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Systematic study on QCD interactions of heavy mesons with ρ meson

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Abstract

The strong interactions of the negative-parity heavy mesons with ρ meson may be described consistently in the context of an effective Lagrangian, which is invariant under isospin $SU(2)$ transformation. Four coupling constants $g_{HH\rho}$, $f_{H^*H\rho}$, $g_{H^*H^*\rho}$ and $f_{H^*H^*\rho}$ enter the effective Lagrangian, where H (H^*) denotes a pseudoscalar bottom or charm meson (the corresponding vector meson). Using QCD light cone sum rule (LCSR) method and, as inputs, the hadronic parameters updated recently, we give an estimate of $g_{H^*H^*\rho}$ and $f_{H^*H^*\rho}$, about which little was known before, and present an improved result for $g_{HH\rho}$ and $f_{H^*H\rho}$. Also, we examine the heavy quark asymptotic behavior of these nonperturbative quantities and assess the two low energy parameters β and λ of the corresponding effective chiral Lagrangian.

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1. Introduction

At present, we have the two well-established theoretical frameworks for describing a large class of two body hadronic decays of B mesons, that is, QCD factorization (QCDF) [1] approach and soft collinear effective theory (SCET) [2]. Long-distance parameters enter inevitably, however, as important inputs in their phenomenological applications. One is yet confronted with the difficult task to cope with the nonperturbative problems. Numerous theoretical works are devoted to this subject. Among all the existing nonperturbative approaches, QCD light cone sum rules (LCSR's) [3,4] have proved to be particularly powerful. This is due to the facts: (1) Contrary to conventional sum rule calculations [5] on the form factors for heavy to light decays, LCSR results turn out to be consistent in the heavy quark limit $m_Q \to \infty$ [6]; (2) LCSR approach allows us to consistently explore the dependence of the form factors on the momentum transfer q^2 in the whole kinematically accessible range [7–10], by combing its results, which are valid for small and intermediate q^2, with the pole model description for the form factors at large q^2. The LCSR method has extensively been applied to study the semileptonic [4,6–12] and hadronic [13] decays of heavy mesons. A recent LCSR reanalysis of heavy-to-light transitions can be found in [12].

Along with the great progresses in the experiment on B physics, we are confronting a more formidable challenge to deal with the nonperturbative dynamics. The data accumulated at B factories and CLEO give a hint that there may be large contributions from the final state interactions (FSI's), which are typically nonperturbative, in some of two body hadronic B decays. In the absence of a rigorous approach to FSI's, one may either resort to Regge theory [14] to estimate their effects [15], or mimic them by the soft rescattering of two intermediate particles so that they could be viewed as a one particle exchange process calculable at the hadron level. In comparison, the latter is of more intuitive physical picture and thus is accepted more readily. Employing the one particle

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exchange model, calculations of the FSI's in both B and D decays have been undertaken many a time in the literature [16–18]. For a recent application of this approach, ones are referred to [17,18]. A precondition of doing such a calculation, however, is that the related couplings are supposed to be known, which parameterize the strong interactions among the underlying meson fields. The most interesting is the situation that heavy mesons interact with a light meson, where the corresponding couplings are also crucial to determine the normalization of heavy to light form factors at large momentum transfer in the pole dominance models [7–10]. These couplings have to be assessed adopting a certain phenomenological method, except that few of them can be extracted directly from the experimental data. In the case where the light meson concerned is a pseudoscalar meson, the related couplings have undergone a systematic investigation, in the frameworks of both LCSR's [7,10,19] and conventional sum rules [20]. In contrast, the existing discussion is incomplete about the interactions of heavy mesons with a light vector meson, despite some efforts being made [16,21–23].

The strong interactions can be described between the negative-parity heavy mesons and ρ meson by constructing an effective Lagrangian which respects the $SU(2)$ symmetry in the isospin space. Letting us define an isospin doublet B composed of the pseudoscalar bottom meson fields B^+ and B^0 and the corresponding vector doublet B^*_μ:

$$B = \begin{pmatrix} B^+ \\ B^0 \end{pmatrix}, \qquad B^*_\mu = \begin{pmatrix} B^{*+}_\mu \\ B^{*0}_\mu \end{pmatrix},$$

with the Hermitian conjugate forms

$$B^\dagger = \begin{pmatrix} B^- & \bar{B}^0 \end{pmatrix}, \qquad B^{*\dagger}_\mu = \begin{pmatrix} B^{*-}_\mu & \bar{B}^{*0}_\mu \end{pmatrix},$$

and representing the isospin triplet of the ρ meson field by

$$P_\mu = \begin{pmatrix} \rho^0_\mu & \sqrt{2}\rho^+_\mu \\ \sqrt{2}\rho^-_\mu & -\rho^0_\mu \end{pmatrix},$$

we can build the effective Lagrangian of the required symmetry as

$$\mathcal{L} = i g_{BB\rho} \operatorname{Tr}\big[\big(B^\dagger \overleftrightarrow{\partial}_\mu B\big) P^\mu\big] - 2 f_{B^*B\rho} \varepsilon^{\mu\nu\alpha\beta} \operatorname{Tr}\big[\big(B^\dagger \overleftrightarrow{\partial}_\mu B^*_\nu - B^{*\dagger}_\nu \overleftrightarrow{\partial}_\mu B\big)\partial_\alpha P_\beta\big] + i g_{B^*B^*\rho} \operatorname{Tr}\big[\big(\bar{B}^{*\dagger}_\mu \overleftrightarrow{\partial}_\nu B^{*\mu}\big) P^\nu\big]$$
$$+ 4 i f_{B^*B^*\rho} m_{B^*} \operatorname{Tr}\big[\big(B^{*\dagger}_\mu B^*_\nu\big)\big(\partial^\mu P^\nu - \partial^\nu P^\mu\big)\big] \tag{1}$$

and analogous one for the charm mesons. In the established effective Lagrangian, four coupling constants $g_{BB\rho}$, $f_{B^*B\rho}$, $g_{B^*B^*\rho}$ and $f_{B^*B^*\rho}$ are introduced, as a result of $SU(2)$ isospin asymmetry, to describe the strength of the strong interactions among the related meson fields. Whereas $g_{BB\rho}$ and $f_{B^*B\rho}$ serve as describing the $BB\rho$ and $B^*B\rho$ interactions respectively, the other two as characterizing the $B^*B^*\rho$ interactions. As explained clearly later, these couplings are of definite physical meaning and in the limit $m_Q \to \infty$, they coincide, up to a prefactor, with one of the two low energy parameters β and λ, which parameterize the effective chiral Lagrangian for the heavy mesons and light vector resonances [23]. Such that the effective description formulated in (1) is consistent with the effective chiral Lagrangian approach. In term of these couplings the relevant hadronic matrix elements are parameterized as:

$$\big\langle \bar{B}^0(p)\rho^-(q,\epsilon)\big|i\mathcal{L}\big|B^-(p+q)\big\rangle = 2\sqrt{2} i g_{BB\rho}\, p\cdot\epsilon^*, \tag{2}$$

$$\big\langle \bar{B}^{*0}(p,\eta)\rho^-(q,\epsilon)\big|i\mathcal{L}\big|B^-(p+q)\big\rangle = -4\sqrt{2} i f_{B^*B\rho} \epsilon_{\mu\alpha\beta\gamma} \eta^{*\mu} q^\alpha \epsilon^{*\beta} p^\gamma, \tag{3}$$

$$\big\langle \bar{B}^{*0}(p,\eta)\rho^-(q,\epsilon)\big|i\mathcal{L}\big|B^{*-}(p+q,\xi)\big\rangle = -2\sqrt{2} i g_{B^*B^*\rho}(\eta^*\cdot\xi)(p\cdot\epsilon^*)$$
$$- 4\sqrt{2} i f_{B^*B^*\rho} m_{B^*}\big[(\eta^*\cdot\epsilon^*)(\xi\cdot q) - (\xi\cdot\epsilon^*)(\eta^*\cdot q)\big], \tag{4}$$

where the momentum and polarization vector assignment is specified in brackets. These hadronic matrix elements, as those parameterizing the strong interaction processes of heavy mesons and a pionic meson, would play a prominent role in the phenomenological study of heavy flavor physics. As aforementioned, however, for the time being we are devoid of an all-around knowledge of them. The previous LCSR calculation is just confined to the case of $g_{BB\rho}$ and $f_{B^*B\rho}$ [21], and the effective parameters β and λ are merely investigated on the basis of the vector dominance assumption [17,23]. On the other hand, the existing LCSR results call for a recalculation with an updated hadronic parameter.

In this Letter, we intend to give a LCSR estimate of $g_{B^*B^*\rho}$ ($g_{D^*D^*\rho}$) and $f_{B^*B^*\rho}$ ($f_{D^*D^*\rho}$), along with an improved numerical prediction of $g_{BB\rho}$ ($g_{DD\rho}$) and $f_{B^*B\rho}$ ($f_{D^*D\rho}$), and then make an investigation into m_Q scaling behavior of the resultant sum rules, present our LCSR results for the effective parameters β and λ.

2. LCSR calculation on strong couplings

We focus on the bottom case and begin with a discussion of the $B^*B^*\rho$ coupling. For implementing a QCD LCSR calculation on $g_{B^*B^*\rho}$ and $f_{B^*B^*\rho}$, it is advisable to make use of the following correlation function:

$$H_{\mu\nu}(p,q,e)=i\int d^4x\, e^{ipx}\langle\rho^-(q,\epsilon)|T\bar{d}(x)\gamma_\mu b(x),\bar{b}(0)\gamma_\nu u(0)|0\rangle$$
$$=H\big(p^2,(p+q)^2\big)g_{\mu\nu}p\cdot\epsilon^*+\tilde{H}\big(p^2,(p+q)^2\big)\big(q_\mu\epsilon^*_\nu-q_\nu\epsilon^*_\mu\big)+\cdots, \tag{5}$$

where ellipses indicate the remaining Lorentz structures. The hadronic form of the correlation function (5) is easily obtained by saturating it with a complete set of intermediate states with the same quantum numbers as the interpolating current operators. However, we need to do it with care, for the vector current operators can also couple with the set of scalar bottom meson with positive-parity, besides that of vector bottom meson. On taking into account all the possible hadronic contributions to $H_{\mu\nu}(p,q,e)$, we find that the invariant functions H and $\tilde{H}$ receive only the contributions from the set of vector bottom meson. Isolating the pole contribution of the lowest B^* meson and parameterizing these from the higher states in a form of double dispersion integral starting with the threshold s_0, we have the desired hadronic forms for $H(p^2,(p+q)^2)$ and $\tilde{H}(p^2,(p+q)^2)$:

$$H^h\big(p^2,(p+q)^2\big)=\frac{-2\sqrt{2}m^2_{B^*}f^2_{B^*}g_{B^*B^*\rho}}{(p^2-m^2_{B^*})[(p+q)^2-m^2_{B^*}]}+\iint\frac{\rho^h(s_1,s_2)\,ds_1\,ds_2}{(s_1-p^2)[s_2-(p+q)^2]}, \tag{6}$$

$$\tilde{H}^h\big(p^2,(p+q)^2\big)=\frac{4\sqrt{2}m^3_{B^*}f^2_{B^*}f_{B^*B^*\rho}}{(p^2-m^2_{B^*})[(p+q)^2-m^2_{B^*}]}+\iint\frac{\tilde{\rho}^h(s_1,s_2)\,ds_1\,ds_2}{(s_1-p^2)[s_2-(p+q)^2]}, \tag{7}$$

with f_{B^*}, as defined usually, being the decay constant of B^* meson and $\rho^h(\tilde{\rho}^h)$ the hadron spectral function.

QCD calculation of the correlator (5) can be carried out for the negative and large values of $p^2-m_Q^2$ and $(p+q)^2-m_Q^2$, which render the operator product expansion (OPE) valid near the light-cone $x^2=0$. Since the underlying heavy quark is sufficiently far off shell in the kinematical regions, in terms of the light-cone expansion the soft gluon emissions from the heavy quark contribute just a higher twist effect, which is concerned with the quark–antiquark–gluon ($q\bar{q}g$) components of the ρ meson distribution amplitudes. As verified by the numerous LCSR calculations, omitting the gluon emission contributions may be considered a better approximation. For the present calculation, we will use the free b quark propagator:

$$\langle 0|Tb(x)\bar{b}(0)|0\rangle=\frac{1}{(2\pi)^4 i}\int d^4k\, e^{-ik\cdot x}\frac{\not{k}+m_b}{m_b^2-k^2}. \tag{8}$$

Substituting (8) in (5) and using the γ algebraic relations

$$\sigma_{\mu\nu}=i(\gamma_\mu\gamma_\nu-g_{\mu\nu}) \tag{9}$$

and

$$\gamma_\mu\sigma_{\rho\lambda}=i(g_{\mu\rho}\gamma_\lambda-g_{\mu\lambda}\gamma_\rho)+\varepsilon_{\mu\rho\lambda\theta}\gamma^\theta\gamma_5, \tag{10}$$

we are led to the light cone wavefunctions of the ρ meson defined by [24–26]

$$\langle\rho(q,e)|\bar{d}(x)\gamma_\mu u(0)|0\rangle=f_\rho m_\rho\Bigg\{\frac{e^{(\lambda)*}\cdot x}{q\cdot x}q_\mu\int_0^1 du\, e^{iuq\cdot x}\bigg[\varphi_\parallel(u,\mu)+\frac{m_\rho^2x^2}{16}A(u,\mu)\bigg]$$
$$+\bigg(e_\mu^{(\lambda)*}-q_\mu\frac{e^{(\lambda)*}\cdot x}{q\cdot x}\bigg)\int_0^1 du\, e^{iuq\cdot x}g_\perp^{(v)}(u,\mu)-\frac{1}{2}x_\mu\frac{e^{(\lambda)*}\cdot x}{(q\cdot x)^2}m_\rho^2\int_0^1 du\, e^{iuq\cdot x}C(u,\mu)\Bigg\}, \tag{11}$$

$$\langle\rho(q,e)|\bar{d}(x)u(0)|0\rangle=-\frac{i}{2}f_\rho^T m_\rho^2\big(e^\lambda\cdot x\big)\int_0^1 du\, e^{iuq\cdot x}h_\parallel^s(u,\mu_b), \tag{12}$$

$$\langle\rho(q,e)|\bar{d}(x)\sigma^{\alpha\beta}u(0)|0\rangle=-if_\rho^T\Bigg\{\big(e_{(\lambda)}^{*\alpha}q^\beta-e_{(\lambda)}^{*\beta}q^\alpha\big)\int_0^1 du\, e^{iuq\cdot x}\bigg[\varphi_\perp(u)+\frac{1}{16}m_\rho^2x^2A_T(u)\bigg]+\big(q^\alpha x^\beta-q^\beta x^\alpha\big)\frac{e^*_{(\lambda)}\cdot x}{(q\cdot x)^2}m_\rho^2$$
$$\times\int_0^1 du\, e^{iuq\cdot x}B_T(u)+\frac{1}{2}\big(e_{(\lambda)}^{*\alpha}x^\beta-e_{(\lambda)}^{*\beta}x^\alpha\big)\frac{m_\rho^2}{q\cdot x}\int_0^1 du\, e^{iuq\cdot x}C_T(u)\Bigg\}, \tag{13}$$

$$\langle\rho(q,e)|\bar{d}(x)\gamma_\mu\gamma_5 u(0)|0\rangle = \frac{1}{4} f_\rho m_\rho \varepsilon_{\mu\alpha\beta\gamma} q^\alpha e^\beta x^\gamma \int_0^1 du\, e^{iuq\cdot x} g_\perp^{(a)}(u,\mu_b), \tag{14}$$

where f_ρ stands for the usual decay constant of the ρ meson, and f_ρ^T is defined as $\langle 0|\bar{u}\sigma_{\mu\nu}d|\rho\rangle = if_\rho^T(e_\mu^{(\lambda)}q_\nu - e_\nu^{(\lambda)}q_\mu)$; both $\varphi_\parallel(u,\mu)$ and $\varphi_\perp(u,\mu)$ denote the leading twist-2 distribution amplitudes, $g_\perp^{(v)}(u,\mu)$, $g_\perp^{(a)}(u,\mu)$ and $h_\parallel^s(u,\mu_b)$ refer to the twist-3 ones, and the others are all of twist-4. With all these expressions, a straightforward calculation yields the following QCD forms for $H(p^2,(p+q)^2)$ and $\tilde{H}(p^2,(p+q)^2)$:

$$\begin{aligned} H^{\mathrm{QCD}}\big(p^2,(p+q)^2\big) = &-\left\{ f_\rho m_\rho \int_0^1 du \frac{\varphi_\parallel(u)}{m_b^2-(p+uq)^2} + f_\rho^T m_\rho^2 m_b \int_0^1 du \frac{h_\parallel^{(s)}(u,\mu_b)}{[m_b^2-(p+uq)^2]^2} \right. \\ &\left. - \frac{1}{2} f_\rho m_\rho^3 \int_0^1 du \left[\frac{A(u)}{[m_b^2-(p+uq)^2]^2} + \frac{m_b^2(A(u)+8\tilde{C}(u))}{[m_b^2-(p+uq)^2]^3} \right] \right\}, \end{aligned} \tag{15}$$

$$\begin{aligned} \tilde{H}^{\mathrm{QCD}}\big(p^2,(p+q)^2\big) = {}& m_b f_\rho^T \int_0^1 du \frac{\varphi_\perp(u)}{m_b^2-(p+uq)^2} \\ &+ \frac{1}{4} f_\rho m_\rho \int_0^1 du \left[\frac{g_\perp^{(a)}(u) + 4(u\frac{dg_\perp^{(a)}}{du}(u) + g_\perp^{(v)}(u))}{m_b^2-(p+uq)^2} + \frac{2m_b^2 g_\perp^{(a)}(u)}{[m_b^2-(p+uq)^2]^2} \right] \\ &- f_\rho^T m_b m_\rho^2 \int_0^1 du \left[\frac{m_b^2 A_T(u)}{2[m_b^2-(p+uq)^2]^3} + \frac{2\tilde{B}_T(u) + u\bar{C}_T(u)}{[m_b^2-(p+uq)^2]^2} \right] \\ &+ \frac{1}{4} f_\rho m_\rho^3 \int_0^1 du \left[\frac{\bar{A}(u) - 8u\tilde{C}(u)}{[m_b^2-(p+uq)^2]^2} + \frac{2m_b^2\bar{A}(u)}{[m_b^2-(p+uq)^2]^3} \right]. \end{aligned} \tag{16}$$

In the derivations of (15) and (16) we have introduced two auxiliary functions $\bar{f}(u) = \int_0^u f(v)\,dv$ and $\tilde{f}(u) = \int_0^u \bar{f}(v)\,dv$. It is needed to convert both the QCD expressions into a form of double dispersion integral, for matching them onto the individual hadronic forms. However, we note that it is sufficient to do it only for the twist-2 and 3 parts. The relevant QCD spectral densities are easily obtained with the standard method [29]. To this end, the following formula is useful:

$$\hat{B}_{M_1^2}\hat{B}_{M_2^2} \frac{(l-1)!}{[m_b^2-(p+uq)^2]^l} = \frac{(M^2)^{2-l}}{M_1^2 M_2^2} e^{-1/M^2[m_b^2+m_\rho^2 u_0(1-u_0)]}\delta(u-u_0), \tag{17}$$

where $\hat{B}_{M_1^2}$ and $\hat{B}_{M_2^2}$ are the Borel operators, the Borel parameters M_1^2 and M_2^2 are associated with p^2 and $(p+q)^2$ respectively, $M^2 = M_1^2 M_2^2/(M_1^2+M_2^2)$ and $u_0 = M_1^2/(M_1^2+M_2^2)$. Simultaneously, we can set $M_1^2 = M_2^2$, because of the symmetry of the correlator, so that the distribution amplitudes entering the QCD spectral densities take only their values at the symmetry point $u_0 = 1/2$. Here we omit the final expressions for the QCD spectral functions to save some spaces.

To proceed, we perform the double Borel transformation $-p^2 \to M_1^2$, $-(p+q)^2 \to M_2^2$ for both the hadronic and QCD representations. The use of the quark–hadron duality results in the final sum rules for the products $f_{B^*}^2 g_{B^*B^*\rho}$ and $f_{B^*}^2 f_{B^*B^*\rho}$:

$$\begin{aligned} f_{B^*}^2 g_{B^*B^*\rho} = {}& \frac{\sqrt{2}}{4m_{B^*}^2} e^{\frac{m_{B^*}^2}{M^2}} \left\{ f_\rho m_\rho M^2 \left[e^{-\frac{1}{M^2}(m_b^2+\frac{1}{4}m_\rho^2)} - e^{-\frac{s_0}{M^2}} \right] \varphi_\parallel(1/2) \right. \\ &\left. + \frac{1}{4} m_\rho^2 e^{-\frac{1}{M^2}(m_b^2+\frac{1}{4}m_\rho^2)} \left[4m_b f_\rho^T h_\parallel^{(s)}(1/2) - f_\rho m_\rho \left(1+\frac{m_b^2}{M^2}\right) A(1/2) - \frac{8 f_\rho m_\rho m_b^2}{M^2} \tilde{C}(1/2) \right] \right\}, \end{aligned} \tag{18}$$

$$\begin{aligned} f_{B^*}^2 f_{B^*B^*\rho} = {}& \frac{\sqrt{2}}{8m_{B^*}^3} e^{\frac{m_{B^*}^2}{M^2}} \left\{ M^2 \left(e^{-\frac{1}{M^2}(m_b^2+\frac{1}{4}m_\rho^2)} - e^{-\frac{s_0}{M^2}} \right) \right. \\ &\times \left[f_\rho^T m_b \varphi_\perp(1/2) + \frac{1}{8} f_\rho m_\rho \left(2g_\perp^{(a)}(1/2) + \frac{dg_\perp^{(a)}}{du}(1/2) + 4g_\perp^{(v)}(1/2) \right) \right] \\ &+ \frac{1}{2} m_\rho e^{-\frac{1}{M^2}(m_b^2+\frac{1}{4}m_\rho^2)} \left[f_\rho m_b^2 g_\perp^{(a)}(1/2) - 2 f_\rho^T m_b m_\rho \left(\frac{m_b^2}{4M^2} A_T(1/2) + 2\tilde{B}_T(1/2) + \frac{1}{2}\bar{C}_T(1/2) \right) \right. \end{aligned}$$

$$+ f_\rho m_\rho^2\left(\left(1+\frac{m_b^2}{2M^2}\right)\bar{A}(1/2)-\frac{1}{2}\tilde{C}(1/2)\right)\Bigg]\Bigg\}. \tag{19}$$

We proceed to the numerical computation of the sum rules. To consistently specify the input, we take [7] $m_b = 4.7 \pm 0.1$ GeV, $m_{B^*} = 5.325$ GeV, $f_{B^*} = 160$ MeV and $s_0 = 35 \pm 1$ GeV for the bottom channels. Some of the parameters related to the ρ meson are chosen as: $m_\rho = 770$ MeV, $f_\rho = 216 \pm 3$ MeV and $f_\rho^T(\mu = 1\ \text{GeV}) = 165 \pm 9$ MeV [27]. The most important sources of uncertainty are the light cone wavefunctions of ρ meson. It is demonstrated that the wavefunctions can be expanded in terms of matrix elements of conformal operators. Based on this expansion, the first attempt was made in [28] to understand the twist-2 distribution amplitudes of light vector mesons. Since a modified result was put forward [24] the model wavefunctions of light vector mesons, up to twist-4, have undergone a successive examination and improvement [25–27]. Very recently, a more systematic inspection was made of the existing model parameters and the updated results were reported in [27]. Here, we will make use of the findings of [27], for the related distribution amplitudes of ρ meson. Certainly, in the present applications the appropriate normalization scale should be set at the typical virtuality of the b quark: $u_b = \sqrt{m_B^2 - m_b^2}$. At this scale, the numerical values of the nonperturbative quantities involved, which contain the model parameters and f_ρ^T, can be reached by use of the renormalization group equations. Using the inputs fixed, the range of the Borel variable M^2 can be determined by demanding that the 4-twist parts contribute less than 10%, while the higher resonance and continuum contributions do not excess 30%. In both the cases, the Borel interval to satisfy the above criteria is $6 \leqslant M^2 \leqslant 12$ GeV^2. From the sum rule "windows", it follows that $f_{B^*}^2 g_{B^*B^*\rho} = 0.048 \pm 0.013$ GeV^2 and $f_{B^*}^2 f_{B^*B^*\rho} = 0.021 \pm 0.007$ GeV. The uncertainties quoted are in view of the variations of the b quark mass m_b, threshold s_0 and Borel parameter M^2. Dividing these two sum rules by $f_{B^*}^2$ yields $g_{B^*B^*\rho} = 1.88$, $f_{B^*B^*\rho} = 0.82$ GeV^{-1}, where we give only the central values of the numerical results.

With a definition different from the present ones by a constant factor, the remaining two couplings $g_{BB\rho}$ and $f_{B^*B\rho}$ have been computed in the same approach [21]. However, the numerical results are not straightforwardly available for a consistent discussion, because they are derived with the inputs, most of which, including the model wavefunctions, are other than those used here and improved to a certain extent. An updated estimate is obligatory. In passing, it is deserving of mention that there is an unfortunate error checked out by us in the previous LCSR calculation on the $B^*B\rho$ coupling (where the factor of 3/4 in the term proportional to $A_T(1/2)$ should be modified as $-1/4$), but with a small numerical impact. With this corresponding change, the LCSR expressions of the present concern can be achieved trivially from (17) and (23) of [21], for the products $f_B^2 g_{BB\rho}$ and $f_{B^*} f_B f_{B^*B\rho}$:

$$f_B^2 g_{BB\rho} = \frac{\sqrt{2}m_b^2}{4m_B^4}e^{\frac{m_B^2}{M^2}}\Bigg\{f_\rho m_\rho M^2\big[e^{-\frac{1}{M^2}(m_b^2+\frac{1}{4}m_\rho^2)} - e^{-\frac{s_0}{M^2}}\big]\varphi_\parallel(1/2)$$
$$+\frac{1}{4}m_\rho^2 e^{-\frac{1}{M^2}(m_b^2+\frac{1}{4}m_\rho^2)}\left[4m_b f_\rho^T h_\parallel^{(s)}(1/2) - f_\rho m_\rho\left(1+\frac{m_b^2}{M^2}\right)\left(A(1/2)+8\tilde{C}(1/2)\right)\right]\Bigg\}, \tag{20}$$

$$f_{B^*} f_B f_{B^*B\rho} = \frac{\sqrt{2}m_b}{8m_{B^*}m_B^2}e^{\frac{m_{B^*}^2+m_B^2}{2M^2}}\Bigg\{f_\rho^T M^2\big(e^{-\frac{1}{M^2}(m_b^2+\frac{1}{4}m_\rho^2)} - e^{-\frac{s_0}{M^2}}\big)\varphi_\perp(1/2)$$
$$+\frac{1}{4}m_\rho e^{-\frac{1}{M^2}(m_b^2+\frac{1}{4}m_\rho^2)}\left[2m_b f_\rho g_\perp^{(a)}(1/2) - m_\rho f_\rho^T\left(1+\frac{m_b^2}{M^2}\right)A_T(1/2)\right]\Bigg\}, \tag{21}$$

where the two additional parameters, m_B and f_B, for the bottom meson channels are taken as $m_B = 5.279$ GeV and $f_B = 140$ MeV. Our observation is that these two sum rules can share one Borel range, which is the about same as that for the $B^*B^*\rho$ case, and provide the numerical predictions: $f_B^2 g_{BB\rho} = 0.037 \pm 0.008$ GeV^2 and $f_{B^*} f_B f_{B^*B\rho} = 0.019 \pm 0.005$ GeV, from whose central values we have $g_{BB\rho} = 1.89$, $f_{B^*B\rho} = 0.85$ GeV^{-1}.

A physical interpretation is in order on the LCSR predictions presented above. As shown explicitly, there exist the approximate sum rule relations $g_{BB\rho} \approx g_{B^*B^*\rho}$ and $f_{B^*B\rho} \approx f_{B^*B^*\rho}$, for the coupling constants appearing in the effective Lagrangian (1). This may be accounted for intuitively by observing the construction of the effective Lagrangian: The terms proportional to $g_{BB\rho}$ and $g_{B^*B^*\rho}$ can be identified as describing the charge interactions between the $B(B^*)$ and ρ meson fields, while the other two parts may be interpreted as indicating the magnetic interactions of the underlying bottom mesons with ρ meson fields. It is not surprising, therefore, that the relations $g_{BB\rho} = g_{B^*B^*\rho}$ and $f_{B^*B\rho} = f_{B^*B^*\rho}$ should hold exactly in the limit $m_Q \to \infty$, because of the heavy quark spin symmetry. In the following section, we are going to return to this problem, putting it the test whether the LCSR calculations could precisely give the asymptotic relations deduced from the heavy quark spin symmetry.

Situations of the charm mesons can in parallel be discussed, using the LCSR formulae (18)–(21) with a replacement of the corresponding inputs. Of course, it is generally believed that in this case the gluon emission corrections from the charm quarks may be relatively important, due to the smaller heavy quark mass. Still, we omit them for a consistent purpose. The parameters for the charm channels are set as [7]: $m_c = 1.3$ GeV, $m_D = 1.87$ GeV, $m_{D^*} = 2.01$ GeV, $f_D = 170$ MeV, $f_{D^*} = 240$ MeV and $s_0 = 6$ GeV^2. In addition, we need to set the proper scale at $\mu_c = \sqrt{m_D^2 - m_c^2}$. Along the same line as in the bottom case, the numerical analysis can be performed. Subject to an evaluation of uncertainty, the yielded sum rule results for the couplings are summarized as: $g_{DD\rho} = 1.63$, $g_{D^*D^*\rho} = 1.68$, $f_{D^*D\rho} = 0.81\ \text{GeV}^{-1}$, and $f_{D^*D^*\rho} = 0.78\ \text{GeV}^{-1}$.

We would like to compare the present LCSR predictions with the ones, which are obtained with the inputs proposed earlier in [24,25] for the ρ meson parameters, to see that to what extent LCSR calculations have been improved with the updated parameters. It is demonstrated that in the bottom case, using the updated parameters can increase the LCSR evaluations by about 20% and a few percent, respectively, for the charge and magnetic interactions. The corresponding changes in the charm case amount to an order of 10% and of few percent, respectively.

A further improvement on the LCSR results proposed here is expected, since in the present case the $q\bar{q}g$ components of ρ meson do not enter in consideration, in addition to the QCD radiative corrections, and a further update is possible on the nonperturbative inputs, in particular, the light-cone wavefunctions of ρ meson. If confining LCSR computation to the present accuracy, the numerical results signify that the heavy quark spin symmetry can kept well for both the charge and the magnetic interactions of the negative-parity heavy mesons with ρ meson, but the heavy flavor symmetry suffers from a violation of different degree in the two interaction situations. To quantify size of the effects from the heavy flavor symmetry breaking, we consider a ratio of the corresponding sum rule results in the bottom and charm cases. We observe that whereas the estimated ratios, for the charge interactions, are of a deviation of about 20% from 1, the resulting breaking effect is at a level of a few percent in the cases of the magnetic interaction.

3. Heavy quark limit and determination of β and λ

In this section, we want to take a closer look at the behavior of the strong couplings in the heavy quark limit, checking up the consistency of the LCSR results with the predictions of heavy quark spin symmetry, and providing an assessment of the low energy effective parameters β and λ.

The desired asymptotic forms are achievable from the corresponding sum rules for the finite quark mass, by working explicitly out m_Q scaling behavior of the relevant parameters depending on the heavy degree of freedom. To be specific, we need to substitute in the sum rule results (18)–(21) the standard expansions of the $B(B^*)$ meson mass $m_B(m_{B^*})$, decay constant $f_B(f_{B^*})$, Borel parameter M^2 and threshold s_0. The former two are of the following expansions in inverse m_b:

$$m_B(m_{B^*}) = m_b + \Lambda + \mathcal{O}(1/m_b), \qquad f_B(f_{B^*}) = F/\sqrt{m_b} + \mathcal{O}(1/m_b), \tag{22}$$

with Λ being the binding energy of the light degree of freedom in the static b quark chromomagnetic field, and F a low energy parameter. For the intrinsic parameters in the sum rules M^2 and s_0, we need to rescale them as,

$$M^2 = 2m_b T, \qquad s_0 = m_b^2 + 2m_b\omega_0, \tag{23}$$

with T and ω_0 being the m_Q independent Borel variable and threshold, respectively.

It turns out, with these expansions, that in the limit $m_Q \to \infty$, the sum rules in (18)–(21), as desired, comply precisely with the asymptotic relations $g_{BB\rho} = g_{B^*B^*\rho}$ and $f_{B^*B\rho} = f_{B^*B^*\rho}$, and so boil down to the two dependent expressions. In consequence, the m_Q scaling behavior of the strong couplings are reproduced rightly and a consistent result is obtained with the effective chiral Lagrangian approach, in the LCSR approach. Denoting the asymptotic forms of $g_{BB\rho}(g_{B^*B^*\rho})$ and $f_{B^*B\rho}(f_{B^*B^*\rho})$ by $\mathcal{G}_1$ and $\mathcal{G}_2$, respectively, the resulting sum rules are of the following forms:

$$F^2\mathcal{G}_1 = \frac{\sqrt{2}}{4} e^{\frac{\Lambda}{T}} \left[2\left(1 - e^{-\frac{\omega_0}{T}}\right) f_\rho m_\rho T \varphi_\parallel(1/2) + m_\rho^2 f_\rho^T h_\parallel^{(s)}(1/2) - \frac{1}{8T} f_\rho m_\rho^3 \left(A(1/2) + 8\tilde{C}(1/2)\right)\right], \tag{24}$$

$$F^2\mathcal{G}_2 = \frac{\sqrt{2}}{8} e^{\frac{\Lambda}{T}} \left[2\left(1 - e^{-\frac{\omega_0}{T}}\right) f_\rho^T T \varphi_\perp(1/2) + \frac{1}{2} f_\rho m_\rho g_\perp^{(a)}(1/2) - \frac{1}{8T} f_\rho^T m_\rho^2 A_T(1/2)\right]. \tag{25}$$

The numerical analysis of the above asymptotic sum rules can be made using all the same procedure as in the finite heavy quark mass case. In the first place, the binding energy Λ, as an important input, requires to be fixed in a consistent way to reduce the numerical uncertainty as much as possible. It is easily calculated by taking the logarithmic derivative for one of (24) and (25) with respect to the inverse Borel parameter T. The result from (24), for instance, is $\Lambda = 0.43 \pm 0.15$ GeV with the Borel interval $0.5 \ll T \ll 1.3$ GeV and threshold $\omega_0 = 1.3 \pm 0.1$ GeV. With these inputs, we get $F^2\mathcal{G}_1 = 0.210 \pm 0.031\ \text{GeV}^3$ and $F^2\mathcal{G}_2 = 0.098 \pm 0.013\ \text{GeV}^2$. The variations are depicted in Fig. 1 of the LCSR results with the Borel parameter T. In order to have an assessment of the asymptotic couplings $\mathcal{G}_1$ and $\mathcal{G}_2$, we may use the determination without QCD radiative corrections included [30]: $F = 0.30 \pm 0.05\ \text{GeV}^{3/2}$. Instead of doing that, we prefer to directly substitute in (24) and (25) the sum rule form for F to make the numerical results free of a large uncertainty, yielding $\mathcal{G}_1 = 2.36 \pm 0.32$ and $\mathcal{G}_2 = 1.09 \pm 0.15\ \text{GeV}^{-1}$, a result compatible with the sum rules for the finite heavy quark mass.

Now we are in a position to determine the effective parameters β and λ. In the context of the effective chiral Lagrangian [23], the related hadronic matrix elements obey, at the leading order in the $1/m_{B^{(*)}}$, the following parameterizations:

$$\left\langle \bar{B}^0(p)\rho^-(q,\epsilon)\middle| B^-(p+q)\right\rangle = i\sqrt{2} M_B \beta g_V \epsilon^* \cdot v, \tag{26}$$

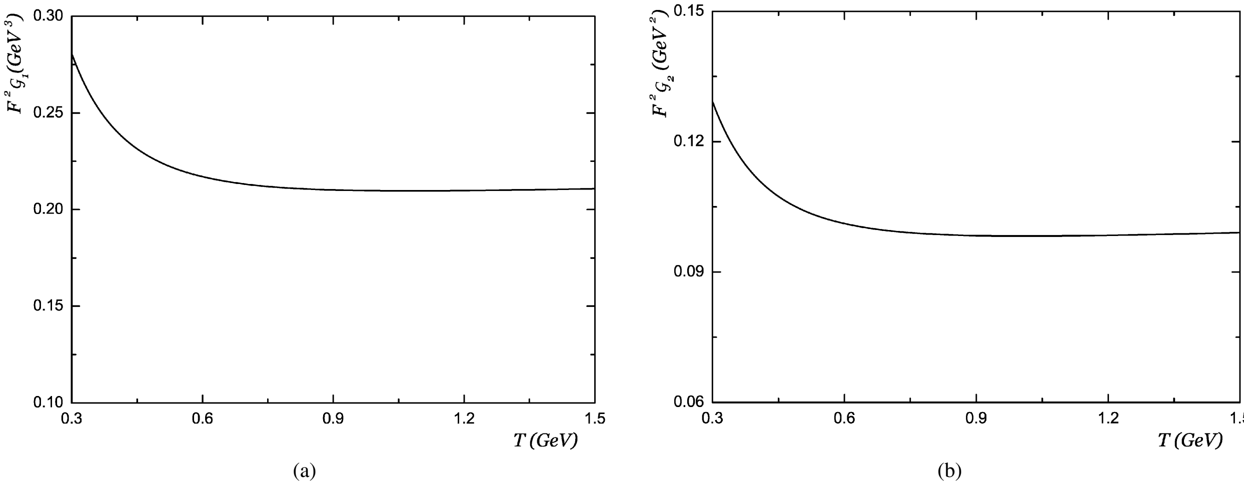

Fig. 1. The stability of the LCSR results for $F^2\mathcal{G}_1$ (a), and $F^2\mathcal{G}_2$ (b), with $\Lambda = 0.43$ GeV and $\omega_0 = 1.3$ GeV.

$$\langle \bar{B}^{*0}(p,\eta)\rho^-(q,\epsilon)\big|B^-(p+q)\rangle = -i2\sqrt{2M_B M_{B^*}}\lambda g_V \epsilon_{\mu\alpha\beta\gamma}\eta^{*\mu}q^\alpha\epsilon^{*\beta}v^\gamma, \tag{27}$$

$$\begin{aligned}\langle \bar{B}^{*0}(p,\eta)\rho^-(q,\epsilon)\big|B^{*-}(p+q,\xi)\rangle &= -i\sqrt{2}M_{B^*}\beta g_V(\eta^*\cdot\xi)(\epsilon^*\cdot v)\\ &\quad - i2\sqrt{2}M_{B^*}\lambda g_V\big[(\eta^*\cdot\epsilon^*)(\xi\cdot q)-(\xi\cdot\epsilon^*)(\eta^*\cdot q)\big]\end{aligned} \tag{28}$$

where $g_V = m_\rho/f_\pi \approx 5.8$ [23] and v indicates the velocity of the underlying heavy mesons. Confronting the hadronic matrix elements in (2)–(4) with those in (26)–(28) respectively, we have the following asymptotic relations:

$$\mathcal{G}_1 = \frac{\beta g_V}{2}, \qquad \mathcal{G}_2 = \frac{\lambda g_V}{2}. \tag{29}$$

Using the above equation and sum rule results for $\mathcal{G}_1$ and $\mathcal{G}_2$, we get $\beta = 0.81 \pm 0.11$ and $\lambda = 0.38 \pm 0.05\ \mathrm{GeV}^{-1}$.

The authors of [17] give an estimate of the effective couplings β and λ. They consider the electromagnetic transition of a heavy pseudoscalar meson and assume that the hadronic matrix element of the light quark current is dominated by the ρ, ω, ϕ vector mesons. Then the current conservation leads automatically to the result $\beta = \frac{\sqrt{2}m_V}{g_V f_V} \approx 0.9$. To order to make an evaluation of the parameter λ, they adopt a combined use of several different approaches. The prescription is to compute one of the $B \to K^*$ form factors at the squared momentum transfer $q^2 = q^2_{\mathrm{max}}$, using the effective chiral Lagrangian and B^*_s dominance model, respectively, and then to equate them for extracting λ which enters the result of the effective theory. The pole model representation for the form factor is determined by identifying, at $q^2 = 17\ \mathrm{GeV}^2$, its result with the corresponding theoretical prediction from the LCSR's and lattice QCD. In such ways one gets $\lambda = 0.57\ \mathrm{GeV}^{-1}$. Also, it is possible to extract λ from the data on the $D \to K^*$ form factor at the largest recoil, by extrapolating the form factor derived at zero recoil in the effective chiral Lagrangian approach by means of the vector dominance [23]. The extracted λ is of a bit smaller central value: $\lambda = 0.41\ \mathrm{GeV}^{-1}$. It is generally agreed that these existing determinations of β and λ would be subject to a large uncertainty, especially in the λ case where a combined use of several different approaches to the form factor would more or less cause the inconsistency in calculation. Concentrating on the central values, we find that QCD LCSR's predict, for the parameter β, a numerical result nearly the same as the one of the pole model. In the λ case, a good numerical agreement is also observed with the result extracted experimentally, whereas there is a numerical deviation of about -30% from the one of [17]. On the whole, our LCSR results for β and λ are compatible with those of other approaches within errors.

4. Summary

The strong interactions of the negative-parity heavy mesons with ρ meson may be described uniformly in the context of an effective Lagrangian observing $SU(2)$ invariance in the isospin space. The established effective Lagrangian contains four independent coupling parameters, which characterize the dynamics of strong interactions among the underlying meson fields. Using the QCD LCSR method and recently updated model parameters for the light-cone distribution amplitudes of ρ meson, we have presented a complete discussion on these couplings. Apart from an updated LCSR result for $g_{BB\rho}$ and $f_{B^*B\rho}$, we give, among others, a detailed LCSR estimate of $g_{B^*B^*\rho}$ and $f_{B^*B^*\rho}$, about which little was known before. Situations of the charm mesons are also inquired into in the same framework, which is especially important for us to understand the FSI effects in B decays. A systematic numerical discussion is made, including a detailed physical interpretation on the sum rule results and a numerical comparison with the LCSR computations using as inputs a model wavefunction given earlier. Also, we examine asymptotic forms of the LCSR results in the heavy quark limit. As shown explicitly, the LCSR approach could reproduce rightly the m_Q scaling behavior of the physical quantities in question, and thus provides a consistent calculation with the results of the heavy quark symmetry. This would, needless to say, enhance considerably our confidence in applying the LCSR method to do calculation of nonperturbative quantities. Finally, we assess the low energy parameters β and λ appearing in the corresponding effective chiral Lagrangian, and draw a numerical parallel between the present and previous calculations.

The effective Lagrangian approaches, using the present findings as inputs and in conjunction with other nonperturbative methods, could help to get a more knowledge of the long distance dynamics in heavy meson weak decays. No doubt, this is beneficial to promote our understanding of the standard model of particle physics.

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