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Finite BRST–antiBRST transformations in Lagrangian formalism

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ABSTRACT

We continue the study of finite BRST–antiBRST transformations for general gauge theories in Lagrangian formalism initiated in [1], with a doublet λ_a, $a=1,2$, of anticommuting Grassmann parameters, and find an explicit Jacobian corresponding to this change of variables for constant λ_a. This makes it possible to derive the Ward identities and their consequences for the generating functional of Green's functions. We announce the form of the Jacobian (proved to be correct in [31]) for finite field-dependent BRST–antiBRST transformations with functionally-dependent parameters, $\lambda_a = s_a\Lambda$, induced by a finite even-valued functional $\Lambda(\phi,\pi,\lambda)$ and by the generators s_a of BRST–antiBRST transformations, acting in the space of fields ϕ, antifields $\phi^*_a, \bar{\phi}$ and auxiliary variables π^a, λ. On the basis of this Jacobian, we present and solve a compensation equation for Λ, which is used to achieve a precise change of the gauge-fixing functional for an arbitrary gauge theory. We derive a new form of the Ward identities, containing the parameters λ_a, and study the problem of gauge-dependence. The general approach is exemplified by the Freedman–Townsend model of a non-Abelian antisymmetric tensor field.

1. Introduction

In our recent work [1], we have proposed an extension of BRST–antiBRST transformations to the case of finite (global and field-dependent) parameters in Yang–Mills and general gauge theories within the Sp(2)-covariant Lagrangian quantization [2,3]; see also [4]. The idea of "finiteness" is based on transformation parameters λ_a which are no longer regarded as infinitesimal and utilizes the inclusion into the BRST–antiBRST transformations [5–7] of a new term, being quadratic in λ_a. First of all, this makes it possible to realize the complete BRST–antiBRST invariance of the integrand in the vacuum functional. Second, the field-dependent parameters $\lambda_a = s_a\Lambda$, induced by a Grassmann-even functional Λ, provide an explicit correspondence (due to the so-called compensation equation for the Jacobian) between the partition function of a theory in a certain gauge, determined by a gauge Boson F_0, and the theory in a different gauge, given by another gauge Boson F. This concept becomes a key instrument to determine, in a BRST–antiBRST manner, the Gribov horizon functional [8] – which is given initially in the Landau gauge within a BRST–antiBRST extension of the Gribov–Zwanziger theory [9] – by utilizing any other gauge, including the R_ξ-gauges used to eliminate residual gauge invariance in the deep IR region. For completeness note that the concept of finite field-dependent BRST transformations has been suggested in [10]; anti-BRST transformations and BRST–antiBRST transformations linear in field-dependent parameters Θ_1, Θ_2 have been considered in [11] and [12], respectively.

The problems listed in Discussion of [1] as unsolved ones include:

1. the study of finite field-dependent BRST–antiBRST transformations for a general gauge theory in the framework of the path integral (2.4);
2. the development of finite field-dependent BRST transformations for a general gauge theory in the BV quantization scheme;
3. the construction of finite field-dependent BRST–antiBRST transformations in the Sp(2)-covariant generalized Hamiltonian quantization [13,14].

The second problem within the BV quantization scheme [15], based on the principle of BRST symmetry [16,17], has been examined in [18], and earlier in [19]. The third problem has been recently solved [20] for arbitrary dynamical systems subject to

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first-class constraints, together with an explicit construction of the parameters λ_a generating a change of the gauge in the path integral for Yang–Mills theories within the class of R_ξ-like gauges in Hamiltonian formalism. For the sake of completeness, notice that, in the case of BRST–BFV symmetry [21], a study of finite field-dependent BRST–BFV transformations in the generalized Hamiltonian formalism [22,23] has been presented in [24]. Therefore, it is only the first item in the list of the above-mentioned problems that remains unsolved. In this connection, the main purpose of the present work is to prove that the ansatz for finite BRST–antiBRST transformations within the path integral (2.4) proposed in [1], using formulae (6.1)–(6.5), holds true. We illustrate our general approach by a well-known gauge theory of non-Yang–Mills type proposed by Freedman and Townsend [25].

The work is organized as follows. In Section 2, we remind the definition of a finite Lagrangian BRST–antiBRST transformation for general gauge theories. In Section 3, we obtain an explicit Jacobian corresponding to this change of variables for global finite BRST–antiBRST transformations and prove the invariance of the integrand in the partition function. In Section 4, we obtain the Ward identities with the help of finite BRST–antiBRST transformations. In Section 5, we consider the reducible gauge theory of Freedman–Townsend (the model of antisymmetric non-Abelian tensor field). In Discussion, we announce the explicit Jacobian of finite field-dependent BRST–antiBRST transformations with functionally-dependent parameters, formulate the corresponding compensation equation, present its solution, which amounts to a precise change of the gauge-fixing functional, derive the Ward identities, depending on the parameters λ_a, and study the problem of gauge dependence. We use the notation of our previous work [1]. In particular, derivatives with respect to the (anti)fields are taken from the (left)right; $\delta_l/\delta\phi^A$ denotes the left-hand derivative with respect to ϕ^A. The raising and lowering of Sp(2) indices, $s^a = \varepsilon^{ab}s_b$, $s_a = \varepsilon_{ab}s^b$, is carried out with the help of a constant antisymmetric tensor ε^{ab}, $\varepsilon^{ac}\varepsilon_{cb} = \delta^a_b$, subject to the normalization condition $\varepsilon^{12} = 1$.

2. Finite BRST–antiBRST transformations

Let Γ^p be the coordinates

$$\Gamma^p = \left(\phi^A, \phi^*_{Aa}, \bar{\phi}_A, \pi^{Aa}, \lambda^A\right) \tag{2.1}$$

in the extended space of fields ϕ^A, antifields ϕ^*_{Aa}, $\bar{\phi}_A$ and auxiliary fields π^{Aa}, λ^A, with the following distribution of Grassmann parity and ghost number:

$$\varepsilon\left(\phi^A, \phi^*_{Aa}, \bar{\phi}_A, \pi^{Aa}, \lambda^A\right) = (\varepsilon_A, \varepsilon_A + 1, \varepsilon_A, \varepsilon_A + 1, \varepsilon_A), \tag{2.2}$$

$$\begin{aligned}&\mathrm{gh}\left(\phi^A, \phi^*_{Aa}, \bar{\phi}_A, \pi^{Aa}, \lambda^A\right)\\ &\quad= \left(\mathrm{gh}\left(\phi^A\right), (-1)^a - \mathrm{gh}\left(\phi^A\right), -\mathrm{gh}\left(\phi^A\right),\right.\\ &\qquad \left.(-1)^{a+1} + \mathrm{gh}\left(\phi^A\right), \mathrm{gh}\left(\phi^A\right)\right).\end{aligned} \tag{2.3}$$

The contents of the configuration space ϕ^A, containing the classical fields A^i and the Sp(2)-symmetric ghost–antighost and Nakanishi–Lautrup fields, depend on the irreducible [2] or reducible [3] nature of a given gauge theory.

The generating functional of Green's functions $Z_F(J)$, depending on external sources J_A, with $\varepsilon(J_A) = \varepsilon_A$, $\mathrm{gh}(J_A) = -\mathrm{gh}(\phi^A)$,

$$\begin{aligned}&Z_F(J) = \int d\Gamma\, \exp\left\{(i/\hbar)\left[\mathcal{S}_F(\Gamma) + J_A\phi^A\right]\right\},\\ &\mathcal{S}_F = S + \phi^*_{Aa}\pi^{Aa} + (\bar{\phi}_A - F_{,A})\lambda^A - (1/2)\varepsilon_{ab}\pi^{Aa}F_{,AB}\pi^{Bb}\end{aligned} \tag{2.4}$$

and the corresponding partition function $Z_F \equiv Z_F(0)$ are determined by a Bosonic functional $S = S(\phi, \phi^*, \bar{\phi})$ and by a gauge-fixing Bosonic functional $F = F(\phi)$ with vanishing ghost numbers, the functional S being a solution of the generating equations

$$\frac{1}{2}(S,S)^a + V^aS = i\hbar\Delta^aS \quad \Leftrightarrow \quad \left(\Delta^a + \frac{i}{\hbar}V^a\right)\exp\left(\frac{i}{\hbar}S\right) = 0, \tag{2.5}$$

where $\hbar$ is the Planck constant, and the boundary condition for S in (2.5) for vanishing antifields ϕ^*_a, $\bar{\phi}$ is given by the classical action $S_0(A)$. The extended antibracket $(F,G)^a$ for arbitrary functionals F, G and the operators Δ^a, V^a are given by

$$\begin{aligned}&(F,G)^a = \frac{\delta F}{\delta\phi^A}\frac{\delta G}{\delta\phi^*_{Aa}} - \frac{\delta_r F}{\delta\phi^*_{Aa}}\frac{\delta_l G}{\delta\phi^A}, \qquad \Delta^a = (-1)^{\varepsilon_A}\frac{\delta_l}{\delta\phi^A}\frac{\delta}{\delta\phi^*_{Aa}},\\ &V^a = \varepsilon^{ab}\phi^*_{Ab}\frac{\delta}{\delta\bar{\phi}_A}.\end{aligned} \tag{2.6}$$

The integrand $\mathcal{I}^{(F)}_\Gamma = d\Gamma\exp[(i/\hbar)\mathcal{S}_F(\Gamma)]$ for $J_A = 0$ is invariant, $\delta\mathcal{I}^{(F)}_\Gamma = 0$, under the global infinitesimal BRST–antiBRST transformations (2.7), $\delta\Gamma^p = (s^a\Gamma^p)\mu_a$, with the corresponding generators s^a,

$$\begin{aligned}\delta\Gamma^p &= \left(s^a\Gamma^p\right)\mu_a = \Gamma^p\overleftarrow{s}^a\mu_a = \delta\left(\phi^A, \phi^*_{Ab}, \bar{\phi}_A, \pi^{Ab}, \lambda^A\right)\\ &= \left(\pi^{Aa}, \delta^a_b S_{,A}(-1)^{\varepsilon_A}, \varepsilon^{ab}\phi^*_{Ab}(-1)^{\varepsilon_A+1}, \varepsilon^{ab}\lambda^A, 0\right)\mu_a,\end{aligned} \tag{2.7}$$

where the invariance at the first order in μ_a is established by using the generating equations (2.5).

The above infinitesimal invariance is sufficient to determine *finite BRST–antiBRST transformations* $\Gamma^p \to \Gamma^p + \Delta\Gamma^p$ with anticommuting parameters λ_a, $a = 1,2$, which were introduced in [1] as follows:

$$\begin{aligned}&\mathcal{I}^{(F)}_{\Gamma+\Delta\Gamma} = \mathcal{I}^{(F)}_\Gamma, \qquad \Delta\Gamma^p\frac{\overleftarrow{\partial}}{\partial\lambda_a}\bigg|_{\lambda=0} = \Gamma^p\overleftarrow{s}^a \quad \text{and}\\ &\Delta\Gamma^p\frac{\overleftarrow{\partial}}{\partial\lambda_b}\frac{\overleftarrow{\partial}}{\partial\lambda_a} = \frac{1}{2}\varepsilon^{ab}\Gamma^p\overleftarrow{s}^2, \quad \text{where } s^2 = s_as^a,\ \overleftarrow{s}^2 = \overleftarrow{s}^a\overleftarrow{s}_a.\end{aligned} \tag{2.8}$$

Thus determined finite BRST–antiBRST symmetry transformations for the integrand $\mathcal{I}^{(F)}_\Gamma$ in a general gauge theory, with the help of the notation

$$\begin{aligned}&X^{pa} \equiv \Gamma^p\overleftarrow{s}^a \quad \text{and} \quad Y^p \equiv (1/2)X^{pa}_{,q}X^{qb}\varepsilon_{ba} = -(1/2)\Gamma^p\overleftarrow{s}^2,\\ &\quad\text{with } G_{,p} \equiv \frac{\delta G}{\delta\Gamma^p},\end{aligned} \tag{2.9}$$

can be represented in the form

$$\begin{aligned}&\Delta\Gamma^p = X^{pa}\lambda_a - \frac{1}{2}Y^p\lambda^2 = \Gamma^p\left(\overleftarrow{s}^a\lambda_a + \frac{1}{4}\overleftarrow{s}^2\lambda^2\right)\\ &\quad\Rightarrow \quad \mathcal{I}^{(F)}_{\Gamma+\Delta\Gamma} = \mathcal{I}^{(F)}_\Gamma.\end{aligned} \tag{2.10}$$

Equivalently, in terms of the components, (2.10) is given by

$$\begin{aligned}&\Delta\phi^A = \pi^{Aa}\lambda_a + \frac{1}{2}\lambda^A\lambda^2, \qquad \Delta\bar{\phi}_A = \varepsilon^{ab}\lambda_a\phi^*_{Ab} + \frac{1}{2}S_{,A}\lambda^2,\\ &\Delta\pi^{Aa} = -\varepsilon^{ab}\lambda^A\lambda_b, \qquad \Delta\lambda^A = 0,\\ &\Delta\phi^*_{Aa} = \lambda_aS_{,A} + \frac{1}{4}(-1)^{\varepsilon_A}\\ &\qquad\times\left(\varepsilon_{ab}\frac{\delta^2S}{\delta\phi^A\delta\phi^B}\pi^{Bb} + \varepsilon_{ab}\frac{\delta S}{\delta\phi^B}\frac{\delta^2S}{\delta\phi^A\delta\phi^*_{Bb}}(-1)^{\varepsilon_B}\right.\\ &\qquad\left. - \phi^*_{Ba}\frac{\delta^2S}{\delta\phi^A\delta\bar{\phi}_B}(-1)^{\varepsilon_B}\right)\lambda^2.\end{aligned} \tag{2.11}$$

In order to make sure that $\mathcal{I}_\Gamma^{(F)}$ is invariant under the finite BRST–antiBRST transformations (2.10) with constant λ_a, one has to find the Jacobian corresponding to this change of variables.

3. Jacobian of finite global BRST–antiBRST transformations

Let us examine the change of the integration measure $d\Gamma \to d\check{\Gamma}$ in (2.4) under the finite transformations $\Gamma^p \to \check{\Gamma}^p = \Gamma^p + \Delta\Gamma^p$ given by (2.10). To this end, taking account of (2.5), we present the invariance of the integrand $\mathcal{I}_\Gamma^{(F)}$ under the infinitesimal transformations $\delta\Gamma^p = \Gamma^p \overleftarrow{s}^a \mu_a = X_a^{pa}\mu_a$ given by (2.7) in the form

$$\mathcal{S}_{F,p}X^{pa} = i\hbar X^{pa}_{,p}, \quad \text{where } X^{pa}_{,p} = -\Delta^a S. \tag{3.1}$$

The consideration of (2.10) implies that we are interested in

$$\mathrm{Str}\left(M - \frac{1}{2}M^2\right), \quad \text{for } M^p_q \equiv \frac{\delta(\Delta\Gamma^p)}{\delta\Gamma^q} \text{ with } \frac{\delta}{\delta\Gamma^q} \equiv \frac{\delta_r}{\delta\Gamma^q}, \tag{3.2}$$

since, in view of the nilpotency $\lambda_a\lambda_b\lambda_c \equiv 0$, we have

$$d\check{\Gamma} = d\Gamma\,\mathrm{Sdet}\left(\frac{\delta\check{\Gamma}}{\delta\Gamma}\right) = d\Gamma \exp\left[\mathrm{Str}\ln(\mathbb{I}+M)\right] \equiv d\Gamma\exp(\Im),$$

$$\Im = \mathrm{Str}\ln(\mathbb{I}+M) = -\,\mathrm{Str}\left(\sum_{n=1}^{\infty}\frac{(-1)^n}{n}M^n\right) = \mathrm{Str}\left(M - \frac{1}{2}M^2\right).$$

Explicitly,

$$\begin{aligned} M^p_q &= \frac{\delta(\Delta\Gamma^p)}{\delta\Gamma^q} = \frac{\delta}{\delta\Gamma^q}\left(X^{pa}\lambda_a - \frac{1}{2}Y^p\lambda^2\right) \\ &= (-1)^{\varepsilon_q}X^{pa}_{,q}\lambda_a - \frac{1}{2}Y^p_{,q}\lambda^2, \\ &\text{with } \mathrm{Str}(M) = X^{pa}_{,p}\lambda_a - \frac{1}{2}(-1)^{\varepsilon_p}Y^p_{,p}\lambda^2 \end{aligned} \tag{3.3}$$

and

$$\begin{aligned} M^p_r M^r_q &= (-1)^{\varepsilon_r}X^{pa}_{,r}\lambda_a(-1)^{\varepsilon_q}X^{rb}_{,q}\lambda_b = X^{pa}_{,r}X^{rb}_{,q}\lambda_b\lambda_a \\ &= -\frac{1}{2}\varepsilon_{ba}X^{pa}_{,r}X^{rb}_{,q}\lambda^2, \\ &\text{with } \mathrm{Str}(M^2) = -\frac{1}{2}(-1)^{\varepsilon_p}X^{pa}_{,q}X^{qb}_{,p}\varepsilon_{ba}\lambda^2. \end{aligned} \tag{3.4}$$

Therefore,

$$\begin{aligned} &\mathrm{Str}\left(M - \frac{1}{2}M^2\right) \\ &= X^{pa}_{,p}\lambda_a - \frac{1}{2}(-1)^{\varepsilon_p}Y^p_{,p}\lambda^2 - \frac{1}{2}\left(-\frac{1}{2}(-1)^{\varepsilon_p}X^{pa}_{,q}X^{qb}_{,p}\varepsilon_{ba}\lambda^2\right) \\ &= X^{pa}_{,p}\lambda_a - \frac{1}{2}(-1)^{\varepsilon_p}Y^p_{,p}\lambda^2 + \frac{1}{4}(-1)^{\varepsilon_p}X^{pa}_{,q}X^{qb}_{,p}\varepsilon_{ba}\lambda^2 \\ &= X^{pa}_{,p}\lambda_a - \frac{1}{2}(-1)^{\varepsilon_p}\left(Y^p_{,p} - \frac{1}{2}X^{pa}_{,q}X^{qb}_{,p}\varepsilon_{ba}\right)\lambda^2. \end{aligned} \tag{3.5}$$

Considering

$$\begin{aligned} &Y^p_{,p} - \frac{1}{2}X^{pa}_{,q}X^{qb}_{,p}\varepsilon_{ba} \\ &= \frac{1}{2}\varepsilon_{ba}\left(X^{pa}_{,qp}X^{qb}(-1)^{\varepsilon_p(\varepsilon_q+1)} + X^{pa}_{,q}X^{qb}_{,p}\right) - \frac{1}{2}\varepsilon_{ba}X^{pa}_{,q}X^{qb}_{,p} \\ &= \frac{1}{2}\varepsilon_{ba}\left(X^{pa}_{,qp}X^{qb}(-1)^{\varepsilon_p(\varepsilon_q+1)} + X^{pa}_{,q}X^{qb}_{,p} - X^{pa}_{,q}X^{qb}_{,p}\right) \\ &= \frac{1}{2}\varepsilon_{ba}X^{pa}_{,pq}X^{qb}(-1)^{\varepsilon_p}, \end{aligned} \tag{3.6}$$

we arrive at

$$\mathrm{Str}\left(M - \frac{1}{2}M^2\right) = X^{pa}_{,p}\lambda_a + \frac{1}{4}\varepsilon_{ab}X^{pa}_{,pq}X^{qb}\lambda^2, \tag{3.7}$$

where (3.1) implies

$$\begin{aligned} &X^{pa}_{,p} = -\Delta^a S, \qquad X^{pa}_{,pq}X^{qb} = -\left(\Delta^a S\right)_{,p}X^{pb} = -s^b\left(\Delta^a S\right), \\ &\text{with } G_{,p}X^{pa} = G_{,p}\left(s^a\Gamma^p\right) = s^a G. \end{aligned} \tag{3.8}$$

Hence, (3.7) takes the form

$$\begin{aligned} \mathrm{Str}\left(M - \frac{1}{2}M^2\right) &= -\left(\Delta^a S\right)\lambda_a - \frac{1}{4}\varepsilon_{ab}\left(\Delta^a S\right)_{,p}X^{pb}\lambda^2 \\ &= -\left(\Delta^a S\right)\lambda_a - \frac{1}{4}\left(s_a\Delta^a S\right)\lambda^2. \end{aligned} \tag{3.9}$$

Consider now the change of the integrand

$$\mathcal{I}_\Gamma \equiv \mathcal{I}_\Gamma^{(F)} = d\Gamma\exp\left[(i/\hbar)\mathcal{S}_F(\Gamma)\right] \tag{3.10}$$

under the transformations (2.10),

$$\begin{aligned} &\mathcal{I}_{\Gamma+\Delta\Gamma} = d\Gamma\,\mathrm{Sdet}\left(\frac{\delta\check{\Gamma}}{\delta\Gamma}\right)\exp\left[\frac{i}{\hbar}\mathcal{S}_F(\Gamma+\Delta\Gamma)\right], \\ &\mathrm{Sdet}\left(\frac{\delta\check{\Gamma}}{\delta\Gamma}\right) = \exp\left\{\frac{i}{\hbar}\left[-i\hbar\,\mathrm{Str}\left(M - \frac{1}{2}M^2\right)\right]\right\} \\ &\qquad = \exp\left\{\frac{i}{\hbar}\left[i\hbar\Delta^a S\lambda_a + \frac{i\hbar}{4}\left(s_a\Delta^a S\right)\lambda^2\right]\right\}, \end{aligned} \tag{3.11}$$

$$\mathcal{S}_F(\Gamma+\Delta\Gamma) = \mathcal{S}_F(\Gamma) + s^a\mathcal{S}_F(\Gamma)\lambda_a + \frac{1}{4}s^2\mathcal{S}_F(\Gamma)\lambda^2, \tag{3.12}$$

where any functional $G(\Gamma)$ expandable as a power series in Γ^p,

$$\begin{aligned} G(\Gamma+\Delta\Gamma) &= G(\Gamma) + G_{,p}(\Gamma)\Delta\Gamma^p + (1/2)G_{,pq}(\Gamma)\Delta\Gamma^q\Delta\Gamma^p \\ &\equiv G(\Gamma) + \Delta G(\Gamma), \end{aligned}$$

transforms under (2.10) as

$$\begin{aligned} \Delta G &= G_{,p}X^{pa}\lambda_a - \frac{1}{2}G_{,p}Y^p\lambda^2 + \frac{1}{2}G_{,pq}X^{qb}\lambda_b X^{pa}\lambda_a \\ &= \left(G_{,p}X^{pa}\right)\lambda_a + \frac{1}{2}\left(\frac{1}{2}\varepsilon_{ab}G_{,qp}X^{pa}X^{qb}(-1)^{\varepsilon_q} - G_{,p}Y^p\right)\lambda^2 \\ &= \left(s^a G\right)\lambda_a + \frac{1}{4}\left(s^2 G\right)\lambda^2. \end{aligned} \tag{3.13}$$

From (3.11), (3.12), it follows that

$$\begin{aligned} \mathcal{I}_{\Gamma+\Delta\Gamma} &= d\Gamma\exp\left\{\frac{i}{\hbar}\left[i\hbar\left(\Delta^a S\right)\lambda_a + \frac{i\hbar}{4}\left(s_a\Delta^a S\right)\lambda^2\right]\right\} \\ &\quad\times\exp\left\{\frac{i}{\hbar}\left[\mathcal{S}_F + \left(s^a\mathcal{S}_F\right)\lambda_a + \frac{1}{4}\left(s^2\mathcal{S}_F\right)\lambda^2\right]\right\} \\ &= d\Gamma\exp\left(\frac{i}{\hbar}\mathcal{S}_F\right)\exp\left[\frac{i}{\hbar}\left(s^a\mathcal{S}_F + i\hbar\Delta^a S\right)\lambda_a\right. \\ &\quad\left. + \frac{i}{4\hbar}s_a\left(s^a\mathcal{S}_F + i\hbar\Delta^a S\right)\lambda^2\right] \\ &= d\Gamma\exp\left(\frac{i}{\hbar}\mathcal{S}_F\right) = \mathcal{I}_\Gamma, \end{aligned} \tag{3.14}$$

since $s^a\mathcal{S}_F + i\hbar\Delta^a S = 0$, due to (3.1), which proves that the change of variables $\Gamma^p \to \Gamma^p + \Delta\Gamma^p$ in (2.10) realizes finite BRST–antiBRST transformations. By virtue of (3.9), the Jacobian of finite BRST–antiBRST transformations (2.10) with constant parameters λ_a equals to

$$\exp(\Im) = \exp\left[-\left(\Delta^a S\right)\lambda_a - \frac{1}{4}\left(\Delta^a S\right)\overleftarrow{s}_a\lambda^2\right]. \tag{3.15}$$

4. Ward identities

We can now apply the finite global BRST–antiBRST transformations to obtain the Ward (Slavnov–Taylor) identities for the generating functional of Green's functions (2.4). Namely, using the Jacobian (3.15) of finite BRST–antiBRST transformations with constant parameters λ_a, we make a change of variables (2.10) in the integrand (2.4) for $Z_F(J)$ and arrive at

$$\left\langle\left[1+\frac{i}{\hbar}J_A\phi^A\left(\overleftarrow{s}^a\lambda_a+\frac{1}{4}\overleftarrow{s}^2\lambda^2\right)\right.\right.\\ \left.\left.-\frac{1}{4}\left(\frac{i}{\hbar}\right)^2 J_A\phi^A\overleftarrow{s}^a J_B\left(\phi^B\right)\overleftarrow{s}_a\lambda^2\right]\right\rangle_{F,J}=1. \quad (4.1)$$

Here, the symbol "$\langle\mathcal{O}\rangle_{F,J}$" for a quantity $\mathcal{O}=\mathcal{O}(\Gamma)$ stands for the source-dependent average expectation value corresponding to a gauge-fixing $F(\phi)$, namely,

$$\langle\mathcal{O}\rangle_{F,J}=Z_F^{-1}(J)\int d\Gamma\,\mathcal{O}(\Gamma)\exp\left\{\frac{i}{\hbar}\left[\mathcal{S}_F(\Gamma)+J_A\phi^A\right]\right\},\\ \text{with } \langle 1\rangle_{F,J}=1. \quad (4.2)$$

The relation (4.1) is a Ward identity depending on a doublet of arbitrary constants λ_a and on sources J_A. Using an expansion in powers of λ_a, we obtain, at the first order, the usual Ward identities

$$J_A\left\langle\phi^A\overleftarrow{s}^a\right\rangle_{F,J}=0 \quad (4.3)$$

and a new Ward identity, at the second order:

$$\left\langle J_A\phi^A\left[\overleftarrow{s}^2-\overleftarrow{s}^a(i/\hbar)J_B\left(\phi^B\overleftarrow{s}_a\right)\right]\right\rangle_{F,J}=0. \quad (4.4)$$

5. Freedman–Townsend model

In this section, we illustrate the above construction of finite BRST–antiBRST transformations in general gauge theories by using the example of a well-known theory of non-Yang–Mills type, being the reducible gauge model [25] suggested by Freedman and Townsend, whose Lagrangian quantization and investigation of the unitarity problem have been considered in the BRST [26,27] and BRST–antiBRST [28,29] symmetries. To this end, let us consider the theory of a non-Abelian antisymmetric tensor field $\mathfrak{B}^m_{\mu\nu}$ given in Minkowski space $\mathbb{R}^{1,3}$ by the action [25]

$$S_0(A,\mathfrak{B})=\int d^4x\left(-\frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}F^m_{\mu\nu}\mathfrak{B}^m_{\rho\sigma}+\frac{1}{2}A^m_\mu A^{m\mu}\right), \quad (5.1)$$

with the Lorentz indices $\mu,\nu,\rho,\sigma=0,1,2,3$, the metric tensor $\eta_{\mu\nu}=\mathrm{diag}(-,+,+,+)$, the completely antisymmetric structure constants f^{lmn} of the Lie algebra $su(N)$ for $l,m,n=1,\ldots,N^2-1$, where A^m_μ is a vector gauge field with the strength $F^m_{\mu\nu}\equiv\partial_\mu A^m_\nu-\partial_\nu A^m_\mu+f^{mnl}A^n_\mu A^l_\nu$ (the coupling constant is absorbed into the structure coefficients f^{mnl}), and $\varepsilon^{\mu\nu\rho\sigma}$ is a constant completely antisymmetric four-rank tensor, $\varepsilon^{0123}=1$. The action (5.1) is invariant under the gauge transformations

$$\delta\mathfrak{B}^m_{\mu\nu}=D^{mn}_\mu\zeta^n_\nu-D^{mn}_{\nu\mu}\zeta^n\equiv R^{mn}_{\mu\nu\rho}\zeta^{n\rho},\\ \delta A^m_\mu=0,\quad \text{for } D^{mn}_\mu=\delta^{mn}\partial_\mu+f^{mln}A^l_\mu, \quad (5.2)$$

where ζ^m_μ are arbitrary Bosonic functions, and D^{mn}_μ is the covariant derivative with potential A^m_μ. The algebra of the gauge transformations (5.2) is Abelian, and the generators $R^{mn}_{\mu\nu\rho}$ have at the extremals of the action (5.1) the Bosonic zero-eigenvectors $Z^{mn}_\mu\equiv D^{mn}_\mu$,

$$R^{ml}_{\mu\nu\rho}Z^{ln\rho}=\varepsilon_{\mu\nu\rho\sigma}f^{mln}\frac{\delta S_0}{\delta\mathfrak{B}^l_{\rho\sigma}}, \quad (5.3)$$

which are linearly independent. By the generally accepted terminology [15], the model (5.1)–(5.3) is an Abelian gauge theory of first-stage reducibility. In accordance with the Lagrangian Sp(2)-symmetric quantization [3] for reducible gauge theories, the fields ϕ^A and the corresponding antifields ϕ^*_{Aa}, $\bar{\phi}_A$ for the model (5.1)–(5.3) are given by

$$\phi^A=\left(A^{m\mu};\mathfrak{B}^{m\mu\nu},B^{m\mu},B^{ma},C^{m\mu a},C^{mab}\right),\\ \phi^*_{Aa}=\left(A^{m*}_{\mu a};\mathfrak{B}^{m*}_{\mu\nu a},B^{m*}_{\mu a},B^{m*}_{a|b},C^{m*}_{\mu a|b},C^{m*}_{a|bc}\right),\\ \bar{\phi}_A=\left(\bar{A}^m_\mu;\bar{\mathfrak{B}}^m_{\mu\nu},\bar{B}^m_\mu,\bar{B}^m_a,\bar{C}^m_{\mu a},\bar{C}^m_{ab}\right), \quad (5.4)$$

where B^{ma} and C^{mab} are the respective Sp(2)-doublets of fields introducing the gauge and the ghost fields (symmetric second rank Sp(2)-tensors) of the first stage, in accordance with the number of gauge parameters ζ^m for the generators $R^{mn}_{1\mu\nu}\equiv R^{ml}_{\mu\nu\rho}Z^{ln\rho}$. With account taken of (2.2), (2.3), the Grassmann parity and ghost number of the variables $(\phi^A,\phi^*_{Aa},\bar{\phi}_A)$ are given by

$$\varepsilon\left(A^{m\mu};\mathfrak{B}^{m\mu\nu},B^{m\mu},B^{ma},C^{m\mu a},C^{mab}\right)=(0;0,0,1,1,0), \quad (5.5)$$

$$\mathrm{gh}\left(A^{m\mu};\mathfrak{B}^{m\mu\nu},B^{m\mu},B^{ma},C^{m\mu a},C^{mab}\right)\\ =\left(0;0,0,3-2a,3-2a,6-2(a+b)\right). \quad (5.6)$$

A solution $S=S(\phi,\phi^*,\bar{\phi})$ of the generating equations (2.5) with the boundary condition $S|_{\phi^*=\bar{\phi}=0}=S_0$ for the model (5.1)–(5.3) can be represented in the form being quadratic in powers of the antifields,

$$S=S_0+\int d^4x\Big[\mathfrak{B}^*_{\mu\nu a}\left(D^\mu C^{\nu a}-D^\nu C^{\mu a}-\varepsilon^{\mu\nu\rho\sigma}\bar{\mathfrak{B}}_{\rho\sigma}\wedge B^a\right)\\ -\varepsilon^{ab}C^*_{\mu a|b}B^\mu+\bar{\mathfrak{B}}_{\mu\nu}\left(D^\mu B^\nu-D^\nu B^\mu\right)\\ +C^*_{\mu a|b}D^\mu C^{ab}-2\varepsilon^{ab}C^*_{a|bc}B^c-B^*_{\mu a}D^\mu B^a+2\bar{C}_{\mu a}D^\mu B^a\\ +\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\left(\mathfrak{B}^*_{\mu\nu a}\wedge\mathfrak{B}^*_{\rho\sigma b}\right)C^{ab}\Big], \quad (5.7)$$

with the following notation for the fields $A^m\equiv A$, $B^m\equiv B$:

$$A^mB^m\equiv AB,\qquad D_\mu B\equiv\partial_\mu B+A_\mu\wedge B,\\ (A\wedge B)^m=f^{mnl}A^nB^l. \quad (5.8)$$

Choosing the gauge Boson $F=F(\phi)$ in the form of a 3-parametric quadratic functional,

$$F(\alpha,\beta,\gamma)=\int d^4x\left(-\frac{\alpha}{4}\mathfrak{B}_{\mu\nu}\mathfrak{B}^{\mu\nu}-\frac{\beta}{2}\varepsilon_{ab}C^a_\mu C^{\mu b}\right.\\ \left.-\frac{\gamma}{12}\varepsilon_{ab}\varepsilon_{cd}C^{ac}C^{bd}\right),\\ \text{for } \alpha,\beta,\gamma\in\mathbb{R}, \quad (5.9)$$

and integrating in (2.4) over the variables λ, π^a, $\bar{\phi}$, ϕ^*_a, we obtain the generating functional of Green's functions

$$Z_F(J)=\int d\phi\,\Delta_\alpha(\phi)\exp\left\{(i/\hbar)\left[S_0(A)+S_{\rm gf}(\phi)\right.\right.\\ \left.\left.+S_{\rm fp}(\phi)+J_A\phi^A\right]\right\}, \quad (5.10)$$

identical with that of [29] in the case $(\alpha,\beta,\gamma)=(\alpha_0,\beta_0,\gamma_0)\equiv(1,2,1)$, corresponding to $F_0\equiv F(1,2,1)$, where

$$S_{gf} = \int d^4x \Big(\alpha B_\mu D_\nu \mathfrak{B}^{\nu\mu} + \beta \varepsilon_{ab} B^a D_\mu C^{\mu b} - \beta B_\mu B^\mu - \frac{\gamma}{2} \varepsilon_{ab} B^a B^b \Big), \quad (5.11)$$

$$S_{fp} = \int d^4x \Big(\frac{\alpha}{4} G^a_{\mu\nu} M_{ab} K_c^{b[\mu\nu][\rho\sigma]} G^c_{\rho\sigma} - \frac{\beta}{2} \varepsilon_{ab}\varepsilon_{cd} D_\mu C^{ac} D^\mu C^{bd} \Big), \quad (5.12)$$

$$\Delta_\alpha = \int d\mathfrak{B}^* \exp\Big(\frac{2i}{\alpha\hbar} \int d^4x\, \mathfrak{B}^*_{0ib} M^{bc} \mathfrak{B}^*_{0jc} \eta^{ij} \Big). \quad (5.13)$$

In (5.12), (5.13) we have used the notation

$$K_b^{a[\mu\nu][\rho\sigma]} \equiv \frac{1}{2}\big[\delta^a_b \big(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho} \big) + \alpha X^a_b \varepsilon^{\mu\nu\rho\sigma} \big],$$
$$G^a_{\mu\nu} \equiv D_\mu C^a_\nu - D_\nu C^a_\mu - \frac{\alpha}{4} \varepsilon_{\mu\nu\rho\sigma} Y^a \mathfrak{B}^{\rho\sigma}, \quad (5.14)$$

and the matrix M_{ab} is the inverse of M^{ab},

$$M^{ab} \equiv \varepsilon^{ab} - \alpha^2 X^a_c X^b_d \varepsilon^{cd}, \qquad M^{ac} M_{cb} = \delta^a_b, \quad (5.15)$$

while the action of the matrices X^a_b and Y^a on the objects $E \equiv E^m$ carrying the indices m is given by the rule

$$X^a_b E \equiv \varepsilon_{bc} \big(C^{ac} \wedge E \big), \qquad Y^a E \equiv \big(B^a \wedge E \big) = -(-1)^{\varepsilon(E)} E Y^a. \quad (5.16)$$

For the vanishing sources, $J = 0$, the integrand in (5.10) is invariant under the BRST–antiBRST transformations [28] in the space of fields ϕ^A

$$\delta \mathfrak{B}^{\mu\nu} = -\varepsilon^{ab} M_{bc} K_d^{c[\mu\nu][\rho\sigma]} G^d_{\rho\sigma} \mu_a, \qquad \delta A^\mu = 0,$$
$$\delta C^{\mu a} = \big(D^\mu C^{ab} - \varepsilon^{ab} B^\mu \big) \mu_b,$$
$$\delta B^\mu = D^\mu B^a \mu_a, \qquad \delta C^{ab} = B^{\{a} \varepsilon^{b\}c} \mu_c, \qquad \delta B^a = 0. \quad (5.17)$$

Indeed, the quantum action and the integration measure under the change of variables $\phi^A \to \check{\phi}^A = \phi^A + \delta\phi^A$ are transformed as

$$\delta(S_0 + S_{gf} + S_{fp}) = 0,$$
$$d\check{\phi}\Delta_\alpha(\check{\phi}) = d\phi \Delta_\alpha(\check{\phi})\, \mathrm{Sdet}(\delta\check{\phi}/\delta\phi) = d\phi\Delta_\alpha + \delta(d\phi)\Delta_\alpha + d\phi\delta(\Delta_\alpha) = d\phi\Delta_\alpha(\phi),$$

with $\delta(d\phi) = \delta^4(0) \int d^4x\, \mathrm{Tr}\, W$ and

$$\delta\Delta_\alpha = -\Delta_\alpha \delta^4(0) \int d^4x\, \mathrm{Tr}\, W, \quad (5.18)$$

where $\delta^4(0) \equiv \delta(x-y)|_{x=y}$ and we use the notation

$$W \equiv W^{mn} = -3\alpha^2 \varepsilon^{ab} M_{bc} X^c_d Y^d \mu_a, \quad \text{for } \mathrm{Tr}\, W \equiv \sum_{m=1}^{N^2-1} W^{mm}. \quad (5.19)$$

The functional Δ_α in (5.13) is a contribution to the integration measure $d\phi\Delta_\alpha$, being invariant, $\delta(d\phi\Delta_\alpha) = 0$, under the BRST–antiBRST transformations (5.17). At the same time, we notice that these transformations depend explicitly on the parameter α of the gauge Boson F in (5.9). Due to a non-trivial integration measure and BRST–antiBRST transformations depending on a choice of the gauge Boson, the task of connecting (by finite BRST–antiBRST transformations) the generating functionals $Z_F(J)$ and $Z_{F+\Delta F}(J)$ given by different gauges F and $F + \Delta F$ in the representation (5.10) cannot be solved literally on the basis of our approach [1], developed on the basis of a compensation equation for Yang–Mills type theories, and deserves a special analysis [30]. In this connection, we restrict the consideration to the quantum theory (5.7), (5.9), with the generating functional $Z_F(J)$ given by the functional integral (2.4) in the extended space ϕ, ϕ^*_a, $\bar{\phi}$, π^a, λ, where (omitting the $su(N)$ indices m)

$$\pi^{Aa} = \big(\pi^{\mu a}_{(A)}; \pi^{\mu\nu a}_{(\mathfrak{B})}, \pi^{\mu a}_{(B)}, \pi^{a|b}_{(B)}, \pi^{\mu a|b}_{(C)}, \pi^{a|bc}_{(C)} \big),$$
$$\lambda^A = \big(\lambda^\mu_{(A)}; \lambda^{\mu\nu}_{(\mathfrak{B})}, \lambda^\mu_{(B)}, \lambda^a_{(B)}, \lambda^{\mu a}_{(C)}, \lambda^{ab}_{(C)} \big). \quad (5.20)$$

Using cumbersome but simple calculations, one can present the finite transformations (2.11) for the generating functional $Z_F(J)$ in (2.4) for the model under consideration with the quantum action S given by (5.7). At the same time, for the purpose of connecting the integrand $\mathcal{I}^{(F_0+\Delta F)}_\Gamma$ of $Z_{F_0+\Delta F}(J)$ given by a gauge $F_0 + \Delta F$ with the one given by a gauge F_0, so that $\mathcal{I}^{(F_0+\Delta F)}_\Gamma = \mathcal{I}^{(F_0)}_\Gamma$, as suggested in Discussion below, it is sufficient, due to the solution of the compensation equation (6.5), to find the explicit form of $\lambda_a(\phi, \pi, \lambda|\Delta F)$ in (6.7). To this end, let us consider a finite change of the gauge condition:

$$\Delta F = F(\alpha, \beta, \gamma) - F_0 = \int d^4x \Big(-\frac{\alpha - \alpha_0}{4} \mathfrak{B}_{\mu\nu}\mathfrak{B}^{\mu\nu} - \frac{\beta - \beta_0}{2} \varepsilon_{ab} C^a_\mu C^{\mu b} - \frac{\gamma - \gamma_0}{12} \varepsilon_{ab}\varepsilon_{cd} C^{ac} C^{bd} \Big). \quad (5.21)$$

The corresponding field-dependent BRST–antiBRST transformations (2.11) which provide the coincidence of the vacuum functionals, $Z_{F_0+\Delta F} = Z_{F_0}$, are determined by the functionally-dependent odd-valued parameters:

$$\lambda_a(\phi, \pi, \lambda|\Delta F) = -\frac{1}{2i\hbar} \sum_{n=1} \frac{1}{n!} \Big[\frac{1}{4i\hbar} \Delta F \overleftarrow{s}^2 \Big]^n (\Delta F \overleftarrow{s}_a). \quad (5.22)$$

6. Discussion

In the present work, we have proved that the finite BRST–antiBRST transformations for a general gauge theory in Lagrangian formalism announced in [1] are actually invariance transformations for the integrand in the path integral $Z_F(0)$, given by (2.4). To this end, we have explicitly calculated the Jacobian (3.15) corresponding to the given change of variables with constant parameters λ_a. Using the finite BRST–antiBRST transformations, we have obtained the Ward identity (4.1) depending on constant parameters λ_a. The identity contains the usual Sp(2)-doublet of Ward identities, as well as a new Ward identity at the second order in powers of λ_a. We have illustrated the construction of finite BRST–antiBRST transformations in general gauge theories by the example of a reducible gauge model of a non-Abelian antisymmetric tensor field [25].

In conclusion, note that the structure of finite BRST–antiBRST transformations with field-dependent parameters,

$$\Delta\Gamma^p = \Gamma^p \Big(\overleftarrow{s}^a \lambda_a + \frac{1}{4} \overleftarrow{s}^2 \lambda^2 \Big), \quad \lambda_a = s_a \Lambda, \ \Lambda = \Lambda(\phi, \pi, \lambda), \quad (6.1)$$

is the same as in the case of finite field-dependent BRST–antiBRST transformations in the Lagrangian formalism for Yang–Mills theories [1], as well as in the case of the generalized Hamiltonian formalism [20]. Consequently, it is natural to expect that the Jacobian corresponding to this change of variables with functionally-dependent (due to $s^1\lambda_1 + s^2\lambda_2 = -s^2\Lambda$) parameters, inspired by the infinitesimal field-dependent BRST–antiBRST transformations of [1–3], should have the form[1]

$$\exp(\Im) = \exp\left[-(\Delta^a S)\lambda_a - \frac{1}{4}(\Delta^a S)\overleftarrow{s}_a\lambda^2\right]\exp\left[\ln(1+f)^{-2}\right],$$
$$\text{with } f = -\frac{1}{2}\Lambda\overleftarrow{s}^2, \tag{6.2}$$

$$d\check{\Gamma} = d\Gamma\exp\left[\frac{i}{\hbar}(-i\hbar\Im)\right] = d\Gamma\exp\left\{\frac{i}{\hbar}\left[i\hbar(\Delta^a S)\lambda_a + \frac{i\hbar}{4}(\Delta^a S)\overleftarrow{s}_a\lambda^2 + i\hbar\ln\left(1-\frac{1}{2}\Lambda\overleftarrow{s}^2\right)^2\right]\right\}. \tag{6.3}$$

Here, $\Lambda(\phi,\pi,\lambda)$ is a certain even-valued potential with a vanishing ghost number, and the integration measure $d\Gamma$ transforms with respect to the change of variables $\Gamma \to \check{\Gamma} = \Gamma + \Delta\Gamma$ given by (6.1). Hence, a compensation equation required to satisfy the relation

$$Z_{F+\Delta F} = Z_F, \tag{6.4}$$

as one subjects $Z_{F+\Delta F}$ to a change of variables $\Gamma^p \to \check{\Gamma}^p$, according to (6.1), has the form

$$i\hbar\ln\left(1-\frac{1}{2}\Lambda\overleftarrow{s}^2\right)^2 = -\frac{1}{2}\Delta F\overleftarrow{s}^2 \quad\Leftrightarrow\quad \left(1-\frac{1}{2}\Lambda\overleftarrow{s}^2\right)^2 = \exp\left(\frac{i}{2\hbar}\Delta F\overleftarrow{s}^2\right), \tag{6.5}$$

or, equivalently,

$$\frac{1}{2}\Lambda\overleftarrow{s}^2 = 1 - \exp\left(\frac{1}{4i\hbar}\Delta F\overleftarrow{s}^2\right). \tag{6.6}$$

The solution of this equation for an unknown Bosonic functional $\Lambda(\phi,\pi,\lambda)$, which determines $\lambda_a(\phi,\pi,\lambda)$ in accordance with $\lambda_a = \Lambda\overleftarrow{s}_a$, with accuracy up to BRST–antiBRST exact (s^a being restricted to ϕ, π_a, λ) terms, is given by

$$\Lambda(\Gamma|\Delta F) = \frac{1}{2i\hbar}g(y)\Delta F,$$
$$\text{for } g(y) = \left[1-\exp(y)\right]/y \text{ and } y \equiv \frac{1}{4i\hbar}\Delta F\overleftarrow{s}^2, \tag{6.7}$$

whence the corresponding field-dependent parameters have the form

$$\lambda_a(\Gamma|\Delta F) = \frac{1}{2i\hbar}g(y)(\Delta F\overleftarrow{s}_a). \tag{6.8}$$

Making in (2.4) a field-dependent BRST–antiBRST transformation (6.1) and using the relations (4.2) and (6.3), one can obtain a *modified Ward* (*Slavnov–Taylor*) *identity*:

$$\left\langle\left\{1+\frac{i}{\hbar}J_A\phi^A\left[\overleftarrow{s}^a\lambda_a(\Lambda)+\frac{1}{4}\overleftarrow{s}^2\lambda^2(\Lambda)\right] - \frac{1}{4}\left(\frac{i}{\hbar}\right)^2 J_A\phi^A\overleftarrow{s}^a J_B(\phi^B)\overleftarrow{s}_a\lambda^2(\Lambda)\right\}\left(1-\frac{1}{2}\Lambda\overleftarrow{s}^2\right)^{-2}\right\rangle_{F,J} = 1. \tag{6.9}$$

Due to the presence of $\Lambda(\Gamma)$, which implies $\lambda_a(\Lambda)$, the modified Ward identity depends on a choice of the gauge Boson $F(\phi)$ for non-vanishing J_A, according to (6.7), (6.8). Notice that the corresponding Ward identities for Green's functions, obtained by differentiating (6.9) with respect to the sources, contain the functionals $\lambda_a(\Lambda)$ and their derivatives as weight functionals. The Ward identities are readily established due to (6.9) for constant λ_a in the form (4.3), (4.4). Finally, (6.9), with account taken of (6.8), implies the following equation, which describes the gauge dependence for a finite change of the gauge $F \to F + \Delta F$:

$$Z_{F+\Delta F}(J) = Z_F(J)\left\{1+\left\langle\frac{i}{\hbar}J_A\phi^A\left[\overleftarrow{s}^a\lambda_a(\Gamma|-\Delta F)+\frac{1}{4}\overleftarrow{s}^2\lambda^2(\Gamma|-\Delta F)\right] - (-1)^{\varepsilon_B}\left(\frac{i}{2\hbar}\right)^2 J_B J_A(\phi^A\overleftarrow{s}^a)(\phi^B\overleftarrow{s}_a)\lambda^2(\Gamma|-\Delta F)\right\rangle_{F,J}\right\}, \tag{6.10}$$

thereby extending (6.4) to the case of non-vanishing J_A. Note that we have proved our conjecture as to the representation (6.2), (6.3) of the Jacobian for field-dependent BRST–antiBRST transformations with functionally-dependent parameters in [31].

We have shown, on the basis of field-dependent BRST–antiBRST transformations, the way to reach an arbitrary gauge, determined by a quadratic (in the fields) gauge Boson (5.9) for the Freedman–Townsend model in the path integral representation, starting from the reference frame with a gauge Boson F_0 and using finite field-dependent BRST–antiBRST transformations with the parameters $\lambda_a(\phi,\pi,\lambda|\Delta F)$ given by (5.22).

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