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Duality in the color flavor locked spectrum

A.D. Jackson, F. Sannino

The Niels Bohr Institute & NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

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Abstract

We analyze the spectrum of the massive states for the color flavor locked phase (CFL) of QCD. We show that the vector mesons have a mass of the order of the color superconductive gap Δ. We also see that the excitations associated with the solitonic sector of the CFL low energy theory have a mass proportional to F_π^2/Δ and hence are expected to play no role for the physics of the CFL phase for large chemical potential. Another interesting point is that the product of the soliton mass and the vector meson mass is independent of the gap. We interpret this behavior as a form of electromagnetic duality in the sense of Montonen and Olive. Our approach for determining the properties of the massive states is non-perturbative in nature and can be applied to any theory with multiple scales.

A color superconductivity phase is a reasonable candidate for the state of strongly interacting matter for very large quark chemical potential [1–5]. Many properties of such a state have been investigated for two and three flavor QCD. In some cases these results rely heavily on perturbation theory, which is applicable for very large chemical potentials.

In this Letter we seek insight regarding the relevant energy scales of various physical states of the color flavor locked phase (CFL), such as the vector mesons and the solitons. Our results do not support the naive expectation that all massive states are of the order of the color superconductive gap, Δ. Our strategy is based on exploiting the significant information already contained in the low-energy effective theory for the massless states. We transfer this information to the massive states of the theory by making use of the fact that higher derivative operators in the low-energy effective theory for the lightest state can also be induced when integrating out heavy fields. For the vector mesons, this can be seen by considering a generic theory containing vector mesons and Goldstone bosons. After integrating out the vector mesons, the induced local effective Lagrangian terms for the Goldstone bosons must match the local contact terms from operator counting. We find that each derivative in the (CFL) chiral expansion is replaced by a vector field ρ_μ as follows

$$\partial \to \frac{\Delta}{F_\pi}\rho. \tag{1}$$

This relation allows us to deduce, among other things, that the energy scale for the vector mesons is

$$m_v \sim \Delta, \tag{2}$$

E-mail addresses: jackson@alf.nbi.dk (A.D. Jackson), francesco.sannino@nbi.dk (F. Sannino).

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where m_v is the vector meson mass. Our result is in agreement with the findings in [6,7]. We shall see that this also suggests that the KSRF relation holds in the CFL phase.

In the solitonic sector, the CFL chiral Lagrangian [8,9] gives us the scaling behavior of the coefficient of the Skyrme term and thus shows that the mass of the soliton is of the order of

$$M_{\text{soliton}} \sim \frac{F_\pi^2}{\Delta}, \tag{3}$$

which is contrary to naive expectations. This is suggestive of a kind of duality between vector mesons and solitons in the same spirit as the duality advocated some years ago by Montonen and Olive for the $SU(2)$ Georgi–Glashow theory [10]. This duality becomes more apparent when considering the product

$$M_{\text{soliton}} m_v \sim F_\pi^2, \tag{4}$$

which is independent of the scale, Δ. In the present case, if the vector meson self-coupling is $\tilde{g}$, we find that the Skyrme coefficient, $e \sim \Delta/F_\pi$, can be identified with $\tilde{g}$. Thus, the following relations hold:

$$M_{\text{soliton}} \propto \frac{F_\pi}{\tilde{g}} \quad \text{and} \quad m_v \propto \tilde{g} F_\pi. \tag{5}$$

In this notation the electric–magnetic (i.e., vector meson-soliton) duality is transparent. Since the topological Wess–Zumino term in the CFL phase is identical to that in vacuum, we identify the soliton with a physical state having the quantum numbers of the nucleon. If quark–hadron continuity [11] is assumed, we expect that the product of the nucleon and vector meson masses will scale like F_π^2 for any non-zero chemical potential for three flavors. Interestingly, quark–hadron continuity can be related to duality. Testing this relation can also be understood as a quantitative check of quark–hadron continuity. It is important to note that our results are tree level results and that the resulting duality relation can be affected by quantum corrections. Our results have direct phenomenological consequences for the physics of compact stars with a CFL phase. While vector mesons are expected to play a relevant role, solitons can safely be neglected for large values of the quark chemical potential.

The Lagrangian for CFL Goldstones

When diquarks condense for the three flavor case, we have the following symmetry breaking:

$$[SU_c(3)] \times SU_L(3) \times SU_R(3) \times U_B(1) \\ \to SU_{c+L+R}(3).$$

The gauge group undergoes a dynamical Higgs mechanism, and nine Goldstone bosons emerge. Neglecting the Goldstone mode associated with the baryon number and quark masses (which will not be important for our discussion at lowest order), the derivative expansion of the effective Lagrangian describing the octect of Goldstone bosons is [8,9]:

$$\mathcal{L} = \frac{F_\pi^2}{8} \operatorname{Tr}\left[\partial_\mu U \partial^\mu U^\dagger\right] \equiv \frac{F_\pi^2}{2} \operatorname{Tr}\left[p_\mu p^\mu\right], \tag{6}$$

with $p_\mu = \frac{i}{2}(\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi)$, $U = \xi^2$, $\xi = e^{i\frac{\phi}{F_\pi}}$ and ϕ is the octet of Goldstone bosons. U transforms linearly according to $g_L U g_R^\dagger$ and $g_{L/R} \in SU_{L/R}(3)$ while ξ transforms non-linearly:

$$\xi \to g_L \xi K^\dagger(\phi, g_L, g_R) \equiv K(\phi, g_L, g_R) \xi g_R^\dagger. \tag{7}$$

This constraint implicitly defines the matrix, $K(\phi, g_L, g_R)$. Here, we wish to examine the CFL spectrum of massive states using the technique of integrating in/out at the level of the effective Lagrangian. F_π is the Goldstone boson decay constant. It is a non-perturbative quantity whose value is determined experimentally or by non-perturbative techniques (e.g., lattice computation). For very large quark chemical potential, F_π can be estimated perturbatively. It is found to be proportional to the Fermi momentum, $p_F \sim \mu$, with μ the quark chemical potential [12]. Since a frame must be fixed in order to introduce a chemical potential, spatial and temporal components of the effective Lagrangians split. This point, however, is not relevant for the validity of our results.

When going beyond the lowest-order term in derivatives, we need a counting scheme. For theories with only one relevant scale (such as QCD at zero chemical potential), each derivative is suppressed by a factor of F_π. This is not the case for theories with multiple scales. In the CFL phase, we have both F_π and the gap, Δ, and the general form of the chiral expansion is

[12]:

$$L \sim F_\pi^2 \Delta^2 \left(\frac{\vec{\partial}}{\Delta}\right)^k \left(\frac{\partial_0}{\Delta}\right)^l U^m U^{\dagger n}. \tag{8}$$

Following [12], we distinguish between temporal and spatial derivatives. Chiral loops are suppressed by powers of $p/4\pi F_\pi$, and higher-order contact terms are suppressed by p/Δ where p is the momentum. Thus, chiral loops are parametrically small compared to contact terms when the chemical potential is large.

There is also a topological term which is essential in order to satisfy the 't Hooft anomaly conditions [13–15] at the effective Lagrangian level. It is important to note that respecting the 't Hooft anomaly conditions is more than an academic exercise. In fact, it requires that the form of the Wess–Zumino term is the same in vacuum and at non-zero chemical potential. Its real importance lies in the fact that it forbids a number of otherwise allowed phases which cannot be ruled out given our rudimentary treatment of the non-perturbative physics. As an example, consider a phase with massless protons and neutrons in three-color QCD with three flavors. In this case chiral symmetry does not break. This is a reasonable realization of QCD for any chemical potential. However, it does not satisfy the 't Hooft anomaly conditions and hence cannot be considered. Were it not for the 't Hooft anomaly conditions, such a phase could compete with the CFL phase.

Gauging the Wess–Zumino term with to respect the electromagnetic interactions yields the familiar $\pi^0 \to 2\gamma$ anomalous decay. This term [16] can be written compactly using the language of differential forms. It is useful to introduce the algebra-valued Maurer–Cartan one form $\alpha = \alpha_\mu \, dx^\mu = (\partial_\mu U)U^{-1} \, dx^\mu \equiv (dU)U^{-1}$ which transforms only under the left $SU_L(3)$ flavor group. The Wess–Zumino effective action is

$$\Gamma_{\mathrm{WZ}}[U] = C \int_{M^5} \mathrm{Tr}\big[\alpha^5\big]. \tag{9}$$

The price which must be paid in order to make the action local is that the spatial dimension must be augmented by one. Hence, the integral must be performed over a five-dimensional manifold whose boundary (M^4) is ordinary Minkowski space. In [8, 13,17] the constant C has been shown to be the same as that at zero density, i.e.,

$$C = -i \frac{N_c}{240\pi^2}, \tag{10}$$

where N_c is the number of colors (three in this case). Due to the topological nature of the Wess–Zumino term its coefficient is a pure number.

The vector mesons

It is well known that massive states are relevant for low energy dynamics. Consider, for example, the role played by vector mesons in pion–pion scattering [18] in saturating the unitarity bounds. More specifically, vector mesons play a relevant role when describing the low energy phenomenology of QCD and may also play a role also in the dynamics of compact stars with a CFL core. In order to investigate the effects of such states, we need to know their in-medium properties including their gaps and the strength of their couplings to the CFL Goldstone bosons. Except for the extra spontaneously broken $U(1)_B$ symmetry, the symmetry properties of the CFL phase have much in common with those of zero density phase of QCD. This fact allows us to make some non-perturbative but reasonable estimates of vector mesons properties in medium. We have already presented the general form of the chiral expansion in the CFL phase. As will soon become clear, we are now interested in the four-derivative (non-topological) terms whose coefficients are proportional to

$$\frac{F_\pi^2}{\Delta^2}. \tag{11}$$

This must be contrasted with the situation at zero chemical potential, where the coefficient of the four-derivative term is always a pure number before quantum corrections are taken into account. In vacuum, the tree-level Lagrangian which simultaneously describes vector mesons, Goldstone bosons, and their interactions is:

$$L = \frac{F_\pi^2}{2} \mathrm{Tr}\big[p_\mu p^\mu\big] + \frac{m_v^2}{2} \mathrm{Tr}\bigg[\bigg(\rho_\mu + \frac{v_\mu}{\tilde{g}}\bigg)^2\bigg] - \frac{1}{4} \mathrm{Tr}\big[F_{\mu\nu}(\rho) F^{\mu\nu}(\rho)\big], \tag{12}$$

where $F_\pi \simeq 132$ MeV and v_μ is the one form $v_\mu = \frac{i}{2}(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi)$ with $U = \xi^2$ and $F_{\mu\nu}(\rho) = \partial_\mu \rho_\nu -$

$\partial_\nu \rho_\mu + i\tilde{g}[\rho_\mu, \rho_\nu]$. At tree level this Lagrangian agrees with the hidden local symmetry results [19].

When the vector mesons are very heavy with respect to relevant momenta, they can be integrated out. This results in the field contraint:

$$\rho_\mu = -\frac{v_\mu}{\tilde{g}}. \tag{13}$$

Substitution of this relation in the vector meson kinetic term (i.e., the replacement of $F_{\mu\nu}(\rho)$ by $F_{\mu\nu}(v)$) gives the following four-derivative operator with two time derivatives and two space derivatives [20]:

$$\frac{1}{64\tilde{g}^2}\,\mathrm{Tr}\big[[\alpha_\mu, \alpha_\nu]^2\big]. \tag{14}$$

The coefficient is proportional to $1/\tilde{g}^2$. It is also relevant to note that since we are describing physical fields we have considered canonically normalized fields and kinetic terms. This Lagrangian can also be applied to the CFL case. In the vacuum, $\tilde{g}$ is a number of order one independent of the scale at tree level. This is no longer the case in the CFL phase. Here, by comparing the coefficient of the four-derivative operator in Eq. (14) obtained after having integrated out the vector meson with the coefficient of the same operator in the CFL chiral perturbation theory we determine the following scaling behavior of $\tilde{g}$:

$$\tilde{g} \propto \frac{\Delta}{F_\pi}. \tag{15}$$

By expanding the effective Lagrangian with the respect to the Goldstone boson fields, one sees that $\tilde{g}$ is also connected to the vector meson coupling to two pions, $g_{\rho\pi\pi}$, through the relation

$$g_{\rho\pi\pi} = \frac{m_v^2}{\tilde{g}F_\pi^2}. \tag{16}$$

In vacuum $g_{\rho\pi\pi} \simeq 8.56$ and $\tilde{g} \simeq 3.96$ are quantities of order one. Since v_μ is essentially a single derivative, the scaling behavior of $\tilde{g}$ allows us to conclude that each derivative term is equivalent to $\tilde{g}\rho_\mu$ with respect to the chiral expansion. For example, dropping the dimensionless field U, the operator with two derivatives becomes a mass operator for the vector meson

$$F_\pi^2 \partial_\mu^2 \to F_\pi^2 \tilde{g}^2 \rho_\mu^2 \sim \Delta^2 \rho_\mu^2. \tag{17}$$

This demonstrates that the vector meson mass gap is proportional to the color superconducting gap. This non-perturbative result is relevant for phenomenological applications. It is interesting to note that our simple counting argument agrees with the underlying QCD perturbative computations of Ref. [6] and also with recent results of Ref. [7]. In [21], at high chemical potential, vector meson dominance is discussed. However, our approach is more general since it does not rely on any underlying perturbation theory. It can be applied to theories with multiple scales for which the counting of the Goldstone modes is known. Since $m_v^2 \sim \Delta^2$, we find that $g_{\rho\pi\pi}$ scales with $\tilde{g}$ suggesting that the KSRF relation is a good approximation also in the CFL phase of QCD.

CFL-solitons

The low energy effective theory supports solitonic excitations which can be identified with the baryonic sector of the theory at non-zero chemical potential. In order to obtain classically stable configurations, it is necessary to include at least a four-derivative term (containing two temporal derivatives) in addition to the usual two-derivative term. Such a term is the Skyrme term:

$$L^{\mathrm{skyrme}} = \frac{1}{32e^2}\,\mathrm{Tr}\big[[\alpha_\mu, \alpha_\nu]^2\big]. \tag{18}$$

Since this is a fourth-order term in derivatives not associated with the topological term we have:

$$e \sim \frac{\Delta}{F_\pi}. \tag{19}$$

This term is the same as that which emerges after integrating out the vector mesons (see Eq. (14)), and one concludes that $e = \sqrt{2}\,\tilde{g}$ [20]. The simplest complete action supporting solitonic excitations is:

$$\int d^4x \left[\frac{F_\pi^2}{2}\,\mathrm{Tr}\big[p_\mu p^\mu\big] + L^{\mathrm{skyrme}}\right] + \Gamma_{\mathrm{WZ}}. \tag{20}$$

The Wess–Zumino term in Eq. (9) guarantees the correct quantization of the soliton as a spin 1/2 object. Here we neglect the breaking of Lorentz symmetries, irrelevant to our discussion. The Euler–Lagrangian equations of motion for the classical, time independent, chiral field $U_0(\mathbf{r})$ are highly nonlinear partial differential equations. To simplify these equations Skyrme adopted the hedgehog *ansatz* which, suitably generalized for the three flavor case, reads [20]:

$$U_0(\mathbf{r}) = \begin{pmatrix} e^{i\vec{\tau}\cdot\hat{r}F(r)} & 0 \\ 0 & 1 \end{pmatrix}, \tag{21}$$

where $\vec{\tau}$ represents the Pauli matrices and the radial function $F(r)$ is called the chiral angle. The *ansatz* is supplemented with the boundary conditions $F(\infty) = 0$ and $F(0) = 0$ which guarantee that the configuration possesses unit baryon number. After substituting the *ansatz* in the action one finds that the classical solitonic mass is, up to a numerical factor:

$$M_{\text{soliton}} \propto \frac{F_\pi}{e} \sim \frac{F_\pi^2}{\Delta}, \tag{22}$$

and the isoscalar radius, $\langle r^2 \rangle_{I=0} \sim 1/(F_\pi^2 e^2) \sim 1/\Delta^2$. Interestingly, due to the non-perturbative nature of the soliton, its mass turns to be dual to the vector meson mass. It is also clear that although the vector mesons and the solitons have dual masses, they describe two very distinct types of states. The present duality is very similar to the one argued in [10]. Indeed, after introducing the collective coordinate quantization, the soliton (due to the Wess–Zumino term) describes baryonic states of half-integer spin while the vectors are spin one mesons. Here, the dual nature of the soliton with respect to the vector meson is enhanced by the fact that, in the CFL state, $\tilde{g} \sim \Delta/F_\pi$ is expected to be substantially reduced with respect to its value in vacuum. Once the soliton is identified with the nucleon (whose density-dependent mass is denoted with $M_N(\mu)$) and assuming quark–hadron continuity, we predict the following relation to be independent of the matter density:

$$\frac{M_N(\mu) m_v(\mu)}{(2\pi F_\pi(\mu))^2} = \frac{M_N(0) m_v(0)}{(2\pi F_\pi(0))^2} \sim 1.05. \tag{23}$$

In this way, we can relate duality to quark–hadron continuity.

Conclusions

We have shown that the vector mesons in the CFL phase have masses of the order of the color superconductive gap, Δ. On the other hand, the solitons have masses proportional to F_π^2/Δ and hence should play no role for the physics of the CFL phase at large chemical potential. We have noted that the product of the soliton mass and the vector meson mass is independent of the gap. This behavior reflects a form of electromagnetic duality in the sense of Montonen and Olive [10]. Combining duality and quark–hadron continuity we have predicted that the nucleon mass times the vector meson mass scales as the square of the pion decay constant at any non-zero chemical potential. In the presence of two or more scales provided by the underlying theory the spectrum of massive states shows very different behaviors which cannot be obtained by assuming a naive dimensional analysis.

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