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# Cosmological constraints on a power law universe

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## Abstract

Linearly coasting cosmology is comfortably concordant with a host of cosmological observations. It is surprisingly an excellent fit to SNe Ia observations and constraints arising from age of old quasars. In this Letter we highlight the overall viability of an open linear coasting cosmological model. The model is consistent with the latest SNe Ia “gold” sample and accommodates a very old high-redshift quasar, which the standard cold–dark model fails to do.

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## 1. Introduction

In the past there has been a spurt of activity to explain the observed “accelerated expansion” of the universe. Classes of  $\Lambda$ CDM models as well as quintessence models have been designed to accommodate such an expansion deduced from observations on high-redshift Supernovae Ia (SNe Ia) [1].

The SNe Ia look fainter than they are expected in the standard Einstein–de Sitter model, which was the favoured model prior to these observations. In standard cosmology, these results, when combined with the latest CMB data and clustering estimates, are used

to make out a case for a universe in which accelerated expansion is fueled by a self-interacting, unclustered fluid, with high negative pressure, collectively known as Dark Energy (for latest review see [2]), the simplest and the most favoured candidate being the cosmological constant  $\Lambda$ . Consequently, several models with a relic cosmological constant ( $\Lambda$ CDM), have been used to best describe the observed universe. However, most of them suffer from severe fine tuning problems [2,3]. The basic reason is the wide spread belief that the early universe evolved through a cascade of phase transitions, thereby yielding a vacuum energy density which is presently 120 orders of magnitude smaller than its value at the Planck time. Such a discrepancy between theoretical expectations and empirical observations constitute a fundamental problem at interface

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of astrophysics, cosmology and particle physics. In the last few years, several attempts have been made to alleviate the cosmological constant problem. For example, in the so-called dynamical  $\Lambda(t)$  scenarios (or deflationary cosmology), the cosmological term is a function of time and its presently observed value is a remnant of primordial inflationary/deflationary stage [4]. Other examples are scenarios in which the evolution of classical fields are coupled to the curvature of the space–time background in such a way that their contribution to the energy density self-adjusts to cancel the vacuum energy [5], as well as some recent ideas of a SU(2) cosmological instanton dominated universe [6]. At least in the two later examples, an interesting feature is a power-law growth for the cosmological scale factor  $a(t) \approx t^\alpha$ , where  $\alpha$  may be constrained by observations.

In a series of earlier articles, we have explored the viability of a model that has  $a(t) \approx t^\alpha$  with  $\alpha \gtrsim 1$  [7–11]. The motivation for such an endeavor comes from several considerations. Such models do not have a horizon problem. Moreover, the scale factor in such theories does not constrain the density parameter and therefore, they are free from flatness problem. There are also observational motivations for considering power-law cosmologies. For  $\alpha \geq 1$ , the predicted age of the universe is  $t_0 \geq H_0^{-1}$ , i.e., at least 50% greater than the prediction of the standard flat model (without cosmological constant). This makes the universe comfortably in agreement with the recent age estimates of globular clusters and high- $z$  redshift galaxies.

A linear evolution of the scale factor is supported in some alternative gravity theories [7], as well as in standard model with a specially chosen equation of state [12]. As described in Refs. [5–11], a power law coasting in a hyperbolic background, independent of the background equation of state emerges in a class of non-minimally coupled theories [5,7] as well as instanton dominated models [6]. Interestingly, there have been serious misgivings in expecting Einstein’s equations to hold for an averaged distribution for an inhomogeneous universe [13–15]. In [13], an inhomogeneous fractal model with no dark energy was considered to introduce a new solution to the averaging problem by identifying cosmic time as measured by observers in voids. A linear coasting FRW (Milne) model emerges as a consequence!

It was reported by Dev et al. [8] that this model is consistent with gravitational lensing statistics (within  $1\sigma$  level) and the constraints from the ages of old high redshift galaxies. It was also demonstrated that this model is consistent with primordial nucleosynthesis [9]. For  $\Omega = 0.65$  and  $\eta = 7.8 \times 10^{-9}$ , the model with  $\alpha = 1$  yields  $\text{He}^4 = 0.23$  and metallicity of the range  $10^{-7}$  [11]. Linear coasting surprising clears preliminary constraints on structure formation and CMB anisotropy [10].

In this article we explore the concordance of an open linear coasting model with the latest SNe Ia data and bounds from age estimates of old quasars. In Section 2, we give the basic equations for the model adopted. In Section 3.1, we constrain parameter  $\alpha$  by using SNe Ia “Gold Sample”. The lower bound on  $\alpha$  from the age estimates of an old high-redshift quasar is discussed in Section 3.2. We summarize the results in Section 4.

## 2. Linear coasting cosmology

We consider a general power law cosmology with the scale factor given in terms of an arbitrary dimensionless parameter  $\alpha$

$$a(t) = \frac{c}{H_0} \left( \frac{t}{t_0} \right)^\alpha \quad (1)$$

for an open FRW metric

$$ds^2 = c^2 dt^2 - a^2(t) \left[ \frac{dr^2}{1+r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]. \quad (2)$$

Here  $t$  is cosmic proper time and  $r, \theta, \phi$  are comoving spherical coordinates.

The expansion rate of the universe is described by a Hubble parameter,  $H(t) = \dot{a}/a = \alpha/t$ . The *present* expansion rate of the universe is defined by a Hubble *constant*, equal in our model to  $H_0 = \alpha/t_0$  (here and subsequently the subscript 0 on a parameter refers to its present value). The scale factor and the redshift are related to their present values by  $a/a_0 = (t/t_0)^\alpha$ . As usual, the ratio of the scale factor at the emission and absorption of a null ray determines the cosmological redshift  $z$  by

$$\frac{a_0}{a(z)} = 1 + z, \quad (3)$$

and the age of the universe is

$$t_z = \frac{\alpha}{H_0(1+z)^{1/\alpha}}. \quad (4)$$

Using (3), we define the dimensionless Hubble parameter

$$h(z) \equiv \frac{H(z)}{H_0} = (1+z)^{1/\alpha}. \quad (5)$$

The present ‘radius’ of the universe is defined as (see Eq. (1))

$$a_0 = \frac{c}{H_0}. \quad (6)$$

In terms of the parameter  $\alpha$ , the luminosity distance between two redshifts  $z_1$  and  $z_2$  is

$$\begin{aligned} d_L(z_1, z_2) &= \frac{c(1+z_2)}{H_0} \\ &\times \sinh \left[ \frac{\alpha}{\alpha-1} \left\{ (1+z_2)^{\frac{\alpha-1}{\alpha}} - (1+z_1)^{\frac{\alpha-1}{\alpha}} \right\} \right]. \end{aligned} \quad (7)$$

In a limiting case,  $\alpha \rightarrow 1$ , we obtain

$$d_L(z_1, z_2) = \frac{c}{2H_0} \frac{(1+z_2)^2 - (1+z_1)^2}{(1+z_1)}. \quad (8)$$

The look-back time, the difference between the age of the universe when a particular light ray was emitted and the age of the universe now, is

$$\frac{c dt}{dz_L} = \frac{c}{H_0(1+z_L)^{\frac{\alpha+1}{\alpha}}}. \quad (9)$$

### 3. The observational tests

#### 3.1. Constraints from SNe Ia data

Of late, properties of type Ia supernovae (SNe Ia) as excellent cosmological standard candles has elevated the status of the Hubble flow to that of a precision measurement. The magnitude of a ‘‘standard candle’’ is related to its luminosity distance  $d_L$  through

$$m(z) = M + 5 \log_{10} \left[ \frac{d_L}{\text{Mpc}} \right] + 25, \quad (10)$$

where  $M$  is the absolute magnitude and is assumed to be constant for a standard candle like SNe Ia. The apparent magnitude can also be expressed in terms of dimensionless luminosity distance  $\mathcal{D}_L(z)$  as

$$m(z) = \mathcal{M} + 5 \log_{10} \mathcal{D}_L(z), \quad (11)$$

with

$$\mathcal{D}_L(z) = \frac{H_0}{c} d_L \quad (12)$$

and

$$\begin{aligned} \mathcal{M} &= M + 5 \log_{10} \left( \frac{c/H_0}{1 \text{ Mpc}} \right) + 25 \\ &= M - 5 \log_{10} h + 42.38. \end{aligned} \quad (13)$$

For our analysis we use ‘‘gold’’ sample compiled by Reiss et al. [16]. The sample consists of 157 data points which are in the terms of distance modulus

$$\begin{aligned} \mu_{\text{obs}} &= m(z) - M \\ &= 5 \log_{10} \mathcal{D}_L(z) - 5 \log_{10} h + 42.38. \end{aligned} \quad (14)$$

The best fit model to the observations is obtained by using  $\chi^2$  statistics, i.e.,

$$\chi^2 = \sum_{i=1}^{157} \left[ \frac{\mu_{\text{th}}^i - \mu_{\text{obs}}^i}{\sigma_i} \right]^2, \quad (15)$$

where  $\mu_{\text{th}}$  is the predicted distance modulus for a supernova at redshift  $z$  and  $\sigma_i$  is the dispersion of the measured distance modulus due to intrinsic and observational uncertainties in SNe Ia peak luminosity. In order to integrate over the Hubble constant, we use the modified  $\chi^2$  statistics as defined in the Ref. [17]

$$\bar{\chi}^2 = \chi_*^2 - \frac{C_1}{C_2} \left( C_1 + \frac{2}{5} \ln 10 \right) - 2 \ln h^*. \quad (16)$$

Here  $h^*$  is the fiducial value of the dimensionless Hubble constant, and

$$\chi_*^2 \equiv \sum_i \left[ \frac{\mu_{\text{th}}^{*(i)} - \mu_{\text{obs}}^i}{\sigma_i} \right]^2, \quad (17)$$

$$C_1 \equiv \sum_i \frac{\mu_{\text{th}}^{*(i)} - \mu_{\text{obs}}^i}{\sigma_i^2}, \quad (18)$$

$$C_2 \equiv \sum_i \frac{1}{\sigma_i^2}, \quad (19)$$

with

$$\begin{aligned} \mu_{\text{th}}^{*(i)}(z_i, h = h^*) \\ = 5 \log_{10} \mathcal{D}_L(z) - 5 \log_{10} h + 42.38. \end{aligned} \quad (20)$$

For our calculations, we use  $h^* = 0.72$ . We work with the following range of the parameter  $\alpha$ :  $0.0 \leq \alpha \leq 3.0$ . We perform a grid search in the parametric space to find the best fit model. For a one parameter fit, the 68% confidence level (CL) (90% CL) corresponds to  $\Delta \bar{\chi}^2 = 1.0$  (2.71).

Fig. 1 shows variation of  $\bar{\chi}^2$  with  $\alpha$ . We find that the minimum of  $\bar{\chi}^2$ , i.e.  $\bar{\chi}_{\text{min}}^2$  occurs for  $\alpha = 1.04$ , with  $\bar{\chi}_v^2 = 1.23$  ( $\bar{\chi}_v^2 = \bar{\chi}_{\text{min}}^2/\text{degree of freedom}$ ). The SNe Ia data thus provides the following constraints:  $0.98 \leq \alpha \leq 1.11$  at 68% CL and  $0.95 \leq \alpha \leq 1.15$  at 90% CL.

### 3.2. Constraints from age estimates of an old, high- $z$ quasar

The age estimates of old high redshift objects play a very important role in constraining cosmological pa-

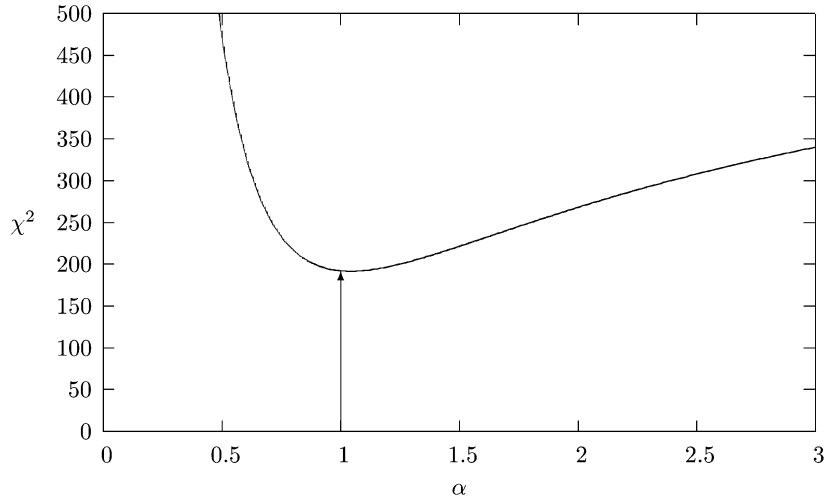


Fig. 1. Variation of  $\bar{\chi}^2$  with  $\alpha$ . The arrow corresponds to the minimum value of  $\bar{\chi}^2$  which occurs for  $\alpha = 1.04$ .

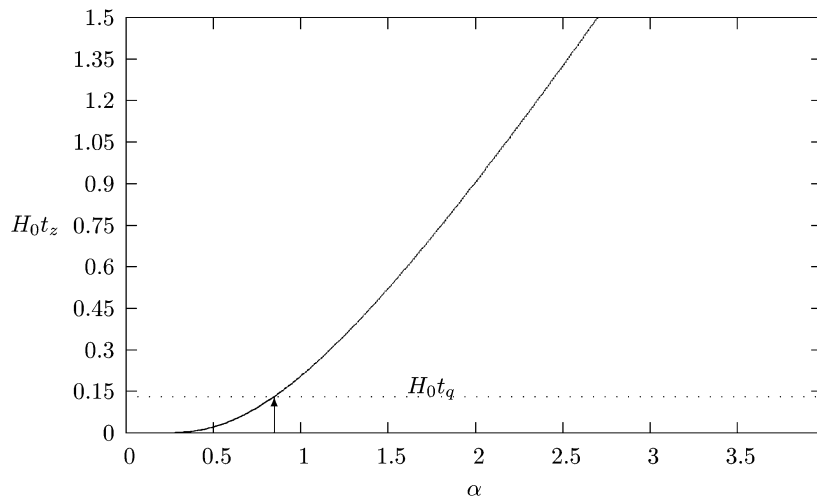


Fig. 2. Variation of dimensionless age parameter,  $H_0 t_z$ , as a function of  $\alpha$  for  $z = 3.91$ . The dotted line corresponds to the dimensionless age parameter,  $H_0 t_q$ , of the old quasar at this redshift. It is clear that the lower bound on value of  $\alpha$  to accommodate the quasar is 0.85.

rameters [18]. The recently discovered quasar APM 08279+5255 at a redshift of  $z = 3.91$  is very important object in this regard [19]. Conservative estimates of its age have been made from iron enrichment in detailed chemodynamical modelling and give a staggering value of at least 2 Gyr for this object. Standard flat FRW models with cosmological constant fail to accommodate this old, high-redshift quasar [20]. In this Letter we use this quasar to put limits on the  $\alpha$  in power law cosmologies.

The age-redshift relationship in power-law cosmology is given as

$$t_z = \frac{\alpha}{H_0(1+z)^{1/\alpha}}.$$

In order to constrain  $\alpha$  from the age estimate of the above mentioned quasar we follow Ref. [18]. The age of the universe at a given redshift has to be greater than or at least equal to the age of its oldest objects at that redshift. In a power law cosmology the age of the universe increases with increasing  $\alpha$ . Hence, this test provides lower bound on  $\alpha$ . This can be checked if we define the dimensionless ratio:

$$\frac{t_z}{t_q} = \frac{f(\alpha, z)}{H_0 t_q} \geq 1, \quad (21)$$

where  $t_q$  is the age of an old object (here the quasar) at a given redshift and  $f(\alpha, z) = \alpha/(1+z)^{1/\alpha}$ , is a dimensionless factor. For every high-redshift object,  $T_q = H_0 t_q$  is a dimensionless age parameter. The error bar on  $H_0$  determines the extreme value of  $T_q$ . The lower limit on  $H_0$  is updated to nearly 10% of accuracy by Freedman [21]:  $H_0 = 72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . So the 2 Gyr old quasar at  $z = 3.91$  gives  $T_q = 2.0 H_0$  Gyr and hence  $0.131 \leq T_q \leq 0.163$ . We use minimal value of the Hubble constant,  $H_0 = 64 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , to get strong conservative limit. It thus follows that  $T_q \geq 0.131$ . Only those values of  $\alpha$  are allowed for which the age of the universe at  $z = 3.91$  equals to or is greater than the age of the quasar at that redshift, i.e.,  $H_0 t_z(z = 3.91) \geq H_0 t_q$ .

Fig. 2 shows the variation of dimensionless age parameter  $H_0 t_z(z = 3.91)$  as a function of  $\alpha$ . The horizontal line in the figure corresponds to the age of the quasar which is  $T_q = 0.131$ . It is clear from the figure that  $\alpha$  should be at least 0.85 in order to allow this quasar to exist in power law cosmology.

Table 1  
Constraints on  $\alpha$  from various cosmological tests

Method	Reference	$\alpha$
Lensing statistics (Optical sample)		
(i) $n_L$	Dev et al. [8]	$1.09 \pm 0.3$
(ii) Likelihood analysis	Dev et al. [8]	$1.13^{+0.4}_{-0.3}$
OHRG	Dev et al. [8]	$\geq 0.8$
SNe Ia (Gold sample)	This Letter	$1.04^{+0.07}_{-0.06}$
Old quasar	This Letter	$\geq 0.85$

#### 4. Discussions

Recent observations of Type Ia supernovae lead to the discovery of an accelerating universe. This accelerated expansion has been attributed to a dark energy component with high negative pressure. The simplest model for dark energy is the cosmological constant that comes with its theoretical and fine tuning problems. We have been exploring alternative models of universe which have the potential of explaining these observations.

The main results of this Letter along with the constraints obtained from the gravitational lensing statistics of the optical sample and age estimates of old high-redshift galaxies [8] are summarized in Table 1. The motivation for our work was to establish the viability of a linear coasting cosmology  $a(t) = t$ . Using SNe Ia data, we find that such a model is well accommodated within  $1\sigma$ :  $0.98 \leq \alpha \leq 1.15$ . The age estimates of the old, high-redshift quasar at  $z = 3.91$  give the lower bound  $\alpha \geq 0.85$  for a power law cosmology. We thus find that  $\alpha = 1.0$  is in concordance with the observational tests listed in Table 1. It is interesting to observe that the SNe Ia data and the age estimates of the old, high-redshift quasar rule out an Einstein–de Sitter universe ( $\alpha = 2/3$ ). We conclude that the coasting cosmology with strictly linear evolution of scale factor,  $a(t) = t$ , is in excellent agreement with these observations.

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