Bearing Capacity Calculation of Rock Foundation based on Nonlinear Failure Criterion

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Abstract

Conventional foundation bearing capacity calculation is based on Mohr–Coulomb linear failure criterion. But it is verified that almost all kinds of rock’s strength envelope is nonlinear with normal stress through tests and is in compliance with modified Hoek-Brown nonlinear failure criterion. Therefore, program is composed by using Matlab software and nonlinear Sequential Quadratic Programming method to calculate bearing capacity and analyze its affect factors according to the upper limit theory of limit analysis adopting Hoek-Brown nonlinear failure criterion and multi-tangential method. The result shows that the main affect factors of rock foundation’s bearing capacity are GSI and $m_i$ of the rock, however, the dead weight $\gamma$, over load $q$ and excavation disturbance coefficient $D$ affect the bearing capacity largely when GSI is small; after comparison with formers’ research, it is found that the bearing capacity is overestimated and having greater risk by using “single-tangential method” while the “multi-tangential method is more rigorous in theory and whose result is more close to the actual value and more applicable.

Keywords: bearing capacity of rock foundation; Hoek-Brown nonlinear failure criterion; sequential quadratic programming method

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1. Forewords

Foundation stability is an important factor needs to be considered in foundation design; it not only
dominates the safety of the building, but also impacts the economics of the construction. So it is necessary to
rationally determine the foundation bearing capacity and finds out its affect factors in building, hydropower,
railway and highway engineering projects. There are normally three methods to determine the bearing
capacity, respectively are in-situ test, theoretical calculation and look up in bearing capacity table. However,
the character of nonlinear strength of Geo-materials is obvious and cannot be ignored which affects the
mechanical behavior heavily and researches on bearing capacity under nonlinear strength criterion are rare.
Someone suggested determining the bearing capacity by Mohr-Coulomb linear criterion after simply linear
equalization of the nonlinear strength criterion. Later, someone suggest determining the bearing capacity
linearly after linearization of the nonlinear strength criterion by single tangential method while the strength
index varies as point of tangency changes. This is actually assuming that the normal stresses of the solving
region are identical and is not in appliance with fact. Reference [1] presents the boundary finite element
solution of bearing capacity under Hoek-Brown failure criterion and they are very close, so their average value
could be considered to be close to the theoretical solution. It is complicated to handle the linear strength
criterion during the boundary finite element solution process while even more complicated to handle the
nonlinear strength criterion and the article does not explain how to handle it. The affect factors of bearing
capacity are complex and boundedness somehow exists no matter in theoretical calculation or in-situ test and
there are few theoretical researches on rock foundation bearing capacity. Therefore, this paper studies the
affect factors of rock bearing capacity by using upper limit theory and multi rigid slider failure mechanism
with nonlinear failure criterion of Geo-materials and proposes a multi-tangential method to handle nonlinear
strength issues. It fully considers the actual unevenly distributed stress and is a nonlinear limit analysis method
in the strict sense.

2. Modified nonlinear Hoek-Brown failure criterion

To avoid effection of rock anisotropy, this paper mainly deals with integrated rock mass and smashed rock
mass because they fit the requirements of Hoek-Brown criterion. Hoek,E and Brown,E.T presented the
nonlinear failure criterion through lots of test and make it improved and developed which is now popular
accepted. Its mathematical expression is:

\[ \sigma_1 = \sigma_3 + \sigma_c \left( \frac{m \sigma_3}{\sigma_c} + s \right)^a \]  

(1)

Where: \( m \), \( s \) are the parameters of rock character and is determined by GSI; \( \sigma_c \) is the uniaxial
compressive strength of rock ; \( \sigma_1 \) and \( \sigma_3 \) is the maximum and minimum principal stress respectively when
rock breaks, \( a \) is a parameter in association with the rock integrity.

\[ \frac{m}{m_i} = \exp \left( \frac{GSI - 100}{28 - 14D} \right) \]  

(2)

\[ s = \exp \left( \frac{GSI - 100}{9 - 3D} \right) \]  

(3)

\[ a = \frac{1}{2} + \frac{1}{6} \left[ \exp \left( -\frac{GSI}{15} \right) - \exp \left( -\frac{20}{3} \right) \right] \]  

(4)

Parameter \( m_i \) can be gained by triaxial test under different ambient pressure and varies between 4~33. \( D \) is
the disturbance coefficient of rock, it equals 0 for integrated rock and 1.0 for smashed rock, interpolation can
be used to gain the value for other situation. To fit traditional analysis method, parameters of Hoek-Brown needs to be transfer in to parameters of Mohr-Coulomb criterion $\phi, c$. Tangential method is used to establish the relation between $\phi, c$ for the ease of calculation, that is, pick a point on the Hoek-Brown curve and draw its tangent line as shown in Fig.1, the equation of the tangent line is:

$$\tau = c_t + \sigma_n \tan \varphi_t$$  \hspace{1cm} (5)

As the value of $\phi, c$ is concerned with the normal stress $\sigma_n$ on the sliding surface, so $\phi, c$ varies as $\sigma_n$ varies at different location in the foundation. Therefore, $\phi, c$ are instantaneous values other than constants and should be marked as $c_t$ and $\varphi_t$. Their relation could be deduced as:

$$c_t = \frac{\sigma_n \cos \varphi_t}{2} \left[ \frac{mn(1 - \sin \varphi_t)}{2 \sin \varphi_t} \right]^{-\frac{1}{m}} - \frac{\sigma_n \tan \varphi_t}{m} \left( \frac{1 + \sin \varphi_t}{n} \right) \left[ \frac{mn(1 - \sin \varphi_t)}{2 \sin \varphi_t} \right]^{-\frac{1}{m}} + \frac{\sigma_n s}{m} \tan \varphi_t \hspace{1cm} (6)$$

3. Analysis on maximum bearing capacity of rock foundation

Multi rigid slider and multi tangential line methods are used in this paper to study the bearing capacity of rock foundation. The strength gained from tangential method exceeds the actual strength of the material and its solution is the upper limit of ultimate load. Nonlinear Hoek-Brown failure criterion is used after the foundation is partitioned into rigid slider as shown in Fig.2. only one side partition is shown because the partition pattern of both sides of rock is the same and so do the calculation. From geometric relation we can have that $L_i = \frac{L_i}{\sum_{j=1}^{n} \sin(\alpha_j + \beta_j)}$, $L_i' = \frac{L_i}{\sum_{j=1}^{n} \sin(\alpha_j + \beta_j)}$. $C_i$ and $\varphi_i$ is induced through multi-tangential method, that is, proper tangential lines of the nonlinear strength curve should be determined and induces couples of $C_i$ and $\varphi_i$ to ensure the rock bearing capacity is the lowest.

According to orthogonal flow rule of relevant flowing, the displacement velocity of rigid blocks has $\varphi_i$ angulations with surface when translating. However, the value of $\varphi$ differs with different stress states at different boundaries, so the instantaneous friction angle is $\varphi_i$, cohesive force is $c_i$ on the common line of triangle while that of bottom line is $\varphi'_i$ and $c'_i$. Due to the surface of discontinuity of velocity exists on common line of the triangle, the velocity on each common line is relative and assumes that the dissipation of plastic work only happens on common lines and bottom lines. The shape of every rigid triangle is confined by side length $L_i, L_i'$ and intersection angle $\alpha_i, \beta_i$. $V_i$ represents the absolute velocity of the $i$th rigid block, $V_{i-1,i}$ represents the relative velocity between the $i-1$th and $i$th block. The velocity field should fit in the geometric relation as shown in Fig.4, we can gain that from Fig.2, Fig.3 and Fig.4:

$$V_i = \frac{V_{1,i} \prod_{j=1}^{n} \sin(\alpha_j + \beta_j - \varphi_j - \varphi_{j+1})}{\prod_{j=1}^{n} \sin(\beta_{j+1} - \varphi_{j+1} - \varphi_j)} \hspace{1cm}, \hspace{1cm} V_{i-1,i} = \frac{V_{1,i} \prod_{j=1}^{n} \sin(\alpha_j + \beta_j - \varphi_j - \varphi_{j+1})}{\prod_{j=1}^{n} \sin(\beta_{j+1} - \varphi_{j+1} - \varphi_j)}$$
3.1. Dissipation rating of internal energy

Because the dissipation of inter energy only happens on surface of velocity discontinuity, which is the common line and bottom line of triangle elements, so the dissipation rating of internal energy is:

\[
\dot{W}_{int} = \sum_{i=2}^{k} L_i C_i V_{i-1,i} \cos \phi_{i-1} + \sum_{i=2}^{k} L_i c_i V_i \cos \phi' = 2(f_i + f_2) L_i V_i 
\]  

\[
f_i = \sum_{i=2}^{k} c_i \cos \phi_i \times \frac{\sin(\alpha_{i-1} - \beta_i + \beta_{i-1} + \varphi_i - \varphi_{i-1})}{\sin(\alpha_{i-1} + \beta_i - \varphi_i - \varphi_{i-1})} \quad \prod_{j=1}^{i-1} \frac{\sin \beta_j \sin(\alpha_j + \beta_j - \varphi_{j+1} - \varphi_{j+1})}{\sin(\alpha_j + \beta_j - \varphi_{j+1} - \varphi_{j+1})}
\]
3.2. Working power of external forces

Involving the self weight of the rock, the external forces include gravity, overload and ultimate load. Their powers are calculated respectively as follows:

3.2.1 power of gravity

Assuming the width of the foundation is $B$, the volume-weight of the rock is $Y$, then the power of gravity is:

$$
W_{gra} = \frac{B\gamma}{4} L_2 \sin \alpha_i V_1 + \sum_{i=2}^{k} L_i L_i \sin \beta_i \left( \sum_{j=1}^{i-1} \alpha_j - \beta_i + \varphi \right) W_i \gamma = \frac{B\gamma}{4} L_2 \sin \alpha_i V_1 + L_i^2 V_i \gamma \cdot f_3
$$

(9)

3.2.2 power of overload

Assuming uniformly distributed load on the surface is $q$, the distributed area is the side length $L_{k+1}$ of the $k$th triangle and the displacement orientation is in accordance with the absolute velocity of $k$th triangle element. Then the power of overload is:

$$
W_{ol} = qL_{k+1}V_k \sin(\alpha_k + \beta_k - \varphi_k) = qL_k V_4
$$

(10)

3.2.3 power of ultimate load

The ultimate load act on 1# element and the orientation is identical with the absolute velocity $V_1$, its power is:

$$
W_{qu} = \frac{B}{2} q u V_1
$$

(11)

The total power of external forces is:

$$
W_{ext} = \frac{B\gamma}{4} L_2 \sin \alpha_i V_1 + L_i^2 V_i \gamma \cdot f_3 + qL_k V_4 \frac{B}{2} q u V_1
$$

(12)

From virtual power principle, we have:

$$
\int W_{ext} - W_{int} = \int W_{int} = 0
$$

(13)

Then:

$$
q_u = (f_1 + f_2) - q_0 f_4 - \frac{B\gamma}{2} f_3 - \frac{B \sin \beta_i \sin \alpha_i \gamma}{2 \sin(\alpha_i + \beta_i)}
$$

(14)

3.3. The solving of upper limit solution of limit analysis

The $B, \gamma, q$ in equation 17 are known, so the bearing capacity is only related with $f_1, f_2, f_3, f_4$. It can be learned that $f_1, f_2, f_3, f_4$ are the function of all geometric parameters of triangle elements and physical parameters of rocks and depend on the value of $\alpha_i, \beta_i, \varphi_i$, $C_i$ and $C_i$. So when the rock is partitioned into
K elements, there are 4k variables. But from equation 5 and 6, we know that $\varphi_i, \varphi'_i,$ and $C_i, C'_i$ meet certain conditions and they also meet following geometric and velocity confining condition:

$$
\sum_{i=1}^{k} \alpha_i = \pi, \quad \alpha_i + \beta_i \geq 0, \quad 0 \leq \varphi_i \leq \pi, \quad 0 \leq \varphi'_i \leq \pi, \quad 0 \leq \beta_i - \varphi_i - \varphi'_i \leq \pi.
$$

Different variables and variable groups could have different upper limit solution of ultimate soil pressure. The smaller the upper limit solution of bearing capacity of rock foundation is, the closer to the actual ultimate load as the upper limit solution of limit analysis is always larger than actual ultimate load, that is, the solving of rock bearing capacity is a matter of finding minimum value. Therefore, program is composed by Matlab software and nonlinear Sequential Quadratic Programming method to calculate the upper limit solution of rock bearing capacity.

4. Feasibility of the arithmetic

A coefficient of foundation bearing capacity $N_\sigma$ is used to evaluate the value of ultimate bearing capacity. For the strip foundation shown in Fig.2, the width is B, the axial compressive strength and volume weight of rock foundation material is $c_i$ and $\gamma$, the Geological intensity index of rock is measured by GSI, then the ultimate bearing capacity could be demonstrated as:

$$
q_u = c_i N_\sigma
$$

(18)

If the self weight of the rock material is ignored, then $N_\sigma$ could be replaced by $N_{\sigma 0}$. The feasibility is verified by comparison with former research results, the consequences are shown in table.1 and the solution of our method is quite close to the theoretical solution and satisfies the requirements of engineering. It is easier than the boundary finite element method and has certain practical value.

Table 1. Bearing capacity coefficient under different GSI and $m_i$ ($q_0=0$, $D=0$ $\gamma = 0$)

<table>
<thead>
<tr>
<th>GSI</th>
<th></th>
<th>7</th>
<th>10</th>
<th>15</th>
<th>17</th>
<th>25</th>
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<td>Literature 1</td>
<td>Literature 9</td>
<td>This method</td>
<td>Literature 1</td>
<td>Literature 9</td>
</tr>
<tr>
<td>10</td>
<td>0.057</td>
<td>0.056</td>
<td>0.101</td>
<td>0.079</td>
<td>0.077</td>
<td>0.161</td>
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<tr>
<td>20</td>
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<td>0.285</td>
<td>0.213</td>
<td>0.209</td>
<td>0.428</td>
</tr>
<tr>
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<td>0.306</td>
<td>0.300</td>
<td>0.534</td>
<td>0.405</td>
<td>0.397</td>
<td>0.776</td>
</tr>
<tr>
<td>40</td>
<td>0.514</td>
<td>0.504</td>
<td>0.857</td>
<td>0.671</td>
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<tr>
<td>50</td>
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<td>1.037</td>
<td>1.828</td>
</tr>
<tr>
<td>60</td>
<td>1.267</td>
<td>1.247</td>
<td>1.917</td>
<td>1.618</td>
<td>1.597</td>
<td>2.674</td>
</tr>
</tbody>
</table>

5. Analysis of the calculation result

The effect of geological strength index GSI, rock integrated coefficient $m_i$, self weight $\gamma$, over load $q$ and disturbance coefficient D on bearing capacity is studied by method mentioned above. We can learn that from the result: the bearing capacity coefficient grows with any parameter among the $m_i$, GSI, $q$ and $\gamma$ changes when other condition is fixed. When GSI is low, effect of $\gamma$, $q$ on bearing capacity is obvious while opposite when GSI is high, that is, $\gamma$, $q$ contributes to the bearing capacity to a certain extent. However, the bearing capacity of rock is dominated by its GSI and $m_i$. When the rock’s integration is good, the cohesive force and friction angle are relatively higher, so is the bearing capacity. When other condition is fixed, the bearing capacity coefficient decreases with the disturbance coefficient increases. As the increasing of GSI, the effect
of disturbance coefficient on bearing capacity is continuously decreasing, when GSI reaches 100, the effect is almost none.

6. Conculusion

On the basis of upper limit theory, program is composed by using Matlab software and nonlinear Sequential Quadratic Programming method to calculate bearing capacity and analyze its affect factors. The result shows that:

(1) The solution of this paper is quite close to the boundary finite element method and satisfies the requirements of engineering with practical value.

(2) Single tangential line method did not involve that $C_t$, $\phi_t$ varies as $\sigma_n$ varies at different location in the foundation, so it is risky in practice due to its larger result.

(3) the bearing capacity of rock is mainly concerned with the GSI and $m_i$ of the rock, self weight $\gamma$, over load $q$ and disturbance coefficient $D$ have certain effect on bearing capacity. When GSI is low, effect of $\gamma$, $q$ on bearing capacity is obvious while opposite when GSI is high.

References


