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Stability tests of urban physical form indicators: the case of European cities

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Abstract

Due to the lack of specific tools designed for the purpose of measuring urban physical form, adopting a variety of indicators from different disciplines is necessary. In this paper, for one of these possible dimensions of form, several indicators are proposed, and using stability tests, this study will focus on identifying the most suitable indicators for estimating degrees of evenness in the spread of urban development in 30 European cities. Our tests show that, among five proposed indicators for measuring degrees of evenness, Atkinson Index (AI) with weighted parameter (β) of 0.1 is relatively the most stable indicator.

Keywords: urban physical form; indicators; configuration; composition; evenness; stability tests;

1. Introduction

Measurement is an essential step in providing knowledge about underlying structures and processes which at work in city systems [1]. Urban form – with respect to spatial distribution pattern among land-uses – is the main way in which a city expresses itself [2], apart from its non-physical aspects, and the measurement of form supplies useful information for urban policy planning. Unlike other usual physical objects, there is an absence of specifically mathematical means used to measuring urban form. Hence, a variety of indicators from various ranges of disciplines are adopted to overcome such difficulty and, most of the time, one single dimension of form can be estimated by more than one indicators. In order to identify the most suitable indicator, we propose the stability test which is suitable for equations that spatially analyses among sub-areas (sub-area 1 versus 2 until n, sub-area 2 versus 3 until n,…, sub-area n-1 versus n) or between sub-areas and the entire area (sub-are 1versus the entire city,…, sub-area n versus the entire city) as can be seen in the research framework (Fig. 1). This study expands urban form into six conceptual spatial characteristics through the approach of landscape ecology; however, only five potential indicators of evenness will be tested in this study.

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Nomenclature

IOD	Index of Dissimilarity
GINI	Gini Coefficient
IEI	Information Entropy Index
REI	Relative Entropy Index
AI	Atkinson Index

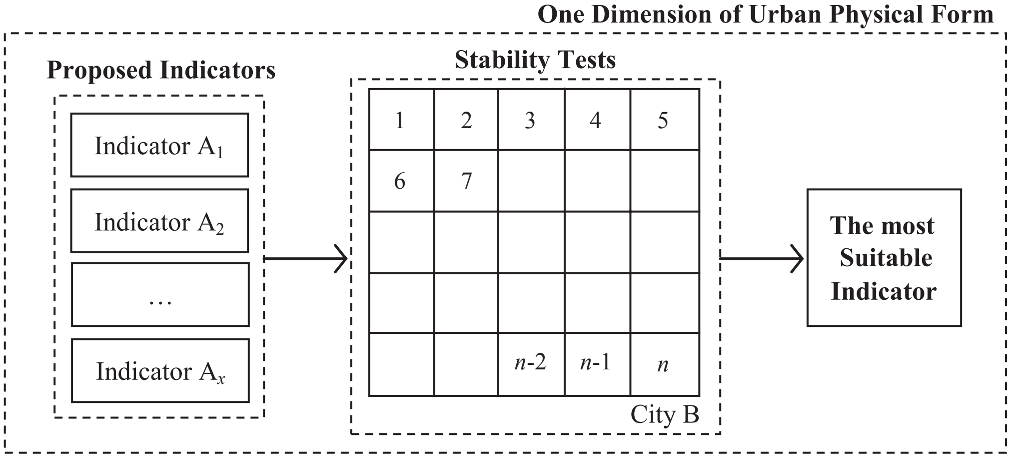


Fig. 1. The research framework

2. Dimensions of urban form

Ecology has a long tradition of application to the geographical distribution of system components including size, shape, amount, type and arrangement of land uses in spatial diverse landscapes [3]. The indicators that it throws up enable us to identify two main conceptual spatial distributions – 1) Configuration and 2) Composition [4]. Composition stresses the level of heterogeneity of land-uses while configuration refers to the geometry or shape of the urban plots and subdivisions. A series of the conceptual characteristics of land-use distribution, their interpretations and potential indicators are summarised in Table 1.

As mentioned earlier, the proposed indicators are adopted from other disciplines such as economics, physics and so on; consequently input variables are varied by contexts of investigation. While the inputs using in this study are residential, non-residential, urbanised and developable land area as the distribution of the residential land-use – one of the main services of the city – is mainly observed. Symbols and identifiers used in the equations are shown in Fig. 2.

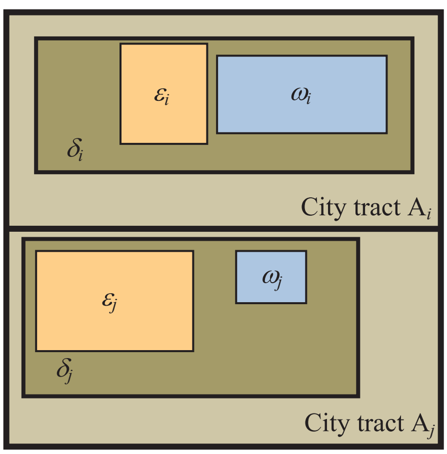
Table 1. Conceptual characteristics of land-uses distribution, measurement and potential indicator

Spatial Distribution Characteristics	Measurement	Potential Indicator
Configuration		
1) Complexity	Extent to which the city fills its two-dimensional area	1.1) Perimeter/Area Ratio 1.2) Fractal Dimension [1]
2) Clustering	Degree to which components of interests are clustered or randomly distributed	2.1) Moran's I [5] 2.2) Geary Coefficient [6] 2.3) Index of Absolute Clustering [7] 2.4) Index of Spatial Proximity [7] 2.5) Index of Relative Clustering [7]
3) Centralisation	Degree of closeness to designated centre	3.1) Absolute Centralisation [7] 3.2) Relative Centralisation [7]
Composition		
4) Evenness	Differential distribution of groups of interests among areal units	4.1) Index of Dissimilarity [7] 4.2) Gini Coefficient [7] 4.3) Atkinson Index [7, 8, 9] 4.4) Entropy Index [7] 4.5) Relative Entropy Index [6]
5) Concentration	Relative amount of physical space occupied by interested components	5.1) Delta [7] 5.2) Absolute Concentration Index [7] 5.3) Relative Concentration Index [7]
6) Exposure	Degree of potential interaction between groups of interests	6.1) Interaction Index [7] 6.2) Eta Squared Index [7]

$$\psi_i = \varepsilon_i + \omega_i$$
$$\phi_i = \varepsilon_i / \psi_i$$

$$\varepsilon = \varepsilon_i + \varepsilon_j$$
$$\omega = \omega_i + \omega_j$$
$$\psi = \psi_i + \psi_j$$
$$\delta = \delta_i + \delta_j$$

$$\psi_j = \varepsilon_j + \omega_j$$
$$\phi_j = \varepsilon_j / \psi_j$$



i, j : Tract number
δ : Total developable land area
δ_i : Area of total developable land in tract *i*
ε : Total residential land area
ε_i : Area of total residential land in tract *i*
ω : Total non-residential land area
ω_i : Area of non-residential land in tract *i*
ψ : Total developed land area
ψ_i : Area of developed land in tract *i*
ϕ : Total residential proportion
ϕ_i : Residential proportion in tract *i*

Fig. 2. Symbols and identifiers used in estimating the degree of evenness

3. An example case: dimension of evenness and its indicators

The spatial characteristics of ‘evenness’ addresses the degree to which development is equally distributed across an entire area or is agglomerated in a relatively few sub-areas. Evenness is maximised, with the value of zero, when all sub-areas have the same residential proportion of developments (ϕ_i) as the city’s average residential proportion (ϕ) as a whole while it is conversely minimized, with a value of one, when residential and non-residential developments are not found together in any sub-areas. According to the aforementioned concept, the indicators of evenness generally vary from 0 to 1. The proposed indicators for measuring evenness are summarised in Table 2.

Table 2. Indicator of evenness, measurement and equation

Indicator and Measurement	Equation	
Index of Dissimilarity The divergence from evenness by taking the weighted mean absolute difference of every unit’s proportion of residential and developed land area from the city’s average proportion	$\mathrm{IOD} = \sum_{i=1}^{n} \frac{\psi_i \lvert \phi_i - \phi \rvert}{2\psi\phi(1-\phi)}$	(1)
Gini Coefficient The mean absolute difference between proportion of residential and developed land weighted across all pairs of areal units	$\mathrm{GINI} = \sum_{i=1}^{n} \frac{\psi_i \psi_j \lvert \phi_i - \phi_j \rvert}{\psi^2 \phi(1-\phi)}$	(2)
Atkinson Index Resemble to the concept of Gini Coefficient but allow the researcher to specify the weight (β) on areal units, over or below the city-wide residential proportion, that contribute to the increments of unevenness	$\mathrm{AI} = 1 - \left(\frac{\phi}{1-\phi} \cdot \left\lvert \sum_{i=1}^{n} \frac{(1-\phi_i)^{1-\beta} \phi_i^{\beta} \psi_i}{\phi\psi} \right\rvert^{\frac{1}{1-\beta}} \right)$	(3)
Information Entropy Index The weighted average deviation of each unit’s entropy from the city-wide entropy, expressed as a fraction of the city’s total entropy	Information Entropy Index: $\mathrm{IEI} = \sum_{i=1}^{n} \frac{\psi(\mathrm{E} - \mathrm{E}_i)}{\mathrm{E} \cdot \psi}$	(4)
	Unit's Entropy: $\mathrm{E}_i = \left(\frac{\phi_i}{\sum_{i=1}^{n} \phi_i} \cdot \log \frac{\sum_{i=1}^{n} \phi_i}{\phi_i} \right) + \left(\left(1 - \frac{\phi_i}{\sum_{i=1}^{n} \phi_i} \right) \cdot \log \frac{1}{1 - \frac{\phi_i}{\sum_{i=1}^{n} \phi_i}} \right)$	(4.1)
	City's Entropy: $\mathrm{E} = \left(\frac{\phi}{\sum_{i=1}^{n} \phi_i} \cdot \log \frac{\sum_{i=1}^{n} \phi_i}{\phi} \right) + \left(\left(1 - \frac{\phi}{\sum_{i=1}^{n} \phi_i} \right) \cdot \log \frac{1}{1 - \frac{\phi}{\sum_{i=1}^{n} \phi_i}} \right)$	(4.2)
Relative Entropy Index Similar to Shannon’s entropy with the elimination of the effect of number of sub-areas	$\mathrm{REI} = \frac{1}{\log N} \cdot \sum_{i=1}^{n} \left(\frac{\phi_i}{\sum_{i=1}^{n} \phi_i} \cdot \log \frac{\sum_{i=1}^{n} \phi_i}{\phi_i} \right)$	(5)

The focal concept of evenness measurement is the transfer principle which can obviously be seen in the structure of equations (1) and (2). A lower degree of evenness means that lesser proportion of developments have to relocate themselves so as the entire city could achieve an even distribution of different land-uses, while a development with a higher degree provides a sense of developments agglomerating in few sub-units. Another explanation can be expressed through the Lorenz curve [7,9] which plots the cumulative percentage of residential land against the cumulative percentage of non-residential land across areal units ordered from smallest to largest amount. The degree of evenness is represented by the maximum vertical distance between the diagonal line of evenness and the generated curve.

The notion of entropy which is related to spatial distribution of a range of phenomena [10] is also introduced and applied in the estimation of the degree of evenness. The characterisation of information entropy most commonly used in spatial analysis research is arisen from Shannon (1948) [11] and its formula can be written as

$$H(r) = -\sum_i p_i \cdot \ln p_i \tag{6}$$

where r is a discrete random variable and p_i is the probability of the event occurring in r_i. The most probable state is the event that even distribution of land-uses takes place and gives maximum entropy of $\ln N$. The reason why $1/\ln N$ takes part in calculating REI is to adjust its maximum value to 1. As can be seen in equation (4.1), (4.2) and (5), the ratio between sub-area's residential proportion and the sum of every sub-area's residential proportion can be applied to this calculation since such ratio perfectly reflects the concept of probability which serves as one of the main ideas of entropy.

For Atkinson Index, β is a shape parameter that determines how to weight the increments to unevenness contributed by different divisions of the Lorenz curve. For $0<\beta<.5$, areal units where $\phi_i<\phi$ contribute more to unevenness; whereas for $.5<\beta<1$, areal units where $\phi_i>\phi$ give larger promotion of unevenness. When $\beta = .5$, units of residential proportion over- and underrepresentation supply equivalently in estimating the evenness index. With values of β specified between zero to one, three different values of β – .1, .5 and .9 – are tested in this study.

4. Stability test of evenness's indicators

The nature of this analysis is a comparison between sub-areas with the whole city and among its sub-areas that can be achieved by superimposing the land-use map as a grid square tessellation of the urban landscape. In calculating any indicators, changing of grid size normally gives different values; despite these changes to a smaller or bigger grid size, stability in the direction of changing values of the indicators – persistently getting smaller or larger – is expected so such indicator could be considered as a reliable index. For example, the value of the GINI for 'city A' gets smaller when changing grid size of analysis from 1x1 to 2x2 km^2, so lower degree of GINI is expected when applying any grid sizes that is greater than 2x2 km^2 to the same case study.

Land cover maps, for the year 2000, from COoRdination of INformation on the Environment (CORINE), the European Environment Agency (EEA) is the main data source used to create base-maps of 30 European cities served as case studies in this analysis. Five different grid sizes – 1x1, 2x2, 3x3, 4x4 and 5x5 kilometre grid squares – are overlaid on every base-map to test the research assumption. Two examples of base-maps are presented in Fig. 3. Then, the degree of IOD, GINI, AI, IEI and REI are calculated using MATLAB© scripting.

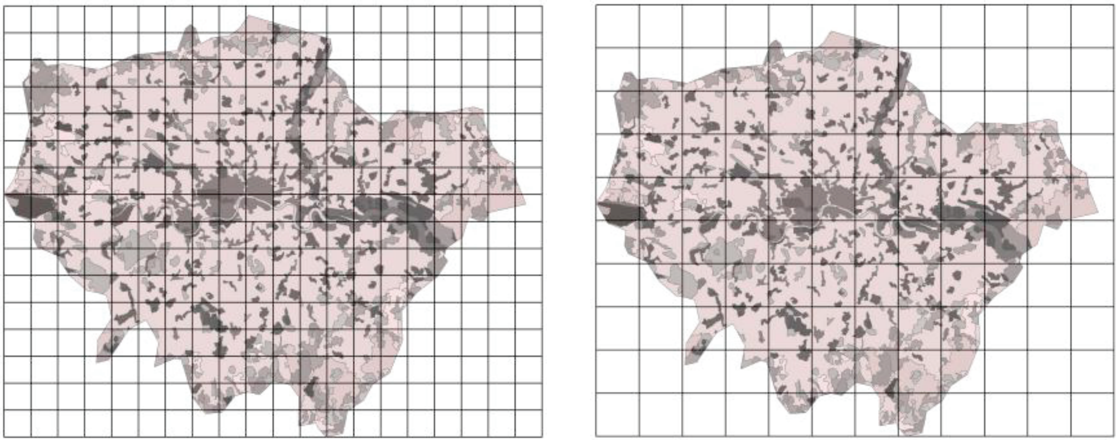


Fig. 3. (a) Land cover map of London superimposed by 3x3 kilometre grid square tessellation (not to scale); (b) Land cover map of London superimposed by 5x5 kilometre grid square tessellation (not to scale)

As the pattern of stability can be discovered easier, degree of an indicator calculated by using 5 different grid sizes are plotted in the same graph. The y-axis is the degree of an indicator while each individual case study stands on the x-axis; so it should be noted that different points or distance from zero on horizontal axis do not represent any values. For ease of interpretation, case studies that analysed by using the same grid size are linked altogether and, then form a continuous line. Consequently, 5 different colour-lines are displayed in each graph. Results are shown in Fig. 4 to 7.

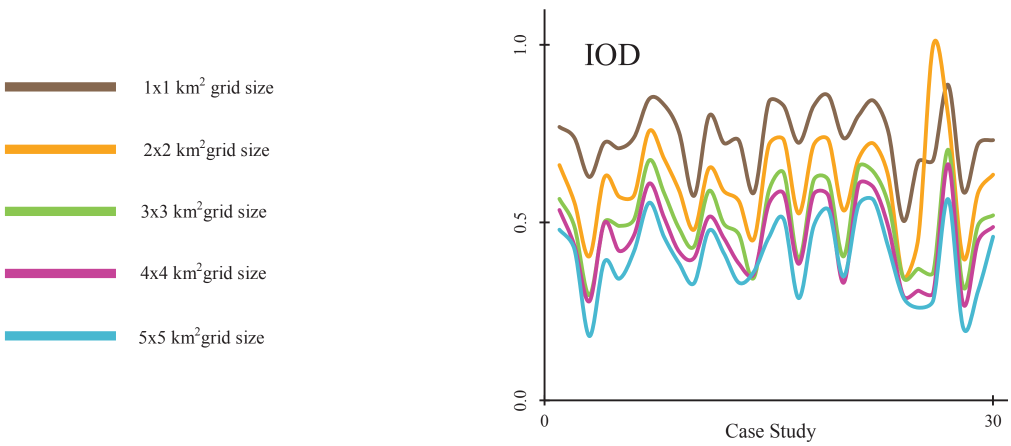


Fig. 4. (a) Legend applied to Fig. 4 to Fig.7 and Fig. 9; (b) Stability Test of IOD

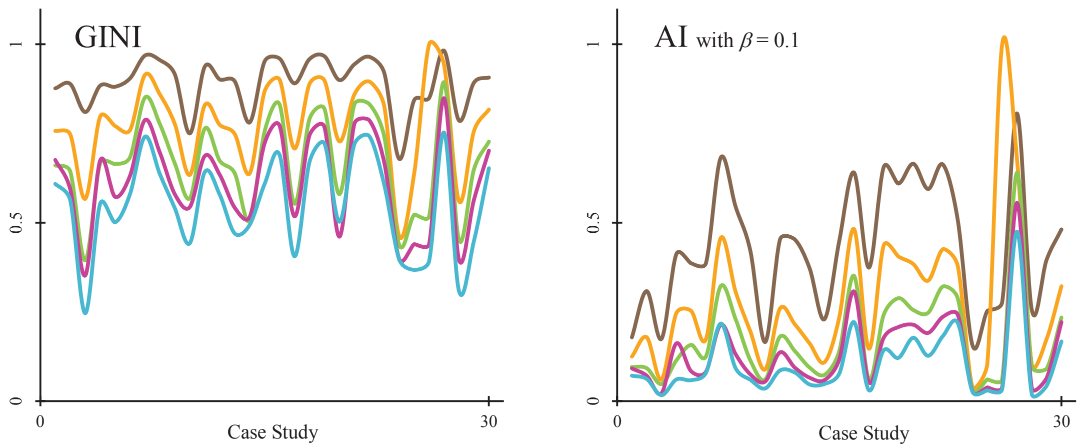


Fig. 5. (a) Stability Test of GINI; (b) Stability Test of AI with $\beta = 0.1$

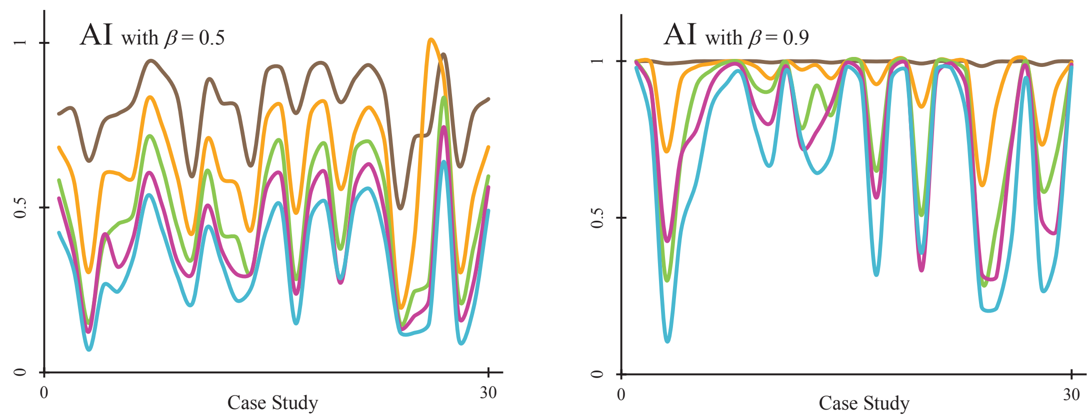


Fig. 6. (a) Stability Test of AI with $\beta = 0.5$; (b) Stability Test of AI with $\beta = 0.9$

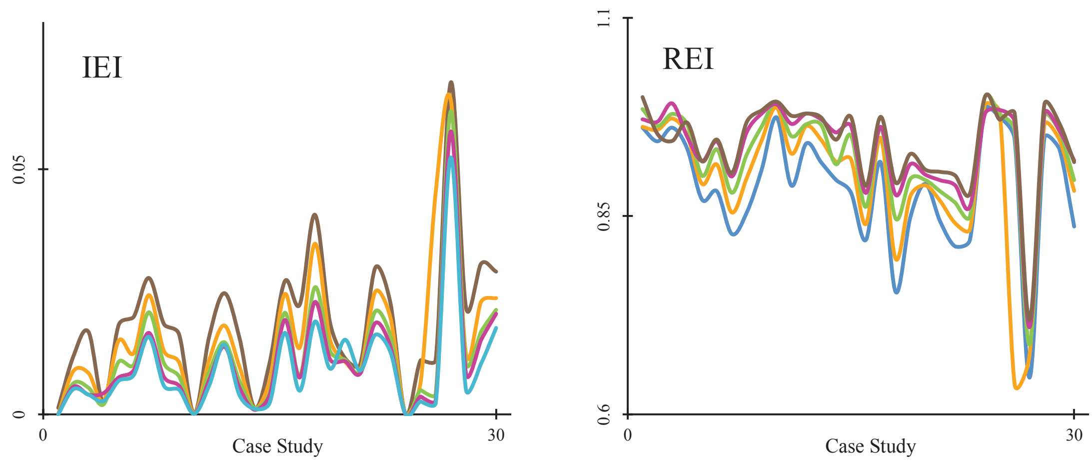


Fig. 7. (a) Stability Test of IEI; (b) Stability Test of REI

5. Analysis and conclusion

Results displayed in Fig. 4 to 7 help assuring the research assumption as, at the overall picture, the degrees of evenness's indicators vary in the same direction when decreasing or increasing grid sizes of analysis. All tested indicators, except REI, get smaller in their degrees when changing grid sizes of analysis from smaller to bigger.

Applying five different grid sizes in estimation of an indicator, one single case study provides 5 degrees of evenness and sorting them, ascending or descending in relation to grid sizes applied to the analysis, allows 10 comparative pairs of data benefitting in judging the stability of such indicator (Fig. 8). From data set of an estimated indicator, every possible pair-data from 5 calculated data in one case study are verified with the changing-degree pattern of its entire dataset. In testing 30 case studies, 300 pairs of data can be compared in one tested indicator. Numbers of pair-data that violate its pattern of changing-degree (Ω) are counted and its summation is compared to other indicators. More numbers of pair-data that fail to agree with the changing-degree pattern of its entire dataset implies less stability in such indicator relative to others. From Table 3, it is shown that AI with $\beta = 0.1$ is relatively the least fluctuant indicator, with $\Omega = 3$.

Another topic deserved discussion is identifying the most suitable grid size. Using too small grid size in the analysis might fail to capture the overall pattern of land-use distribution while applying too large grid size might overlook the fine details of urban phenomenon. Consistency in the shifted degree of the entire series of data, when changing grid size, is exercised as a criterion in searching for the optimal gird size and such criterion satisfied by less intersection between two lines of data set. Ideally, two lines that run into parallel perfectly meet this standard. In the case of AI with $\beta = 0.1$, lines of data set using 2x2 and 3x3 kilometre grid square tessellations run without crossing each other (Fig. 9a). Since greater numbers of cells are created from employing a smaller grid size which implies more work in computation, larger grid size is more preferable. In brief conclusion, in measuring the degree of evenness, the most suitable indicator and grid size are Atkinson Index (AI) with $\beta = 0.1$ and grid size of 3x3 km^2.

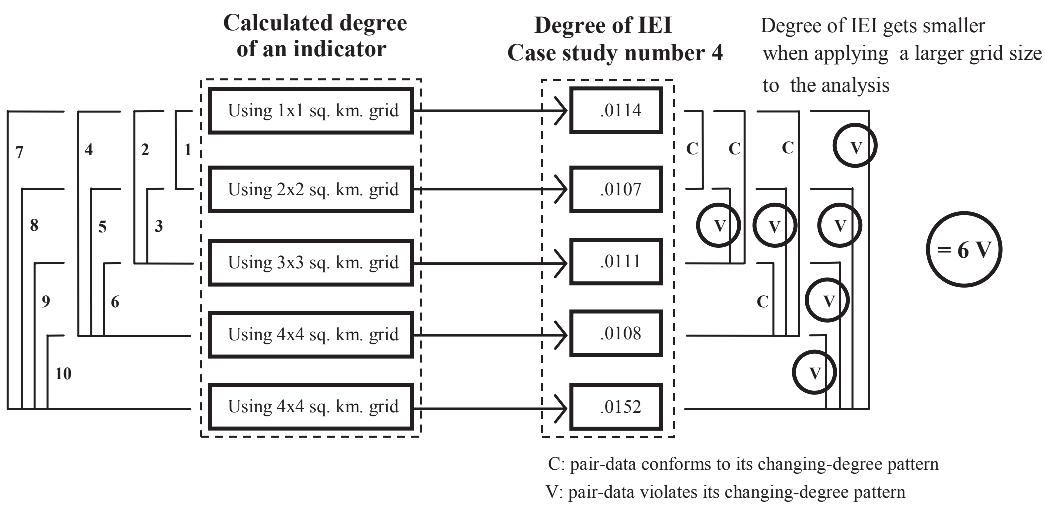


Fig. 8. Methodology in identifying numbers of pair-data that violate its pattern of changing-degree (Ω) and example of calculated degree of REI on the case study number 4

Table 3. Numbers of pair-data that violate its pattern of changing-degree (Ω) in each tested indicator

Indicator	**Numbers of pair-data that violate its pattern of changing-degree (Ω)**
IOD	5
GINI	5
AI with $\beta = 0.1$	3
AI with $\beta = 0.5$	4
AI with $\beta = 0.9$	7
IEI	17
REI	19

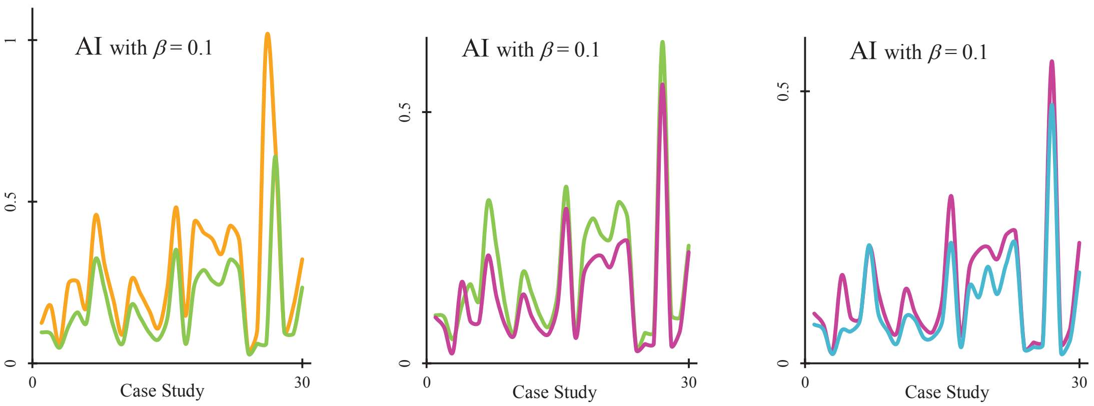


Fig. 9. Comparisons of calculated Atkinson Index (AI) with $\beta = 0.1$between grid size (a) 2x2 km^2 vs. 3x3 km^2; (b) 3x3 km^2 vs. 4x4 km^2 and (c) 4x4 km^2 vs. 5x5 km^2 to identify the optimal grid size

It should be noticed that case study number 26 gives a jumping value shown in almost every tested indicator analysed with 2x2 square-kilometer grids. Further qualitative study or outlier analysis in such case is worth investigated in future study. Stability tests using more case studies and applied in other contexts are advised for future works, moreover, greater grid sizes should be tried out and different locations in overlaying the grid square tessellation upon the map are important issues that this study has not yet covered.

References

[1] Lam NS, Cola LD. *Fractals in Geography*. New Jersey: PTR Prentice-Hall A Simon & Schuster; 1993.

[2] Steadman P, Bruhns HR, Holtier S, Gakivic Brarislav. A Classification of Built Forms. *Environment and Planning B: Planning and Design* 2000; **27**(1): 73-91.

[3] Turner MG. Landscape Ecology: The Effect of Pattern on Process. *Annual Review of Ecological Systems* 1989; **20**: 171-97.

[4] Torrens PM, Alberti M. *Measuring Sprawl*. CASA Working Paper 2000.

[5] Malpezzi S, Guo WK. *Measuring "sprawl": Alternative Measures of Urban Form in U.S. Metropolitan Areas*. 2001.

[6] Tsai YH. Quantifying Urban Form: Compactness versus "Sprawl." *Urban Studies* 2005; **42**(1): 141-61.

[7] Massey SD, Denton AN. The Dimensions of Residential Segregation. *Social Forces* 1988; **67**(2): 281-315.

[8] Allison PD. Measure of Inequality. *American Sociological Review* 1978; **43**(6): 865-80.

[9] Schwartz J, Winship C. The Welfare Approach to Measuring Inequality. *Sociological Methodology* 1980; **11**: 1-36.

[10] Batty M. Entropy and Spatial Geometry. *Area* 1972; **4**(4): 230-36.

[11] Shannon CE. A Mathematical Theory of Communication. *The Bell System Techical Journal* 1948; **27**: 379-423, 623-56.

Appendix A. Degrees of Atkinson Index (AI), with β = 0.1, of 30 European cities calculated by applying 5 different grid sizes†

Case Study No.	Country	City	Atkinson Index with $\beta = 0.1$				
			1x1 km^2	2x2km^2	3x3 km^2	4x4 km^2	5x5 km^2
1	Austria	Graz	0.1786	0.1246	0.0956	0.0913	0.0711
2	Austria	Vienna	0.3080	0.1788	0.0920	0.0710	0.0619
3	Belgium	Brussels	0.1744	0.0604	0.0478	0.0219	0.0166
4	Denmark	Copenhagen	0.4118	0.2462	0.1138	0.1615	0.0604
5	Finland	Helsinki	0.3862	0.2527	0.1575	0.0836	0.0585
6	France	Lyon	0.3864	0.1734	0.1249	0.0822	0.0790
7	France	Marseille	0.6823	0.4570	0.3223	0.2142	0.2171
8	France	Nantes	0.5451	0.3068	0.2299	0.1316	0.0922
9	France	Paris	0.4206	0.1948	0.1138	0.0747	0.0595
10	Germany	Berlin	0.1655	0.0885	0.0601	0.0547	0.0353
11	Germany	Frankfurt	0.4528	0.2593	0.1806	0.1365	0.0858
12	Germany	Hamburg	0.4529	0.2153	0.1403	0.0937	0.0800
13	Germany	Dusseldorf	0.3721	0.1615	0.0938	0.0654	0.0464
14	Germany	Munich	0.2293	0.1087	0.0733	0.0587	0.0479
15	Germany	Stuttgart	0.4486	0.2346	0.1399	0.1164	0.0769
16	Greece	Athens	0.6404	0.4818	0.3510	0.3071	0.2212
17	Italy	Milan	0.3740	0.1466	0.0607	0.0526	0.0303
18	Italy	Bologna	0.6556	0.4361	0.2415	0.1774	0.1423
19	Italy	Rome	0.6097	0.4050	0.2883	0.2075	0.1211
20	Netherlands	Amsterdam	0.6651	0.3839	0.2551	0.2138	0.1774
21	Spain	Barcelona	0.5943	0.3375	0.2474	0.1910	0.1268
22	Spain	Madrid	0.6629	0.4256	0.3211	0.2361	0.1823
23	Sweden	Stockholm	0.5075	0.3787	0.2880	0.2419	0.2191
24	United Kingdom	Glasgow	0.1556	0.0369	0.0314	0.0265	0.0259
25	United Kingdom	London	0.2526	0.1061	0.0605	0.0382	0.0299
26	United Kingdom	Manchester	0.2774	1.0000	0.0616	0.0391	0.0338
27	United Kingdom	Newcastle	0.8063	0.6775	0.6400	0.5552	0.4750
28	Czech Republic	Prague	0.2555	0.1020	0.0970	0.0367	0.0197
29	Hungary	Budapest	0.3960	0.1738	0.0905	0.0644	0.0427
30	Poland	Cracow	0.4811	0.3218	0.2342	0.2214	0.1670

† Web link to full appendix: http://www.casa.ucl.ac.uk/STGIS Boontore_Appendix.pdf