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Fermionic anticommutators for open superstrings in the presence of antisymmetric tensor field

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Abstract

We build up the anticommutator algebra for the fermionic coordinates of open superstrings attached to branes with antisymmetric tensor fields. We use both Dirac quantization and the symplectic Faddeev–Jackiw approach. In the symplectic case we find a way of generating the boundary conditions as zero modes of the symplectic matrix by taking a discretized form of the action and adding terms that vanish in the continuous limit. This way boundary conditions can be handled as constraints.

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1. Introduction

Non-commutativity of spacetime and its consequences for quantum field theory have been one of the main objects of interest for theoretical particle physicists in the last years. A general discussion and an important list of references can be found in [1]. An important source of non-commutativity of space in string theory [2,3] is the presence of an antisymmetric constant tensor field along the D-brane [4] world volumes (where the string endpoints are located). The quantization of strings attached to branes involves mixed (combination of Dirichlet and Neumann) boundary conditions. This makes the quantization procedure more subtle since the quantum commutators/anticommutators must be consistent with these boundary conditions. For the bosonic string coordinates, the non-commutativity at end points has already received much attention. Many important aspects have been discussed and the commutators have been explicitly calculated (see, for example, [5–11]).

In contrast, the complete canonical structure (anticommutators) for the fermionic coordinates when antisymmetric tensor fields are present has not yet been presented explicitly, although some important aspects have already been discussed [5,12,13]. As we will see here, the requirement of consistency with boundary conditions affect the structure of the anticommutation relations at the string endpoints even in the absence of any external field. We start calculating the complete Dirac antibrackets for the fermionic coordinates consistent with the boundary conditions

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of strings attached to branes with antisymmetric tensor field. We will do this by introducing a discretization along the string spacelike coordinate. Such a construction make it transparent the behavior of the anticommutators at the end points. The string boundary conditions will lead to a discontinuity in the anticommutator at the endpoints. Also it will emerge that, in contrast to the standard canonical form, the anticommutator between the fermionic components $\psi^{\mu}_{(+)}, \psi^{\mu}_{-}$ is not vanishing at the endpoints even in the absence of the antisymmetric field.

Then we consider the symplectic quantization scheme and develop a procedure of generating the fermionic string boundary conditions as constraints directly from the symplectic matrix. We will consider again a discretization of the string world sheet spatial coordinate. In a previous article [11] we found the boundary conditions for the bosonic string from the corresponding symplectic matrix by means of some field redefinitions. Here we will improve such a procedure making use of the fact that a finite number of terms that vanish in the continuous limit may be added to the discretized form of the action. By choosing appropriate terms we will get the boundary conditions as zero modes of the fermionic symplectic matrix. Then the anticommutators will be calculated in the standard way.

2. The model

Let us start with a superstring coupled to an antisymmetric tensor field living on a brane. Considering just the coordinates along the brane, the action can be represented in superspace as

$$S = \frac{-i}{8\pi\alpha'} \int_{\Sigma} d^2\sigma \, d^2\theta \left(\overline{D} Y^{\mu} D Y_{\mu} + \mathcal{F}_{\mu\nu} \overline{D} Y^{\mu} \rho_5 D Y_{\mu} \right), \tag{1}$$

where the superfield

$$Y^{\mu}\left(\sigma^a, \theta\right) = X^{\mu}\left(\sigma^a\right) + \bar{\theta}\psi^{\mu}\left(\sigma^a\right) + 1/2\bar{\theta}\theta B^{\mu}\left(\sigma^a\right)$$

contains the bosonic and fermionic spacetime string coordinates. In components the action reads[1]

$$S = \frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma \left(\eta_{\mu\nu} \partial_a X^{\mu} \partial^a X^{\nu} + \epsilon_{ab} \mathcal{F}_{ij} \partial_a X^i \partial_b X^j - B^{\mu} B_{\mu} - i \bar{\psi}^{\mu} \rho^a \partial_a \psi_{\mu} + i \mathcal{F}_{\mu\nu} \bar{\psi}^{\mu} \rho_b \epsilon^{ab} \partial_a \psi^{\nu} \right). \tag{3}$$

The bosonic and fermionic sectors decouple. We will consider just the fermionic sector once the bosonic sector was already discussed [5–11]. The fermions are Majorana and can be represented as

$$\psi^{\mu} = \begin{pmatrix} \psi^{\mu}_{(-)} \\ \psi^{\mu}_{(+)} \end{pmatrix}. \tag{4}$$

So that the fermionic sector reads

$$S_0 = \frac{-i}{4\pi\alpha'} \int_{\Sigma} d\tau \, d\sigma \left(\psi^{\mu}_{(-)} \partial_+ \psi_{(-)\mu} + \psi^{\mu}_{(+)} \partial_- \psi_{(+)\mu} - \mathcal{F}_{\mu\nu} \psi^{\mu}_{(-)} \partial_+ \psi^{\nu}_{(-)} + \mathcal{F}_{\mu\nu} \psi^{\mu}_{(+)} \partial_- \psi^{\nu}_{(+)} \right). \tag{5}$$

The minimum action principle $\delta S = 0$ leads to a volume term that vanishes when the equations of motion hold and also to a surface term:

$$\left(\psi^{\mu}_{(-)} (\eta_{\mu\nu} - \mathcal{F}_{\mu\nu}) \delta\psi^{\nu}_{(-)} - \psi^{\mu}_{(+)} (\eta_{\mu\nu} + \mathcal{F}_{\mu\nu}) \delta\psi^{\nu}_{(+)} \right)\Big|_0^{\pi} = 0. \tag{6}$$

[1] Our conventions are

$$\rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \qquad \rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \rho_5 \equiv \rho^0 \rho^1, \qquad \partial_{\pm} = \partial_0 \pm \partial_1. \tag{2}$$

It is not possible to find non-trivial boundary conditions involving $\psi^{\mu}_{(-)}$ and $\psi^{\mu}_{(+)}$ that makes this surface term vanish. However, the solution to this problem shows up when we take into account a result from Ref. [12] (see also [14]). There it was shown that in order to keep supersymmetry unbroken at the string endpoints it is necessary to include a boundary term to the action. Actually, considering the boundary term (6), we realize that it is impossible even to solve the boundary condition unless some extra term is added to the action. The interesting thing is that the same kind of term proposed in [12] in order to restore SUSY at the end points

$$S_{\text{Bound}} = \frac{i}{2\pi\alpha'} \int_{\Sigma} d\tau\, d\sigma\, \left(\mathcal{F}_{\mu\nu}\psi^{\mu}_{(+)}\partial_{-}\psi^{\nu}_{(+)}\right) \tag{7}$$

will make it possible to find a solution to the boundary condition. Adding this term to S_0 the total action reads

$$S = \frac{-i}{4\pi\alpha'} \int_{\Sigma} d\tau\, d\sigma\, \left(\psi^{\mu}_{(-)}\mathbf{E}^{\nu\mu}\partial_{+}\psi_{(-)\nu} + \psi^{\mu}_{(+)}\mathbf{E}^{\nu\mu}\partial_{-}\psi_{(+)\nu}\right), \tag{8}$$

where $\mathbf{E}^{\mu\nu} = \eta^{\mu\nu} + \mathcal{F}^{\mu\nu}$. The corresponding boundary term coming from $\delta S = 0$ is now

$$\left(\psi^{\mu}_{(-)}\mathbf{E}_{\nu\mu}\delta\psi^{\nu}_{(-)} - \psi^{\mu}_{(+)}\mathbf{E}_{\nu\mu}\delta\psi^{\nu}_{(+)}\right)\Big|_0^{\pi} = 0. \tag{9}$$

This condition is satisfied imposing the constraint that preserve supersymmetry [5]

$$\mathbf{E}_{\nu\mu}\psi^{\nu}_{(+)}(0,\tau) = \mathbf{E}_{\mu\nu}\psi^{\nu}_{(-)}(0,\tau), \tag{10}$$

$$\mathbf{E}_{\nu\mu}\psi^{\nu}_{(+)}(\pi,\tau) = \lambda\mathbf{E}_{\mu\nu}\psi^{\nu}_{(-)}(\pi,\tau), \tag{11}$$

at the endpoints $\sigma = 0$ and $\sigma = \pi$, where $\lambda = \pm 1$ with the plus sign corresponding to Ramond boundary condition and the minus corresponding to the Neveu–Schwarz case. We will only consider the endpoint $\sigma = 0$ in our calculations. The results for $\sigma = \pi$ have the same form.

Now considering the total fermionic action S we want to incorporate the boundary conditions (10) in a quantum formulation of the theory. That means: we want to calculate anticommutators that are consistent with these boundary conditions. Following the approach successfully applied to the bosonic sector (see, for example, [6–11]) we will consider a discrete version of the string in which we replace the continuous coordinate σ with range $(0,\pi)$ by a discrete set corresponding to intervals of length ϵ. Representing the fermionic coordinates at the endpoints of the N intervals as: $\psi^{\nu}_{0(-)}, \psi^{\nu}_{1(-)}, \dots, \psi^{\nu}_{N(-)}; \psi^{\nu}_{0(+)}, \psi^{\nu}_{1(+)}, \dots, \psi^{\nu}_{N(+)}$, the discretized form of the Lagrangian reads

$$\begin{aligned} L = \frac{-i}{4\pi\alpha'}\Bigg(&\epsilon\psi^{\mu}_{0(-)}\mathbf{E}_{\nu\mu}\partial_0\psi^{\nu}_{0(-)} + \epsilon\psi^{\mu}_{1(-)}\mathbf{E}_{\nu\mu}\partial_0\psi^{\nu}_{1(-)} + \cdots + \epsilon\psi^{\mu}_{0(-)}\mathbf{E}_{\nu\mu}\frac{\psi^{\nu}_{1(-)} - \psi^{\nu}_{0(-)}}{\epsilon} \\ &+ \epsilon\psi^{\mu}_{1(-)}\mathbf{E}_{\nu\mu}\frac{\psi^{\nu}_{2(-)} - \psi^{\nu}_{1(-)}}{\epsilon} + \cdots + \epsilon\psi^{\mu}_{0+}\mathbf{E}_{\nu\mu}\partial_0\psi^{\nu}_{0+} + \epsilon\psi^{\mu}_{1+}\mathbf{E}_{\nu\mu}\partial_0\psi^{\nu}_{1+} + \cdots \\ &- \epsilon\psi^{\mu}_{0+}\mathbf{E}_{\nu\mu}\frac{\psi^{\nu}_{1+} - \psi^{\nu}_{0+}}{\epsilon} - \epsilon\psi^{\mu}_{1+}\mathbf{E}_{\nu\mu}\frac{\psi^{\nu}_{2+} - \psi^{\nu}_{1+}}{\epsilon} + \cdots\Bigg). \end{aligned} \tag{12}$$

The original theory is recovered by taking the limit $\epsilon \to 0$.

3. Dirac quantization

The equal time canonical antibrackets for the original continuous fermionic fields are:

$$\begin{aligned} &\left\{\psi^{\mu}_{(+)}(\sigma), \psi^{\nu}_{(+)}(\sigma')\right\} = \left\{\psi^{\mu}_{(-)}(\sigma), \psi^{\nu}_{(-)}(\sigma')\right\} = -2\pi i\alpha'\eta^{\mu\nu}\delta(\sigma - \sigma'), \\ &\left\{\psi^{\mu}_{(+)}(\sigma), \psi^{\nu}_{(-)}(\sigma')\right\} = 0. \end{aligned} \tag{13}$$

So that the canonical antibrackets for the corresponding discrete fermionic variables become

$$\begin{aligned} &\{\psi^\mu_{i(+)}, \psi^\nu_{j(+)}\} = \{\psi^\mu_{i(-)}, \psi^\nu_{j(-)}\} = -\frac{2\pi i\alpha' \delta_{ij}\eta^{\mu\nu}}{\epsilon}, \\ &\{\psi^\mu_{i(+)}, \psi^\nu_{j(-)}\} = 0. \end{aligned} \tag{14}$$

The discrete version of the boundary condition (10), that we will impose as a constraint in the Dirac formalism, is

$$\Omega_\mu \equiv \mathbf{E}_{\nu\mu}\psi^\nu_{0(+)} - \mathbf{E}_{\mu\nu}\psi^\nu_{0(-)}.$$

So, the matrix of constraints is

$$M_{\mu\nu} \equiv \{\Omega_\mu, \Omega_\nu\} = \frac{-4\pi i\alpha'}{\epsilon}\left(\eta_{\mu\rho} - \mathcal{F}^\nu_\mu \mathcal{F}_{\nu\rho}\right) = \frac{-4\pi i\alpha'}{\epsilon}\left(\mathbf{1} - \mathcal{F}^2\right)_{\mu\rho}, \tag{15}$$

and the Dirac (anti-) brackets are calculated in the standard way:

$$\{A, B\}_D = \{A, B\} - \{A, \Omega_\mu\} M^{-1}_{\mu\nu}\{\Omega_\nu, B\}. \tag{16}$$

For the coordinates $\psi^\mu_{i\pm}$ with $i \neq 0$, corresponding to points inside the string they will be equal to the Poisson brackets but for the boundary coordinates we get:

$$\{\psi^\mu_{0(+)}, \psi^\nu_{0(+)}\} = \{\psi^\mu_{0(-)}, \psi^\nu_{0(-)}\} = -\frac{\pi i\alpha'\eta^{\mu\nu}}{\epsilon}, \tag{17}$$

$$\{\psi^\mu_{0(+)}, \psi^\nu_{0(-)}\} = -\frac{\pi i\alpha'}{\epsilon}\left(\eta^{\mu\gamma} + \mathcal{F}^{\mu\gamma}\right)\left(\left[1 - \mathcal{F}^2\right]^{-1}\right)_{\gamma\rho}\left(\eta^{\rho\nu} + \mathcal{F}^{\rho\nu}\right). \tag{18}$$

The anticommutators (17) agree with the results found previously in Ref. [5]. Now we can obtain the continuous version of our results. Once the anticommutators of $\psi^\mu_{i(\pm)}$ for $i \neq 0$ are not changed by the Dirac quantization, inside the string ($0 \leqslant \sigma \leqslant \pi$) the anticommutators keep their canonical form. Then, for the boundary points, we use the fact that the mapping between continuous and discrete expressions involve the mapping of Kronecker and Dirac deltas in the following way: $\delta_{ij}/\epsilon \Leftrightarrow \delta(\sigma_i - \sigma_j)$ (note that expressions (17), (18) involve a factor $\delta_{00} = 1$). The anticommutators of the points inside the string and on the boundary may be accommodated in one single expression if we introduce a parameter β such that $\beta = 1/2$ for $\sigma = \sigma' = 0$ or $\beta = 1$ elsewhere. The continuous limit of the Dirac antibrackets is then

$$\{\psi^\mu_{(+)}(\sigma), \psi^\nu_{(+)}(\sigma')\} = \{\psi^\mu_{(-)}(\sigma), \psi^\nu_{(-)}(\sigma')\} = -2\beta\pi i\alpha'\eta^{\mu\nu}\delta(\sigma - \sigma'), \tag{19}$$

$$\{\psi^\mu_{(+)}(\sigma), \psi^\nu_{(-)}(\sigma')\} = -\pi i\alpha'\left(\eta^{\mu\gamma} + \mathcal{F}^{\mu\gamma}\right)\left(\left[1 - \mathcal{F}^2\right]^{-1}\right)_{\gamma\rho}\left(\eta^{\rho\nu} + \mathcal{F}^{\rho\nu}\right)\delta(\sigma - \sigma') \tag{20}$$

for $\sigma = \sigma' = 0$ and zero elsewhere except for the other endpoint $\sigma = \sigma' = \pi$ where the same kind of relation holds but with a sign depending on choosing Ramond or Neveu–Schwarz boundary conditions. It is important to note that the anticommutator (20) does not vanish even in the absence of the antisymmetric tensor field. This result is consistent with the boundary condition (10) that relates $\psi^\mu_{(+)}$ and $\psi^\nu_{(-)}$ at the string endpoints.

4. Symplectic quantization

Let us now see how the fermionic anticommutators can be calculated using the symplectic Faddeev–Jackiw quantization [15]. We need particularly the analysis of constraints and gauge symmetries in the symplectic quantization developed in [16–18].

We consider a Lagrangian that is first order in time derivatives (if the original Lagrangian is not in this form one can introduce auxiliary fields and change it to first order).

$$L^0 = a^0_k(q)\partial_\tau q_k - V(q), \tag{21}$$

where q_k are the generalized coordinates of the system. For bosonic variables the symplectic matrix is defined as

$$f^0_{kl} = \frac{\partial a^0_l}{\partial q_k} - \frac{\partial a^0_k}{\partial q_l}. \tag{22}$$

If it is non-singular we define the commutators of the quantum theory (if there is no ordering problem for the corresponding quantum operators) as

$$\big[A(q), B(q)\big] = \frac{\partial A}{\partial q_k}\big(f^0\big)^{-1}_{kl}\frac{\partial B}{\partial q_l}. \tag{23}$$

If the matrix (22) is singular we find the zero modes that satisfy $f^0_{kl} v^\alpha_l = 0$ and the corresponding constraints:

$$\Omega^\alpha = v^\alpha_l \frac{\partial V}{\partial q_l} \approx 0. \tag{24}$$

Then we introduce new variables λ^α and add a new term to the kinetic part of Lagrangian

$$L^1 = a^0_k(q)\dot{q}_k + \dot{\lambda}^\alpha \Omega^\alpha - V(q) \equiv a^1_r(\tilde{q})\dot{\tilde{q}}_r - V(q), \tag{25}$$

where we introduced the new notation for the extended variables: $\tilde{q}^r = (q^k, \lambda^\alpha)$. We find now the new matrix f^1_{rs}

$$f^1_{rs} = \frac{\partial a^1_s}{\partial \tilde{q}_r} - \frac{\partial a^1_r}{\partial \tilde{q}_s}. \tag{26}$$

If f^1 is not singular we define the quantum commutators as

$$\big[A(\tilde{q}), B(\tilde{q})\big] = \frac{\partial A}{\partial \tilde{q}_r}\big(f^1\big)^{-1}_{rs}\frac{\partial B}{\partial \tilde{q}_s}. \tag{27}$$

This process of incorporating the constraints in the Lagrangian is repeated until a non-singular matrix is found.

In the present case we are dealing with fermionic string coordinates. For fermionic variables Ψ_i we define

$$a_{\Psi_i} = \frac{\partial L}{\partial(\partial_\tau)\Psi_i}, \tag{28}$$

as in the bosonic case, but the symplectic matrix takes the form

$$f_{\Psi_i\Psi_j} = \frac{\partial a_{\Psi_j}}{\partial \Psi_i} + \frac{\partial a_{\Psi_i}}{\partial \Psi_j}. \tag{29}$$

The procedure of incorporating constraints then is the same as in the bosonic case.

In the previous section, the boundary conditions did not show up directly from the Dirac procedure. That means, the method of quantization itself did not generate the boundary conditions. We had to impose them as additional constraints. In the symplectic approach, in contrast, we will find the boundary conditions from the zero modes of the symplectic matrix. We do not get this result if we use directly action (12) as our starting point. However, we note that the individual terms in this discrete form of the action tend to zero in the limit $\epsilon \to 0$. So if we remove or add a finite number of them we do not change the continuous limit $\epsilon \to 0$ corresponding to the original action of Eq. (8). However the symplectic matrix in the discrete variables changes and this will make it possible to find the boundary conditions as zero modes of the symplectic matrix. A possible way to do this is to include in the action the extra term

$$\frac{i}{4\pi\alpha'}\psi^\mu_{0(+)}\mathbf{E}_{\mu\nu}\big(\psi^\nu_{1(-)} - \psi^\nu_{0(-)}\big), \tag{30}$$

that vanishes in the limit $\epsilon \to 0$ and then redefine the variables as

$$\psi^\mu_{(\pm)i} \equiv \frac{\tilde{\psi}^\mu_{(\pm)i}}{\sqrt{\epsilon}} \quad (i \neq 0), \qquad \psi^\mu_{(0)+} \equiv \frac{\tilde{\psi}^\mu_{(0)+}}{\sqrt{\epsilon}}, \qquad \psi^\mu_{(0)(-)} \equiv \tilde{\psi}^\mu_{(0)(-)}$$

in the Lagrangian and we will see that this will make it possible to generate the appropriate boundary condition that mixes the $(+)$ and $(-)$ components.

The symplectic matrix takes the form

$$\begin{array}{c} \\ \tilde\psi^\mu_{0(+)} \\ \tilde\psi^\mu_{0(-)} \\ \tilde\psi^\mu_{1(+)} \\ \tilde\psi^\mu_{1(-)} \\ \tilde\psi^\mu_{2(+)} \\ \tilde\psi^\mu_{2(-)} \\ \cdots \end{array}
\begin{array}{c} \begin{array}{ccccccc} \tilde\psi^\nu_{0(+)} & \tilde\psi^\nu_{0(-)} & \tilde\psi^\nu_{1(+)} & \tilde\psi^\nu_{1(-)} & \tilde\psi^\nu_{2(+)} & \tilde\psi^\nu_{2(-)} & \cdots \end{array} \\
\begin{pmatrix} -2g^{\mu\nu} & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & -2\epsilon g^{\mu\nu} & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & -2g^{\mu\nu} & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & -2g^{\mu\nu} & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & -2g^{\mu\nu} & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & -2g^{\mu\nu} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix} \end{array} \tag{31}$$

(times a factor $1/i4\pi\alpha'$). In the limit $\epsilon \to 0$ this symplectic matrix becomes singular. The zero mode corresponds to the vector

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}, \tag{32}$$

and the corresponding constraints come from

$$\frac{\partial V}{\partial \tilde\psi^\mu_{0(-)}} = 0. \tag{33}$$

Considering the inclusion of the extra crossed term of Eq. (30) in the potential V, we find that in the $\epsilon \to 0$ limit the constraints, returning to the original variables, are

$$\Omega^\mu = \mathbf{E}^{\mu\nu}\psi^\nu_{0(-)} - \mathbf{E}^{\mathrm{Tr}\,\mu\nu}\psi^\nu_{0(+)} = 0. \tag{34}$$

Then we introduce a Lagrange multiplier λ^μ and include the term $\dot\lambda^\mu\Omega_\mu$ in the Lagrangian. Returning to the original fermionic variables, the symplectic matrix becomes

$$\begin{array}{c} \\ \psi^\mu_{0(+)} \\ \psi^\mu_{0(-)} \\ \psi^\mu_{1(+)} \\ \psi^\mu_{1(-)} \\ \psi^\mu_{2(+)} \\ \psi^\nu_{2(+)} \\ \cdots \\ \lambda^\mu \end{array}
\begin{array}{c} \begin{array}{cccccccc} \psi^\nu_{0(+)} & \psi^\nu_{0(-)} & \psi^\nu_{1(+)} & \psi^\nu_{1(-)} & \psi^\nu_{2(+)} & \psi^\nu_{2(-)} & \cdots & \lambda^\nu \end{array} \\
\begin{pmatrix} -2\epsilon g^{\mu\nu} & 0 & 0 & 0 & 0 & 0 & \cdots & -\mathbf{E}^{\mu\nu} \\ 0 & -2\epsilon g^{\mu\nu} & 0 & 0 & 0 & 0 & \cdots & \mathbf{E}^{\nu\mu} \\ 0 & 0 & -2\epsilon g^{\mu\nu} & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & -2\epsilon g^{\mu\nu} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & -2\epsilon g^{\mu\nu} & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & -2\epsilon g^{\mu\nu} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ -\mathbf{E}^{\nu\mu} & +\mathbf{E}^{\mu\nu} & 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \end{array} \tag{35}$$

(again times a factor $1/i4\pi\alpha'$). Inverting this matrix we find the anticommutators

$$\{\psi^\mu_{0(+)}, \psi^\nu_{0(+)}\} = \{\psi^\mu_{0(-)}, \psi^\nu_{0(-)}\} = \frac{-\pi i\alpha'}{\epsilon} g^{\mu\nu}, \tag{36}$$

$$\{\psi^\mu_{0(+)}, \psi^\nu_{0(-)}\} = -\frac{\pi i\alpha'}{\epsilon}\left(\eta^{\mu\gamma} + \mathcal{F}^{\mu\gamma}\right)\left(\left[1-\mathcal{F}^2\right]^{-1}\right)_{\gamma\rho}\left(\eta^{\rho\nu} + \mathcal{F}^{\rho\nu}\right). \tag{37}$$

The anticommutators (36) agree with the previous result from [5] and reproduce the result obtained in the Dirac quantization.

5. Conclusion

We have calculated the fermionic anticommutators at a string endpoint by Dirac and symplectic quantization. In both cases, the discretization of the string spatial coordinate made it more easy to handle the discontinuity in the antibrackets associated to the effect of the boundary conditions. In the symplectic case we found a way of getting the boundary conditions from the symplectic matrix by adding terms that vanish in the continuous limit.

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