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ORIGINAL ARTICLE

Approximation of Rough Soft Set and Its Application to Lattice

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Abstract The approximation of soft set is presented in modified soft rough (MSR) approximation space in this paper, i.e., approximation of an information system with respect to another information one. Besides, the concept of rough soft set is introduced in a modified soft rough approximation space. Various properties are studied like subset, union, intersection on rough soft set with some propositions presented on rough soft set. Moreover, the measure of roughness of soft set is defined in MSR-approximation space and the order relation is introduced on soft set. Furthermore, lattice theory is studied in the MSR-approximation space under a modified rough soft environment. Finally, some realistic examples are considered to usefulness and illustrate of the paper.

Keywords Soft set · Rough set · MSR-approximation space · Lattice

1. Introduction

In recent years, scientists, engineers and mathematicians have shown great interest in uncertainty as it found many fields like decision making, engineering, environmental science, social sciences, and medical science etc. Probability theory, fuzzy set theory

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[1], rough set theory [2, 3] and other mathematical tools have been used successfully to describe uncertainty, but each of these theories has its inherent difficulties [4, 5]. Consequently, in 1999, Molodtsov [4] proposed a novel concept for modeling vagueness and uncertainty called soft set theory which is free from difficulties affecting existing methods. The operations of soft set defined by Maji et al. [6] and redefined by Cağman and Enginoglu [7]. Recently, the properties and applications on soft set have been studied increasingly [8-10]. Rough set was initiated by Pawlak [3] as a formal tool for modeling and processing incomplete information in information system. Every rough sets are associated with two crisp sets, called lower and upper approximations and viewed as the sets of elements which certainly and possibly belong to the set. Pawlak's rough set is mainly based on equivalence relation. But, in practical, it is very difficult to find an equivalence relation among the elements of a set. So, some other general relations such as tolerance ones and dominance ones are considered to define rough set models [11, 12]. It has been successfully applied to knowledge discovery, decision analysis, signal processing, mereology and many other fields [13, 14]. Soft set theory and rough set theory are treated as mathematical tools to deal with uncertainty. A connection between these two has been made by Feng et al. [15] and introduced the notion of soft rough set. In this model, they described the parameterize subset on the universe of discourse. As a result, some unusual situations have occurred, like upper approximation of a non-empty set may be empty. Upper approximation of a subset may not contain the set which does not occur in classical rough set theory. To overcome these difficulties, Shabir et al. [14] redefined a soft rough set model called MSR set.

In this paper, we study the approximations of an information system with respect to another information ones. We approximate a soft set with respect a modified soft rough approximation space and introduce the notion of rough soft set. Here, we endeavor to establish link between soft set and rough set in connection with an application in lattice. Also, we introduce the concept of measure of roughness in a soft set and consequently some propositions and examples are presented here.

2. Preliminaries

Definition 2.1 *An information system (or a knowledge representation system) is a pair (U, A) of non-empty finite sets U and A where U is a set of objects and A is a set of attributes; each attribute $a \in A$ is a function $a : U \to V_a$, where V_a is called set of values of attribute a.*

Let U be a non-empty set of universe and R be an equivalence relation on U. The pair (U, R) is called Pawlak's approximation space. The equivalence relation R is often called indiscernibility relation and related to an information system. An indiscernibility relation $R = I(B)$, $B \subseteq A$ is defined as:

$$(x, y) \in I(B) \Leftrightarrow a(x) = a(y), \quad \forall\ a \in B,$$

where $x, y \in U$, and $a(x)$ denotes the value of attribute a for object x.

Using this indiscernibility relation, one can define the following operations as:

$$A_\star(X) = \{x \in U \mid [x]_R \subseteq X\} \text{ and } A^\star(X) = \{x \in U \mid [x]_R \cap X \neq \psi\}.$$

For any $X \subseteq U$, $A_\star(X)$ and $A^\star(X)$ are called lower and upper approximations of X respectively. If $A_\star(X) \neq A^\star(X)$, then X is called the rough set in the approximation space (U, R). The difference $A^\star(X) - A_\star(X)$ is called boundary region of X and is treated as the area of uncertainty.

Let U be an initial universe of objects and E be the set of parameters and $A \subseteq E$. $P(U)$ is the power set of U.

Definition 2.2 *A pair $S = (F, A)$ is called a soft set over U, where $F : A \to P(U)$ denotes a set valued mapping.*

Definition 2.3 [14] *Let (F, A) be a soft set over U, where F is a mapping from A to $P(U)$, i.e., $F : A \to P(U)$, where $P(U)$ is the power set of U. Let $\psi : U \to P(A)$ be another mapping defined as $\psi(x) = \{a \mid x \in F(a)\}$. Then the pair (U, ψ) is called MSR approximation space and for any $X \subseteq U$, lower MSR-approximation and upper MSR-approximation respectively are defined as follows:*

$$\underline{X}_\psi = \{x \in U \mid \psi(x) \neq \psi(y)\ \forall\ y \in X^c\},$$

where $X^c = U - X$,

$$\overline{X}_\psi = \{x \in U \mid \psi(x) = \psi(y) \text{ for some } y \in X\}.$$

If $\underline{X}_\psi \neq \overline{X}_\psi$, then X is said to be modified soft rough set.

Example 2.1 Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be the set of schools considered as universal set and an attribute set $A = \{e_1, e_2, e_3, e_4\}$. Here e_1 denotes good location, e_2 denotes sufficient teachers, e_3 denotes good maintenance of discipline, e_4 denotes good relation in teacher-student. Let the soft set (F, A) over U is given by the following table:

Table 1: Table represents the soft set.

	e_1	e_2	e_3	e_4
u_1	1	1	0	0
u_2	0	0	1	1
u_3	1	1	1	0
u_4	0	1	1	1
u_5	0	1	0	1

Here 1 and 0 denote 'yes' and 'no' respectively. Then from the definition of MSR set, $\psi : U \to P(A)$ is defined as follows:

$$\psi(u_1) = \{e_1, e_2\}; \psi(u_2) = \{e_2\}; \psi(u_3) = \{e_4\}; \psi(u_4) = \{e_1, e_3\} = \psi(u_5).$$

Let $X = \{u_1, u_3, u_5\}$. Therefore for the MSR-approximation space (U, ψ), we can write

$$\underline{X}_\psi = \{u_1, u_3\} \text{ and } \overline{X}_\psi = \{u_1, u_3, u_4, u_5\}.$$

Clearly, $\underline{X}_\psi \neq \overline{X}_\psi$, so X is a modified soft rough set.

3. Rough Soft Set

In this section, we introduce the notion of rough soft set in modified soft rough approximation space.

Definition 3.1 *Let (F, A) be a soft set over U and (U, ψ) be an MSR-approximation space with respect to A. Let (G, B) be another soft set over U. (G, B) is said to be rough soft set with respect to a parameter $e \in B$ if $\underline{G(e)}_\psi \neq \overline{G(e)}_\psi$, (G, B) is said to be a full rough soft set or a simply rough one if $\underline{G(e)}_\psi \neq \overline{G(e)}_\psi$ $\forall$ $e \in B$ and we denote it by $RsG(e_B)$. We denote rough soft set with respect to e by $RsG(e) = (\underline{G(e)}_\psi, \overline{G(e)}_\psi)$.*

Example 3.1 Considering a universal set of batsman $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ and an attribute set $A = \{e_1, e_2, e_3\}$ where e_1 denotes Bold out, e_2 denotes Catch out, e_3 denotes LBW. Let (F, A) be a soft set representing the record of the player given by the following table:

Table 2: Table represents the data on information system.

	e_1	e_2	e_3
u_1	1	1	1
u_2	1	0	1
u_3	0	1	1
u_4	1	0	1
u_5	0	0	1
u_6	1	0	0
u_7	1	1	0
u_8	0	0	1

Here 1 and 0 denotes 'yes' and 'no' respectively. Then from the definition of MSR set, $\psi : U \to P(A)$ is defined as follows:

$\psi(u_1) = \{e_1, e_2, e_3\};$ $\psi(u_2) = \{e_1, e_3\};$ $\psi(u_3) = \{e_2, e_3\};$ $\psi(u_4) = \{e_1, e_3\};$
$\psi(u_5) = \{e_3\};$ $\psi(u_6) = \{e_1\};$ $\psi(u_7) = \{e_1, e_2\};$ $\psi(u_8) = \{e_2\}.$

Let (G, B) be another soft set defined as

$G(e_1) = \{u_1, u_2, u_4, u_6, u_7\};$ $G(e_2) = \{u_1, u_3, u_4, u_6\};$
$G(e_3) = \{u_2, u_3, u_5, u_6, u_7, u_8\};$ $G(e_4) = \{u_1, u_2, u_3, u_5, u_6, u_7\},$

where e_1 denotes Bold out, e_2 denotes Catch out, e_3 denotes LBW and e_4 denotes Run out. Lower MSR-approximation set and upper MSR-approximation set of (G, B) are

$\underline{G(e_1)}_\psi = \{u_1, u_2, u_4, u_6, u_7\};$ $\overline{G(e_1)}_\psi = \{u_1, u_2, u_4, u_6, u_7\};$
$\underline{G(e_2)}_\psi = \{u_1, u_3, u_6\};$ $\overline{G(e_2)}_\psi = \{u_1, u_2, u_3, u_4, u_6\};$
$\underline{G(e_3)}_\psi = \{u_3, u_5, u_6, u_7, u_8\};$ $\overline{G(e_3)}_\psi = \{u_2, u_3, u_4, u_5, u_6, u_7, u_8\};$
$\underline{G(e_4)}_\psi = \{u_1, u_3, u_5, u_6, u_7\};$ $\overline{G(e_4)}_\psi = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}.$

Clearly, (G, B) is a rough soft set with respect to parameters e_2, e_3 and e_4.

Proposition 3.1 *Let (F, A) be a soft set over U and (U, ψ) be an MSR approximation* *space with respect to A. Let* (G_1, B_1), (G_2, B_2) *be two rough soft sets. Then*

$$H(e) = \begin{cases} G_1(e), & \text{if } e \in B_1 - B_2, \\ G_2(e), & \text{if } e \in B_2 - B_1, \\ G_1(e) \cup G_2(e), & \text{if } e \in B_1 \cap B_2 \end{cases}$$

is a rough soft set if $B_1 \cap B_2 = \phi$.

Proposition 3.2 *Let* (F, A) *be a soft set over* U *and* (U, ψ) *be an MSR approximation space with respect to A. Let* (G_1, B_1), (G_2, B_2) *be two rough soft sets. Then* $\forall e \in B_1 \cap B_2$, $H(e) = G_1(e) \cap G_2(e)$ *is a rough soft set.*

Definition 3.2 *Let* (F, A) *be a soft set over* U *and* (U, ψ) *be an MSR approximation space with respect to A. Let* (G_1, B_1), (G_2, B_2) *be two rough soft sets.* (G_1, B_1) *is said to be rough soft subset of* (G_2, B_2) *if*

(i) $B_1 \subseteq B_2$, *and*

(ii) $\forall e \in B_1$, $\underline{G_1(e)}_{\psi} = \underline{G_2(e)}_{\psi}$ *and* $\overline{G_1(e)}_{\psi} = \overline{G_2(e)}_{\psi}$.

We write $(G_1, B_1) \sqsubseteq (G_2, B_2)$, *where* $\sqsubseteq$ *denotes soft rough subset.*

Definition 3.3 *The union of rough soft sets* $RsG(e_1)$ *and* $RsG(e_2)$ *with respect to the parameters* e_1 *and* e_2 *respectively in MSR-approximation space* (U, ψ) *is denoted by* $RsG(e_1) \sqcup RsG(e_2)$ *and is defined as* $RsG(e_1) \sqcup RsG(e_2) = (\underline{G(e_1)}_{\psi} \cup \underline{G(e_2)}_{\psi}, \overline{G(e_1)}_{\psi} \cup \overline{G(e_2)}_{\psi})$.

The union of rough soft sets $RsG(e_A)$ *and* $RsG(e_B)$ *is defined as* $RsG(e_A) \sqcup RsG(e_B) = (\underline{G(e)}_{\psi} \cup \underline{G(f)}_{\psi}, \overline{G(e)}_{\psi} \cup \overline{G(f)}_{\psi})$ *for all* $e \in A$ *and* $f \in B$.

Definition 3.4 *The intersection of rough soft sets* $RsG(e_1)$ *and* $RsG(e_2)$ *with respect to parameters* e_1 *and* e_2 *respectively in MSR-approximation space* (U, ψ) *is denoted by* $RsG(e_1) \sqcap RsG(e_2)$ *and is defined as* $RsG(e_1) \sqcap RsG(e_2) = (\underline{G(e_1)}_{\psi} \cap \underline{G(e_2)}_{\psi}, \overline{G(e_1)}_{\psi} \cap \overline{G(e_2)}_{\psi})$.

The intersection of rough soft sets $RsG(e_A)$ *and* $RsG(e_B)$ *is defined as* $RsG(e_A) \sqcap RsG(e_B) = (\underline{G(e)}_{\psi} \cap \underline{G(f)}_{\psi}, \overline{G(e)}_{\psi} \cap \overline{G(f)}_{\psi})$ *for all* $e \in A$ *and* $f \in B$.

Proposition 3.3 *Let* (G, B) *be an soft set over* U *and* (U, ψ) *be an MSR-approximation space. Then set* $(RsG(e), \sqcup, \sqcap)$, $\forall e \in B$ *together with* (U, U) *and* (ϕ, ϕ) *form a lattice where the order relation* $\subseteq$ *is defined as* $RsG(e_1) \subseteq RsG(e_2) \Rightarrow \underline{G(e_1)}_{\psi} \subseteq \underline{G(e_2)}_{\psi}$ *and* $\overline{G(e_1)}_{\psi} \subseteq \overline{G(e_2)}_{\psi}$.

Example 3.2 Considering the Example 3.1, for simplicity, we denote the subset of U, other than ϕ and U by sequence of numeric suffices. For example u_1, u_2, u_4, u_6, u_7 is written as 12467. The Hasse diagram for Example 3.1 of lattice under the soft rough set is given in Figure 1.

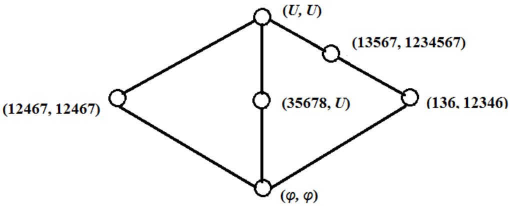

Fig. 1 Lattice under rough soft set

Proposition 3.4 *Let (G_1, B_1) be a soft subset of (G_2, B_2). If (G_2, B_2) is a rough soft set, then (G_1, B_1) is rough soft subset of (G_2, B_2).*

Proof Since (G_1, B_1) is soft subset of (G_2, B_2), therefore $G_1(e) = G_2(e)$ for all $e \in B_1$. Therefore $\underline{G_1(e)}_\psi = \underline{G_2(e)}_\psi$ and $\overline{G_1(e)}_\psi = \overline{G_2(e)}_\psi$ for all $e \in B_1$. Hence (G_1, B_1) is rough soft subset.

Definition 3.5 *Let (F, A) be a soft set over U and (U, ψ) be an MSR approximation space with respect to A. Let (G, B) be another soft set over U. We define*

$\underline{G(e_1)}_\psi = \underline{G(e_2)}_\psi \Leftrightarrow G(e_1) \simeq G(e_2)$,

$\overline{G(e_1)}_\psi = \overline{G(e_2)}_\psi \Leftrightarrow G(e_1) \simeq G(e_2)$,

$\underline{G(e_1)}_\psi = \underline{G(e_2)}_\psi$ *and* $\overline{G(e_1)}_\psi = \overline{G(e_2)}_\psi \Leftrightarrow G(e_1) \approx G(e_2)$.

These binary relations are called lower rough soft, upper rough soft and rough soft equal relations respectively.

Proposition 3.5 *The rough soft equal relation is an equivalence one.*

4. Measure of Roughness of Soft Set

In this section, we study the measure of roughness of a soft set with respect to an MSR-approximation space.

Definition 4.1 *Let (F, A) be a soft set over U and (U, ψ) be an MSR-approximation space. Let (G, B) be another soft set over U. Measure of roughness of (G, B) with respect to parameter $e \in B$ is denoted by $R_{G(e)}$ and is defined as follows:*

$$R_{G(e)} = \frac{|\underline{G(e)}_\psi|}{|\overline{G(e)}_\psi|}.$$

Clearly, $0 \le R_{G(e)} \le 1$. Now, we define binary relation '≡' on soft set (G, B) as $G(e_1) \equiv G(e_2)$ if and only if $R_{G(e_1)} = R_{G(e_2)}$ for $e_1, e_2 \in B$.

Proposition 4.1 *'≡' is an equivalence relation on (G, B). The partition $[G(e)]_\equiv$ has a strict order in its element.*

Proof The measure of roughness of all members of a class is the same. Therefore, each class is characterized by a unique number belonging to interval $[0, 1]$. So there is a strict order among these classes.

Proposition 4.2 *(G, B) forms a chain by the order relation ≡.*

Proof Since relation '≡' partitions the soft set and the partition has a strict order relation, therefore the soft set forms a chain.

Example 4.1 Suppose (F, A) is a soft set over $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ and the set of parameter is $A = \{e_1, e_2, e_3, e_4\}$, where e_1 denotes under stress, e_2 denotes young age, e_3 denotes drug addicted, e_4 denotes healthy. The soft set (F, A) is given in Table 3.

Table 3: Table represents the data on information system.

	e_1	e_2	e_3	e_4
u_1	0	1	0	1
u_2	0	1	0	1
u_3	0	0	0	1
u_4	0	0	0	0
u_5	1	0	0	0
u_6	0	0	0	1

Now (U, ψ) is the modified soft rough approximation space where $\psi : U \to P(A)$ is defined as $\psi(u_1) = \{e_2, e_4\}$; $\psi(u_2) = \{e_2, e_4\}$; $\psi(u_3) = \{e_4\}$; $\psi(u_4) = \{\}$; $\psi(u_5) = \{e_1\}$; $\psi(u_6) = \{e_4\}$. Let (G, B) be another soft set over U and the set of parameter $B = \{e_1, e_2, e_3, e_4\}$, where e_1 denotes smokers; e_2 denotes smokers and drinkers; e_3 denotes men; e_4 denotes people live in city. The soft set (G, B) is given by the following table:

Table 4: Table represents the data on information system.

	e_1	e_2	e_3	e_4
u_1	1	0	1	1
u_2	0	0	1	1
u_3	0	0	1	1
u_4	0	1	0	0
u_5	0	0	0	0
u_6	0	0	0	0

Lower MSR-approximation and upper MSR-approximation of (G, B) are $\underline{G(e_1)}_\psi = \{\}$; $\overline{G(e_1)}_\psi = \{u_1, u_2\}$; $\underline{G(e_2)}_\psi = \{u_4\}$; $\overline{G(e_2)}_\psi = \{u_4\}$; $\underline{G(e_3)}_\psi = \{u_3, u_4, u_5, u_6\}$; $\overline{G(e_3)}_\psi = \{u_1, u_2, u_3, u_4, u_5, u_6\}$; $\underline{G(e_4)}_\psi = \{u_1, u_2\}$; $\overline{G(e_4)}_\psi = \{u_1, u_2, u_3, u_6\}$. Clearly, (G, B) is a rough soft set with respect to parameters e_1, e_3, e_4. Now $R_{G(e_1)} = 0$, $R_{G(e_2)} = 1$, $R_{G(e_3)} = 2/3$, $R_{G(e_4)} = 1/2$.

The Hasse diagram of this chain is given in Figure 2.

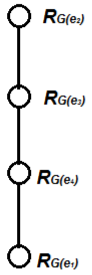

Fig. 2 Chain form by rough soft set

5. Conclusion

Soft and rough sets are two different approaches to uncertainty and rough soft set is a fusion of these two theories. We have introduced the concept of approximation on an information system with respect to another information one based on an MSR-approximation space. We have constructed the rough soft set and studied their properties in MSR-approximation space. Besides, we have established the connection between a rough soft set and a lattice theory by measuring the roughness of a soft set. The theme of this paper may be extended to further results in lattices under different environments of soft-rough relations.

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References

[1] L.A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338-353.
[2] Z. Pawlak, Rough Sets Theoretical Aspects of Reasoning about Data, Academic Publisher, 1991.
[3] Z. Pawlak, Rough sets, International Journal of Computer and Information Sciences 11 (1992) 341-356.
[4] D. Molodtsov, Soft set theory- first results, Computers and Mathematics with Applications 37 (1999) 19-31.
[5] D. Molodtsov, The Theory of Soft Sets, URSS Publishers, Moscow, 2004.
[6] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Computers and Mathematics with Applications 45 (2003) 555-562.
[7] N. Cağman, S. Enginoglu, Soft set theory and uni-int decision making, European Journal of Operational Research 207 (2010) 848-855.
[8] M.I. Ali, F. Feng, X. Liu, W.K. Min, M. Shabir, On some new operations in soft set theory, Computers and Mathematics with Applications 57 (2009) 1547-1553.
[9] Y. Jiang, Y. Tang, Q. Chen, J. Wang, S. Tang, Extending soft sets with description logic, Computers and Mathematics with Applications 59 (2010) 2087-2096.
[10] K. Qin, K. Hong, On soft equality, Computers and Mathematics with Applications 234 (2010) 1347-1355.
[11] Y.Y. Yao, Constructive and algebraic methods of the theory of rough sets, Information Sciences 109 (1998) 21-47
[12] W. Zhu, Generalized rough sets based on relations, Information Sciences 177 (2007) 4997-5011.

[13] T.B. Iwinski, Algebraic approach to rough sets, Bull. Polish. Acad. Sci. (Math) 35 (1987) 673-683.

[14] M. Shabir, M.I. Ali, T. Shaheen, Another approach to soft rough sets, Knowledge-based Systems 40 (2013) 72-78

[15] F. Feng, X. Liu, V. Leoreanu-Fotea, Y.B. Jun, Soft sets and soft rough sets, Information Sciences 181 (2011) 1125-1137

[16] S. Bera, S.K. Roy, Rough modular lattice, Journal of Uncertain Systems 7 (2013) 289-293.

[17] D. Rana, S.K. Roy, Lattice for covering rough approximations, Malaya Journal of Matematik 2 (2014) 222-227.

[18] S.K. Roy, S. Bera, Distributive lattice: A rough set approach, Malaya Journal of Matematik 2 (2014) 273-276.

[19] D. Rana, S.K. Roy, Rough set approach on lattice, Journal of Uncertain Systems 5 (2011) 72-80.