auto and metro, with one path linking the residential location and workplace for each travel mode. The link performance function of auto is an increasing function of demand, e.g. the BPR function.

(H_3): Travelers have different values of time, such that $vot^1 < vot^2 < ... < vot^K$.

Proposition 1 *Under conditions* $(H_0)-(H_3)$*, any changes in railway investment, e.g. headway and/or fare, do not induce any changes in residents' choices on housing types.*

Proof

Let's consider that the change in railway headway ∂hw or fare ∂cp introduces a corresponding change in travel disutility via (1)-(7), denoted as $\Delta\mu^k, k = 1,2,...,K$, which is different for different income group k but is fixed within each income group regardless of the housing type chosen.

Proposition 1 is equivalent to saying that the change in probability of residents' location choice for each housing type v, as defined in (12), is zero, expressed as:

$$\Delta \Pr^{k/v} = \frac{\exp\left(\beta \cdot (WP^{k/v} + \Delta WP^{k/v})\right)}{\sum_{k' \in K} \exp\left(\beta \cdot (WP^{k'/v} + \Delta WP^{k'/v})\right)} - \frac{\exp\left(\beta \cdot WP^{k/v}\right)}{\sum_{k' \in K} \exp\left(\beta \cdot WP^{k'/v}\right)} = 0 . \tag{40}$$

Simplifying (40), we have:

$$\Delta WP^{k/v} = \frac{1}{\beta} \ln \left\{ \frac{\sum_{k' \in K} \exp\left(\beta \cdot (WP^{k'/v} + \Delta WP^{k'/v})\right)}{\sum_{k' \in K} \exp\left(\beta \cdot WP^{k'/v}\right)} \right\}. \tag{41}$$

(41) implies that the resultant change in willingness-to-pay is the same for all income groups as the RHS is independent of a specific income group k, i.e.:

$$\Delta WP^{1/v} = \Delta WP^{2/v} = ... = \Delta WP^{K/v}, \tag{42}$$

According to (10), we have:

$$\Delta WP^{k/v} = \Delta b^k - \Delta\mu^k, \ \forall k, v . \tag{43}$$

In the logit modeling framework, only the relative utility between groups matters. By setting the utility index b^1 of income group $k=1$ as defined in (10) to be zero, i.e. $b^1=0$, we can always find a unique set of $b^k, k=1,2,...,K$ so as to achieve the equilibrium of residential location choice for each given land use and transport supply [13]. As a result, the problem then is to prove that the equilibrium solution b^k after the change in railway investment fulfills (40) for each housing type v.

By setting the utility index b^1 of income group $k=1$ as defined in (10) to be unchanged as a reference, i.e. $\Delta b^1 = b^{1(1)} - b^{1(0)} = 0$, where $b^{1(0)}, b^{1(1)}$, respectively, are the utility indices of b^1 before and after the change, we have:

$$\Delta WP^{1/v} = \Delta b^1 - \Delta\mu^1 = -\Delta\mu^1,\ \forall v, \tag{44}$$

$$\Delta WP^{k/v} = \Delta b^k - \Delta\mu^k = b^{k(1)} - b^{k(0)} - \Delta\mu^k,\ \forall v, \tag{45}$$

where $b^{k(0)}$ and $b^{k(1)}$ refer to the solutions before and after the change, respectively. Using (42), by setting (44) equal to (45), we have:

$$b^{k(1)} = b^{k(0)} + \Delta\mu^k - \Delta\mu^1,\ \forall v. \tag{46}$$

Putting (46) into (45), we have:

$$\Delta WP^{k/v} = -\Delta\mu^1,\ \forall v \tag{47}$$

We can show that (47) fulfills (10). In other words, with the change in travel disutility $\Delta\mu^k$ fixed prior to the housing type decision, the new equilibrium solution after the change can be found by selecting $b^{k(1)}$ according to (46), which satisfies (40) and also all the supply constraints as the housing type decisions by each income group k before and after the change are the same. As a result, the residents' choice on housing types is not changed, expressed as:

$$\Delta q^{vk} = \Psi^v \cdot \Delta \Pr^{k/v} = 0. \tag{48}$$

■

Note that Proposition 1 is only valid for the case of one OD pair. For the case with multiple OD pairs, since the change of travel disutility, $\Delta\mu^{rk}$, is not necessarily the same among different residential location r for the same income group k, i.e. $\Delta\mu^{r'k} \neq \Delta\mu^{r''k},\ \forall r' \neq r'', r' \in R, r'' \in R$. As a result, according to (44)-(46), we may not always be able find a set of $b^{k(1)},\ k=1,2,...,K$ such that $\Delta WP^{1/r} = \Delta WP^{2/r} = ... = \Delta WP^{K/r}$ is satisfied for each residential location r.

Corollary 1 *Under conditions $(H_0)-(H_3)$, any changes in railway investment, e.g. headway and/or fare, induce neither changes in individual consumer surplus nor total consumer surplus.*

Proof

According to (14), the change in housing rent φ^v is:

$$\Delta\varphi^r = \frac{1}{\beta}\ln\left(\frac{\sum_{k'\in K}\exp\left(\beta\cdot(WP^{sk'/r} + \Delta WP^{sk'/r})\right)}{\sum_{k'\in K}\exp\left(\beta\cdot WP^{sk'/r}\right)}\right) = \Delta WP^{k/v} = -\Delta\mu^1. \tag{49}$$

Therefore, the change in housing rent is the same as the change in willingness-to-pay and equals the change in travel disutility of the income group $k=1$. Furthermore, according to (7), if conditions $(H_2)-(H_3)$ are simplified to one travel mode and homogeneous value of time, we conclude that any reduction in travel cost due to transport improvement leads to an equivalent increase in housing rent [13].

According to (15) and (44)-(49), the corresponding change in consumer surplus is:

$$\Delta CS^{vk} = \Delta WP^{k/v} - \Delta\varphi^v = 0. \tag{50}$$

That is, changes in headway and/or fare do not result in changes in individual consumer surplus. In other words, residents do not receive any benefit from the transport improvement. Accordingly, we can conclude that the total consumer surplus is also not changed.

It can also be proved in the following way. According to Proposition 1, we have:

$$\frac{\partial WP^{1/v}}{\partial hw} = \frac{\partial WP^{2/v}}{\partial hw} = ... = \frac{\partial WP^{K/v}}{\partial hw} = \frac{\partial WP^{k'/v}}{\partial hw}, \forall k' \in \{1,2,...,K\}, \tag{51}$$

$$\frac{\partial WP^{1/v}}{\partial cp} = \frac{\partial WP^{2/v}}{\partial cp} = ... = \frac{\partial WP^{K/v}}{\partial cp} = \frac{\partial WP^{k'/v}}{\partial cp}, \forall k' \in \{1,2,...,K\}. \tag{52}$$

Accordingly, we have:

$$\frac{\partial \varphi^v}{\partial hw} = \frac{\partial WP^{k'/v}}{\partial hw}, \forall k' \in \{1,2,...,K\}, \tag{53}$$

$$\frac{\partial \varphi^v}{\partial hw} = \frac{\partial WP^{k'/v}}{\partial hw}, \forall k' \in \{1,2,...,K\}, \tag{54}$$

$$\frac{\partial CS^{vk}}{\partial hw} = \frac{\partial WP^{k/v}}{\partial hw} - \frac{\partial \varphi^v}{\partial hw} = 0, \tag{55}$$

$$\frac{\partial CS^{vk}}{\partial cp} = \frac{\partial WP^{k/v}}{\partial cp} - \frac{\partial \varphi^v}{\partial cp} = 0. \tag{56}$$

$$\frac{\partial \Pr^{k/v}}{\partial hw} = \beta \cdot \Pr^{k/v} \cdot \frac{\partial WP^{k/v}}{\partial hw} - \beta \cdot \Pr^{k/v} \cdot \sum_k \left(\Pr^{k/v} \cdot \frac{\partial WP^{k/v}}{\partial hw} \right) = 0, \tag{57}$$

$$\frac{\partial \Pr^{k/v}}{\partial cp} = \beta \cdot \Pr^{k/v} \cdot \frac{\partial WP^{k/v}}{\partial cp} - \beta \cdot \Pr^{k/v} \cdot \sum_k \left(\Pr^{k/v} \cdot \frac{\partial WP^{k/v}}{\partial cp} \right) = 0. \tag{58}$$

Define the total consumer surplus as:

$$CS = \sum_k \sum_v \left(\left(\Psi \cdot \Pr^v \cdot \Pr^{k/v} \right) \cdot CS^{vk} \right). \tag{59}$$

The partial derivatives of total consumer surplus with respect to headway, hw, and fare, cp, are calculated by:

$$\frac{\partial CS}{\partial hw} = \sum_v \sum_k \Psi \cdot \left(\frac{\partial CS^{vk}}{\partial hw} \cdot \Pr^v \cdot \Pr^{k/v} + CS^{vk} \cdot \Pr^v \cdot \frac{\partial \Pr^{k/v}}{\partial hw} \right) = 0, \tag{60}$$

$$\frac{\partial CS}{\partial cp} = \sum_v \sum_k \Psi \cdot \left(\frac{\partial CS^{vk}}{\partial cp} \cdot \Pr^v \cdot \Pr^{k/v} + CS^{vk} \cdot \Pr^v \cdot \frac{\partial \Pr^{k/v}}{\partial cp} \right) = 0. \tag{61}$$

■

Note that the unchanged consumer surplus for every income group in residential housing choice does not indicate that the overall impact of transport system changes (e.g. headway and fare) on residents of different income groups are the same. Actually, our previous research proved that residents with higher incomes and hence higher values of time benefit more from transport system improvements [13]. Without going into the details of the proof, the intuition is that an improvement in transport infrastructure or reduction in transport cost will lead to an increase in housing rent, $-\Delta\mu^1$ according to (49), which is the same for all income groups and is equal to the travel cost as valued by the lowest income group $k=1$. The same travel time saving would be valued higher by higher income groups due to their higher values of time. Therefore, for higher income groups, the increase in rental cost is smaller than the travel cost saving gained from transport infrastructure improvement. Hence, higher income groups will gain more from transport infrastructure improvements, or a regressive effect.

In the next proposition, we show that any degradation in transport service, either in terms of longer headway or higher fare, will result in higher travel disutility, and consequentially reduction in housing rent. This result seems obvious, but nevertheless, can be established rigorously via the analytical formluation.

Proposition 2 *Under conditions* $(H_0)-(H_3)$*, the travel disutility for every income group is monotonically increasing with headway and railway fare. The housing rent is monotonically decreasing with headway and fare.*

<u>Proof</u>

We first prove the property with respect to headway hw. For the travel cost by railway, according to (1)-(2), we have:

$$\frac{\partial c_{\text{rail}}^k}{\partial hw} = vot^k \cdot \kappa_{cti} > 0. \tag{62}$$

For the travel cost by auto, according to (3)-(5) and condition (H_2), we have:

$$\frac{\partial c_{\text{auto}}^k}{\partial hw} = vot^k \cdot \frac{\partial t_a}{\partial hw} = vot^k \cdot \frac{\partial t_a}{\partial q_{\text{auto}}} \cdot \frac{\partial q_{\text{auto}}}{\partial hw}, \tag{63}$$

where q_{auto} is the auto demand, which can be calculated by:

$$q_{\text{auto}} = \Psi - q_{\text{rail}} = \Psi - \sum_v \sum_k q^{vk} \cdot \text{Pr}_{\text{rail}}^k. \tag{64}$$

Substituting (64) into (63), we have:

$$\frac{\partial c_{\text{auto}}^k}{\partial hw} = -vot^k \cdot \frac{\partial t_a}{\partial q_{\text{auto}}} \cdot \sum_v \sum_k \left(\frac{\partial q^{vk}}{\partial hw} \cdot \text{Pr}_{\text{rail}}^k + \frac{\partial \text{Pr}_{\text{rail}}^k}{\partial hw} \cdot q^{vk} \right). \tag{65}$$

In (65), according to condition (H_2), $\partial t_a / \partial q_{auto} > 0$. The first derivative in the bracket is zero, i.e. $\partial q^{vk} / \partial hw = 0$, according to Proposition 1. In addition, according to (6), since the increase in travel cost by railway will result in a demand shift from railway to auto, it can be concluded that the second derivative term in the bracket is negative, expressed as:

$$\frac{\partial \text{Pr}_{\text{rail}}^k}{\partial hw} < 0. \tag{66}$$

Therefore, in (65), we have $\partial c_{\text{auto}}^k / \partial hw > 0$.

Eventually, according to (7), (10), and (14), we have:

$$\frac{\partial \mu^k}{\partial hw} = \text{Pr}_{\text{rail}}^k \cdot \frac{\partial c_{\text{rail}}^k}{\partial hw} + \text{Pr}_{\text{auto}}^k \cdot \frac{\partial c_{\text{auto}}^k}{\partial hw} > 0, \tag{67}$$

$$\frac{\partial WP^{k/v}}{\partial hw} < 0, \tag{68}$$

$$\frac{\partial \varphi^v}{\partial hw} = \sum_k \left(\text{Pr}^{k/v} \cdot \frac{\partial WP^{k/v}}{\partial hw} \right) < 0. \tag{69}$$

Similarly, we conclude the same results with respect to the railway fare cp, expressed as:

$$\frac{\partial \text{Pr}_{\text{rail}}^k}{\partial cp} < 0, \tag{70}$$

$$\frac{\partial \mu^k}{\partial cp} > 0, \tag{71}$$

$$\frac{\partial WP^{k/v}}{\partial cp} < 0, \tag{72}$$

$$\frac{\partial \varphi^v}{\partial cp} = \sum_k \left(\text{Pr}^{k/v} \cdot \frac{\partial WP^{k/v}}{\partial cp} \right) < 0. \tag{73}$$

■

Corollary 2 *Under conditions* $(H_0)-(H_3)$, *there exists an optimal headway such that the producer surplus of a joint railway and housing development is maximized. More importantly, the producer surplus is monotonically decreasing with railway fare in a multi-modal transport network.*

Proof

First let us check the partial derivative of the producer surplus of a joint railway and housing developer's overall profit as defined in (33), with respect to the headway hw. According to (28)-(32) and (66), we have:

$$\frac{\partial R_H}{\partial hw} = \frac{\partial \left(\sum_v \varphi^v \cdot \Psi^v \right)}{\partial hw} = \Psi \cdot \frac{\partial WP^{k'/v}}{\partial hw} < 0, \forall k' \in \{1, 2, ..., K\}, \tag{74}$$

$$\frac{\partial R_{\tilde{T}}}{\partial hw} = \sum_v \Psi^v \cdot cp \cdot \sum_k \left(\frac{\partial \text{Pr}_{\text{rail}}^k}{\partial hw} \cdot \text{Pr}^{k/v} \right) < 0, \tag{75}$$

$$\frac{\partial B_H}{\partial hw} = 0, \tag{76}$$

$$\frac{\partial B_{\tilde{T}}}{\partial hw} = b_{\tilde{T}O} \cdot \frac{\partial st}{\partial hw} < 0. \tag{77}$$

According to (74)-(77), both the railway revenue and cost are monotonically decreasing with the headway; the housing investment is not changed with the headway, while the housing revenue are monotonically decreasing with the headway. As a result, according to (33), there exists an optimal headway such that the producer surplus of a joint railway and housing developer is maximized.

Similarly, the partial derivative of the producer surplus of a joint railway and housing developer with respect to the railway fare cp is expressed as:

$$\frac{\partial R_H}{\partial cp} = -\sum_v \left(\Psi^v \cdot \sum_k \left(\Pr^{k/v} \cdot \Pr^k_{\text{rail}} \right) \right) = -q_{\text{rail}} < 0, \tag{78}$$

$$\frac{\partial R_{\tilde{T}}}{\partial cp} = q_{\text{rail}} + \sum_v \Psi^v \cdot cp \cdot \sum_k \left(\frac{\partial \Pr^k_{\text{rail}}}{\partial cp} \cdot \Pr^{k/v} \right), \tag{79}$$

$$\frac{\partial B_H}{\partial cp} = \frac{\partial \left(\sum_v b^v_H \cdot \Psi^v \right)}{\partial cp} = 0, \tag{80}$$

$$\frac{\partial B_{\tilde{T}}}{\partial cp} = \frac{\partial (B_{\tilde{T}C} + st \cdot b_{\tilde{T}O})}{\partial cp} = 0. \tag{81}$$

Therefore, according to (70) and (78)-(81), the partial derivative of producer surplus with respect to fare is expressed as:

$$\begin{aligned} \frac{\partial \Pi}{\partial cp} &= \frac{\partial R_H}{\partial cp} + \frac{\partial R_{\tilde{T}}}{\partial cp} - \frac{\partial B_H}{\partial cp} - \frac{\partial B_{\tilde{T}}}{\partial cp} \\ &= \sum_v \Psi^v \cdot cp \cdot \sum_k \left(\frac{\partial \Pr^k_{\text{rail}}}{\partial cp} \cdot \Pr^{k/v} \right) < 0 \end{aligned}. \tag{82}$$

■

As a result, according to Proposition 2 and Corollary 2, the producer surplus is monotonically decreasing with railway fare in the presence of competing travel modes. Therefore, in a joint railway and property development scheme, the developer will keep the railway fare at a low level in order to maintain the producer surplus at a high level; the gain from property rental value gain offsets the reduction in fare revenue.

Note also that in the logit modeling framework, only the relative utility between groups matters. By setting the utility index b^1 of income group $k = 1$ as defined in (10) to be unchanged as a reference, i.e. $\Delta b^1 = 0$, if railway is the only available travel mode, i.e. $\Pr^k_{\text{rail}} = 1$, the total profit will not be affected by the change in railway fare, since any railway fare increase (reduction) will be fully absorbed in the resultant housing rent by the same level of reduction (increase).

2.4 *Sensitivity Analyses*

The properties derived in the previous section are illustrated by sensitivity analyses here. To be consistent, the analyses here consider that there is one residential location and one workplace connected by a railway line and a highway link. Travelers' railway waiting time is assumed to be half of the headway. The BPR function is used to calculate the auto travel time. The housing investment involves two types of units with different lot sizes, Big and Small units. Residents are stratified into two income groups with different values of time, e.g. High and Low. Each income group has a different preference on the lot size. At each equilibrium solution for a set of given housing and transport supply, the utility index of residents of Low income group, i.e. b^L, is set to be unchanged, fixed at zero.

Two sensitivity scenarios are conducted. Scenario 1 studies the combined decisions on headway and housing type, with fixed railway fare. Scenario 2 studies the combined decisions on railway fare and housing type, with fixed headway.

2.4.1 *Developer's cost and revenue*

In Fig. 2-4, the horizontal axis refers to the proportion of Big units to be developed, and the vertical axis refers to headway for the figure on the left, and fare for the figure on the right. This set of figures is to illustrate how housing investment and revenue, shown as contours in Fig. 2, transport investment and revenue, shown as contours in Fig. 3, and producer surplus, shown as contours in Fig. 4, vary with the provision of Big units and transport system attributes.

Firstly, from the housing investment perspective, as shown in Fig. 2, the housing investment increases as the proportion of Big units increases, which is obvious. Meanwhile, for a fixed proportion of Big units, i.e., tracing along a vertical line on either the left or right figure of Fig. 2, the housing revenue (blue or solid contour) decreases as the headway or fare increases, confirming Proposition 2.

Another interesting observation is that, as shown in Fig. 2, for a fixed level of headway or fare, i.e. tracing along a horizontal line on either figure of Figure 2, there is an optimal proportion of Big units such that the total housing revenue is maximized, around the provision of 80% Big units.

Secondly, as shown in Fig. 3, decreases in headway, which require more trains and higher operating costs, result in increases in railway investment cost. Moreover, a shorter headway attract more travelers, which bring in a higher railway revenue. On the other hand, as the railway fare increases, the total railway revenue increases although the total railway demand decreases. Note that the changes in railway investment cost and revenue are not affected by housing type investments, since the travel demand by income group is fixed.

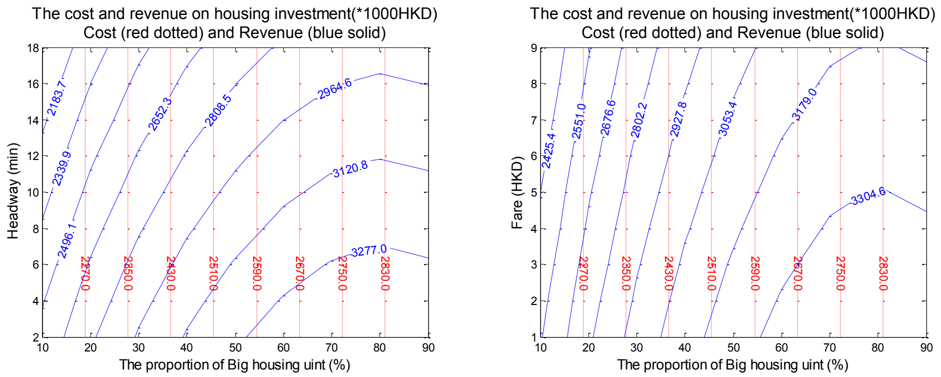

Fig. 2. The cost and revenue of housing investment

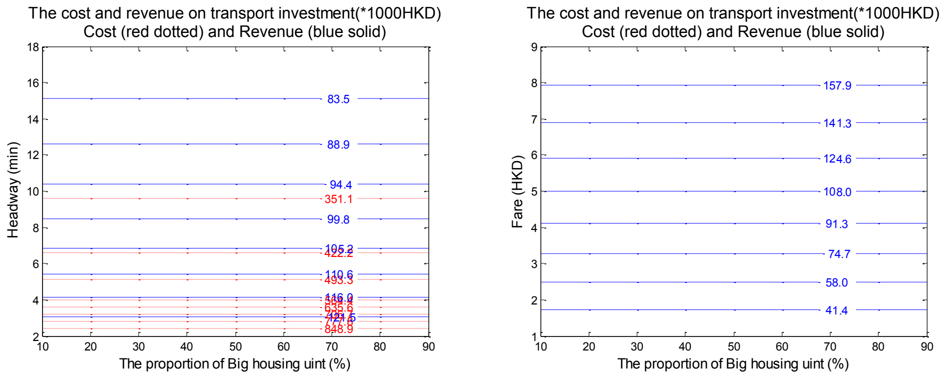

Fig. 3. The cost and revenue of railway investment

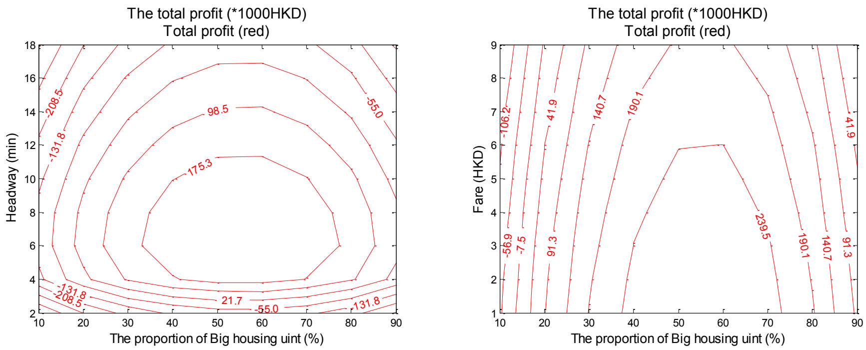

Fig. 4. The total profit or producer surplus

Combining the costs and revenues of housing and railway investments together, we obtain their combined overall profit for a single developer. As shown in Fig. 4, the maximum profit for the combined housing and headway decision occurs when the headway is set at 6-min and the proportion of Big units at 55%. For the combined housing and railway fare decision, the maximum profit occurs at a very low, close to zero fare and the provision of 55% Big units. In this study, the results indicate the same optimal housing investment decision for both scenarios. While we cannot conclude that this outcome will always hold, the more interesting observation is that the optimal joint railway and housing investment decision is to set the fare at a very low level, close to zero. This, actually, should not be surprising, given the earlier results that any reduction in transport cost can be recovered by corresponding increases in housing rents, when both types of revenues belong to the same developer.

2.4.2 *Residents' choices and surplus*

Given different combinations of housing supplies and railway service levels, residents make their location and travel choices accordingly, with different resultant benefit distributions in terms of consumer surplus (CS). As shown in Fig. 5 and Fig. 6, the consumer surpluses for both income groups do not change with any railway service improvement, e.g. reduced headway or fare. That is, any travel cost reductions introduced by transport system improvement lead to corresponding increases in housing rents, thus neutralizing their effects on CS[3]. However, change in housing supply, e.g. proportion of Big housing unit, does change residence choice and resultant CS.

Moreover, as shown in Fig. 5, the CS of the high-income group increases with the provision of more Big units, and vice versa for the low-income group. Combining the CS of the high-income and low-income groups, the optimal provision occurs at the extreme ends, either providing exclusively Big or exclusive Small units. The lowest combined CS occurs at the mid-range, or 50% Big units. This result is interesting and certainly deserves further studies to confirm whether it will hold for a wide range of scenarios or only for the limited study here. It implies that there is a tradeoff between the CS of the high-income group versus that of the low-income group, and society's overall CS.

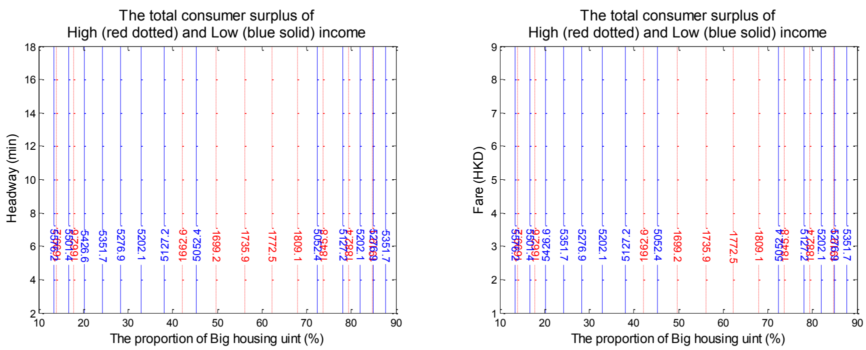

Fig. 5. The total consumer surplus by income group

[3] This result holds based on the assumption that intrinsic housing attributes, and other types of externalities, such as environmental externality, location externality, etc. remain unchanged.

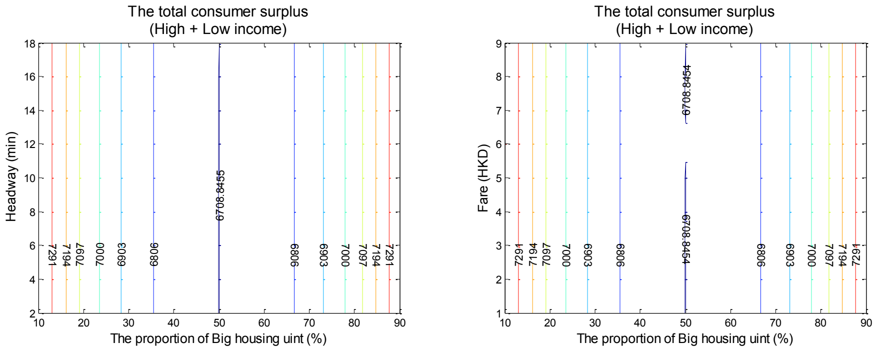

Fig. 6. The total consumer surplus

2.4.2.1 *Comparison with other objectives*

We study two scenarios here. Scenario 1 studies the combined decisions on headway and housing type, with fixed railway fare. Scenario 2 studies the combined decisions on railway fare and housing type, with fixed headway. Summing up the total consumer surplus and total producer surplus, we obtain the overall social welfare, as shown in Fig. 7. We notice that the investment decision that gives rise to the maximum social welfare occurs at 6-min headway and about 100% Big units for Scenario 1, and at zero fare and 100% Big units for Scenario 2.

Fig. 4 and Fig. 7 demonstrate the decisions according to two different planning perspectives for a single decision-maker; Figure 4 shows the result for maximizing total profit by a private developer and Fig. 7 for maximizing social welfare by the government. While the exact optimal decisions are likely to be case specific and parameter dependent, the comparison between these two results shows that both objectives would likely produce similarly good transport system performance, i.e. short headway and low fare. The key difference is in the housing supply market. The profit-seeking objective would create a shortage of large units, whereas the social welfare objective would produce a large amount of large units to drive down the housing rents.

Nowadays, when the government is short of funding for large-scale housing and transport infrastructure investments, the participation of private developers is widely accepted. However, in joint railway and property developments, while the synergy between them is clearly evident, the result here seems to indicate that regulating the housing supply is just as important, if not more so, in this joint development scheme.

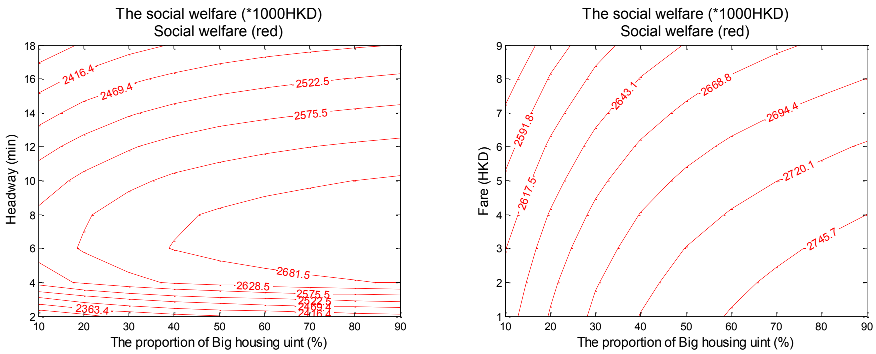

Fig. 7. The social welfare

3. Numerical studies

This section demonstrates the above joint railway and housing development in a generic mono-centric city (one CBD, four residential zones) with two alternative travel modes, e.g. auto and railway. As shown in Fig. 8, Zone 1 and Zone 2 are two existing residential zones connected to the CBD by highway links and an existing railway line R1. Zone 3 and Zone 4 are newly planned residential zones, which will be connected to the CBD by highway links and a planned railway line R2. The developer needs to make joint investment decisions on R2 (including headway and fare) and housing supply (with two housing types) in Zone 3 and Zone 4 so as to maximize the overall profit. Note that the housing supplies in Zone 1 and Zone 2 and the headway of railway Line 1 are fixed. However, the railway fare, e.g. HKD/km, is changeable, the same for both R1 and R2. For the residents side, the underlying assumptions include: a) one worker per household, and he evaluates the residential zones based on his perceived travel costs[4], as defined in (7) and (10); b) each household resides in one housing unit; c) the total housing demand, including both the existing population residing in the old residences and the new residents, are free to choose their new residential locations after the new joint housing and railway development.

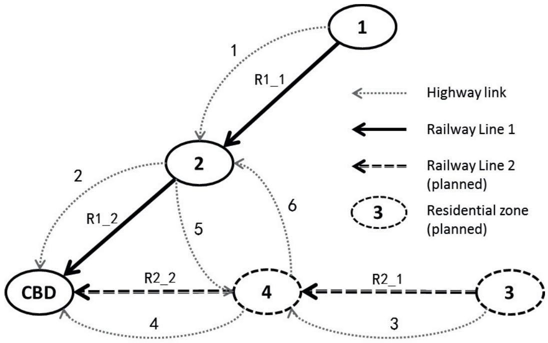

Fig. 8. A TOD network

3.1 The optimal joint railway and housing investments

Table 1 and Table 2 are the resultant housing and railway investment decisions. The new investment decisions choose a relatively shorter headway for the new railway R2 and larger proportions of Small housing units in both Zone 3 and Zone 4. Besides, the developer invests more housing units in Zone 4 which is closer to the CBD.

Table 1. The railway investment decisions

Indicators	Unit	R1	R2
Headway	min	5 (fixed)	1.7
Fare	HKD/km	1.8	1.8

Table 2. The housing investment decisions

Housing type	Old areas (in 1000 units)		New areas (in 1000 units)		
	Zone 1	Zone 2	Zone 3	Zone 4	Sub-total
Big	15	15	2.8 (5.5%)	3.3 (6.5%)	6.1 (12%)
Small	10	10	20.3 (40.7%)	23.6 (47.3%)	43.9 (88%)
Total	25	25	23.1 (46.2%)	26.9 (54.8%)	50 (100%)

3.2 Impacts on residents' choices and land value

Table 3 - Table 6 show the residents' resultant travel and location choices, as well as the impacts on land value and transport system performance. For the transport modal choices, as shown in Table 3, travelers from the low income group prefer to take the railway which has a relatively longer travel time due to their lower value of time. On the other hand, residents living in the newly planned zones are more attracted by the railway because of its shorter headway. Furthermore, residents living in Zone 2 and Zone 4, which are closer to the CBD, prefer to take the railway than auto, because they generally experience high highway congestion, as observed from the highway link volume to capacity (V/C) ratios in Table 4. For the residential location and housing choices, as shown in Table 5, the probabilities of residents from the high income group living in Zone 2 and Zone 4 are higher than those of the low income group. It implies that residents' with higher incomes and higher value of times are more likely to reside in more attractive locations, e.g. higher accessibility and Big units. Meanwhile, the resultant housing rents of both Big and Small units in these two locations are also relatively higher than the

[4] See more discussions in Ma and Lo [13]

ones that are far from the CBD, as shown in Table 6. These observations demonstrate that changes in the transport system not only alter travelers' travel choices but also have significant impact on residents' location choices and land values.

Table 3. Residents' travel mode choices

Income Group	Mode	Zone 1	Zone 2	Zone 3	Zone 4
High	Auto	69%	42%	50%	22%
	Rail	31%	58%	50%	78%
Low	Auto	56%	32%	43%	20%
	Rail	44%	68%	57%	80%

Table 4. The highway link congestion levels (V/C ratio)

Highway	Link 1	Link 2	Link 3	Link 4	Link 5	Link 6
V/C ratio	0.77	1.04	0.53	1.02	0.55	0.35

Table 5. Residents' location and housing choices

Housing type	Income Group	Zone 1	Zone 2	Zone 3	Zone 4
Big	High	7.0 (46%)	5.4 (54%)	1.3 (49%)	1.9 (58%)
	Low	8.0 (54%)	4.6 (46%)	1.4 (51%)	1.4 (42%)
Small	High	4.4 (44%)	7.7 (51%)	9.3 (46%)	13.0 (55%)
	Low	5.6 (56%)	7.3 (49%)	11.0 (54%)	10.6 (45%)

Table 6. The housing rents

Housing rent	Unit	Zone 1	Zone 2	Zone 3	Zone 4
Big	HKD/day	109	120	123	131
Small	HKD/day	108	114	106	114

3.3 The overall system performance

Table 7. Comparisons of the overall system performance

Outcome indicators	Scenario A (Joint rail and housing investment)	Scenario B (Same rail and housing investment)		Scenario C (Same housing but optimized rail investment)	
Total travel time[a]	533	589	10.5%	539	1.1%
Total consumer surplus[b]	1,340	1,237	-7.7%	1,238	-7.6%
Housing investment cost[c]	1,736	1,880	8.3%	1,880	8.3%
Housing revenue[c]	2,049	2,092	2.1%	2,104	2.7%
Railway investment cost[c]	234	212	-9.4%	233	-0.4%
Railway fare revenue[c]	107	62	-42.1%	110	2.8%
Developer's total profit[c]	185	62	-66.5%	102	-44.9%
Social welfare[c]	187	63	-66.3%	103	-44.9%

[a] The unit is thousand minutes/day; [b] The unit is million HKD/day; [c] The unit is million HKD/year.

Table 7 shows the comparisons of the system performance among different investment strategies. Scenario A is the performance resulted from the joint rail and housing investment as formulated in this study. Scenario B is the performance resulted from planning the new area exactly the same as the old area, i.e. the housing unit distribution and railway operations follow the same as the old ones. Scenario C is the performance resulted from optimizing the railway operations alone while keeping the land use layout of the new area same as the old one. Column 4 and Column 6 are the comparisons of performances between the joint investment decision and the other two strategies as measured by their percentage differences. Generally, the performances of Scenario A are better than the other two Scenarios in most aspects. The joint housing and railway investment scheme not only maximizes the developer's total profit, but also results in higher consumer surplus and social welfare. In addition, it also reduces the total travel time. These results show that the joint housing and railway development performs better than separate housing and railway investment decisions, as it is more flexible in internalizing the tradeoff between housing and transport investments and revenues. The results also re-iterate that joint housing and railway development tends to improve the transport sysem performance on its own intiative, as evidenced in the total system travel time, which could be over-compensated by corresponding rental gains.

4. Concluding remarks

This paper developed a modeling framework to study the impact of joint railway and housing development on the system performance and benefit distribution among stakeholders. The problem was formulated as a mathematical programming with equilibrium constraints (MPEC). Analytical results were obtained for a single corridor in a multi-modal transport network. A monotonic relationship was found between changes in land value and railway service quality, i.e. better railway service always leads to higher land value or housing prices, which is consistent with existing empirical studies. A numerical example was performed to find the optimal joint railway and housing development. The results generally showed that joint development strategy is systematically better than treating railway and housing developments separately, in terms of overall social welfare and railway service quality, which confirm that there are benefits to be gained by exploiting the synergy between railway and housing developments.

The study results also indicate that joint railway and housing developments tend to introduce transport system improvement on its own initiative, in return for gains in housing revenue. On the other hand, if the objective is to optimize for social welfare, it is important to regulate the housing supply; the objectives of profit-seeking and social welfare can produce very different housing development patterns. One of the interesting extensions, therefore, is to study planning regulations so as to balance the interests of all the stakeholders.

In the implementation of railway projects, Hong Kong's experience demonstrates the importance of private sector participation, either in the railway or the property side, or both. Public-private partnership (PPP) can be considered as a way to transfer certain inherent risks from the public sector to the private partner who may be able to better manage the risks [20], which could be, for example, alleviated by building in recourse considerations in the development strategy [21]. One initiative of this study is to analyze the benefit distribution among different stakeholders under different types of investments in an integrated and broader analytical framework. Once this problem is formulated and understood, various extensions can be explored, such as comparing the performances with other forms of PPP. In this paper, we studied joint rail and housing development by a single profit-oriented developer, which is one type of PPP, to understand the benefit distribution among all stakeholders. Studying the implications of other types of PPP, together with an understanding of the interactions between the government and private developers, will produce further insights for developing joint railway and housing projects.

Acknowledgement

The study is supported by Public Policy Research Fund HKUST6002-PPR-11of the Research Grants Council of the HKSAR Government.

References

[1] Litman, T. (1995). Land use impact costs of transportation. World Transport Policy & Practice, 1(4), 9-16.

[2] Tang, S. and Lo, H.K. (2008). The impact of public transport policy on the viability and sustainability of mass railway transit – The Hong Kong experience. Transportation Research Part A, 42, 563- 576.

[3] Tang, S. and Lo, H.K. (2010). On the financial viability of mass transit development: The Case of Hong Kong. Transportation, 37, 299-316.

[4] Hass-Klau C., Crampton, G. and Benjari, R. (2004). Economic impact of light rail: The results of 15 urban areas in France, Germany, UK and North America, Environmental & Transport Planning.

[5] Rodríguez, D.A. and Targa, F. (2004). The value of accessibility to Bogotá's bus rapid transit system. Transport Reviews, 24, 587-610.

[6] Munoz-Raskin, R. (2007). Walking accessibility to bus rapid transit: Does it affect property values? The case of Bogotá, Colombia, 11th World Conference on Transport Research, Berkeley, California.

[7] Lo, H.K,, Tang, S., and Wang, D. Z.W. (2008). Managing the Accessibility on Mass Public Transit: the Case of Hong Kong. Journal of Transport and Land Use, 1:2, 23-49.

[8] Waugh, F.V. 1928. Quality factors influencing vegetable prices. Journal of Farm Economics, 10, 185-196.

[9] Rosen, H.S. (1974). Hedonic prices and implicit markets : Product differentiation in pure competition. Journal of Political Economy, 82, 34-55.

[10] Cervero, R. and J. Murakami. (2009). Rail and property development in Hong Kong: Experiences and extensions. Urban Studies, 46(10), 2019–2043.

[11] Martínez, F.J. (1992). The bid - choice land-use model: an integrated economic framework. Environment and Planning A, 24, 871-885.

[12] Martínez, F.J. and Henríquez, R. (2007). A random bidding and supply land use equilibrium model. Transportation Research Part B, 41, 632-651.

[13] Ma, X.S. and Lo, H.K. (2012). Modeling transport management and land use over time. Transportation Research Part B, 46, 678-709.

[14] Daganzo C.F. (1982). Unconstrained external formulation of some transportation equilibrium problems. Transportation Science 16 (3), 332-360.

[15] Yang, H. and Huang, H.J. (2004). The multi-class, multi-criteria traffic network equilibrium and systems optimum problem. Transportation Research Part B, 38 (1), 1-15.

[16] Train, K.J. (2003). Discrete choice methods with simulation. Cambridge University Press.

[17] Lo, H.K. and Chen, A. (2000). Reformulating the traffic equilibrium problem via a smooth gap function. Mathematical and Computer Modelling, 31, 179-195.

[18] Fischer, A. (1992). A special Newton-type optimization method. Optimization, 24, 269-284.

[19] Lo, H.K. and Chen A. (2000). Traffic equilibrium problem with route-specific costs: Formulation and algorithms. Transportation Research Part B, 34, 493-513
[20] Tang, S. and Lo, H.K. (2010). Assessment of Public Private Partnership Models for Mass Rail Transit - An Influence Diagram Approach. Public Transport, 2(1), 111-134.
[21] Ma, X.S. and Lo, H. (2012). Transport supply and demand management strategies with recourse considerations. Transportation Research Part E, submitted.