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Bayesian estimation of a highly parameterized hydraulic conductivity field: a study case

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Abstract

Comprehensive characterization of the spatial variability of aquifer hydraulic parameters is fundamental to correctly design a structure that interacts with the groundwater system, e.g. a detention reservoir. A flood control structure has been planned to dampen the Baganza River floods, upstream the city of Parma (Northern Italy), storing a portion of the flood volume then released at a controlled rate. The location and the geometry of the planned reservoir are already set and the aquifer system beneath the structure is under investigation and monitoring. A large numbers of boreholes identified a sequence of gravel-sand and clayey layers that gives rise to two distinct aquifers (one unconfined and one confined). A numerical model has been developed by means of MODFLOW_2005 in order to simulate the unconfined aquifer under current conditions. The numerical model has been calibrated in steady-state conditions by means of 11 observations of water levels (collected during spring 2013). About 18000 hydraulic conductivity values have been defined and estimated by means of a Bayesian Geostatistical Approach. The inverse methodology estimates the hydraulic conductivity field using both field observations (head data) and prior information on the structure of the unknown parameters. The inverse procedure requires the calculation of the sensitivity of each observation to each of the estimated parameters that has been efficiently evaluated making use of an adjoint state formulation of the forward model MODFLOW_2005-Adjoint. The inverse code is the freeware bgaPEST, a highly parameterized inversion software package developed according to the PEST software concept. The Bayesian Geostatistical Approach is shown to be able to constrain the hydraulic conductivity field with a large number of parameters and just few observations. The results of the inversion are consistent with the alluvial nature of the investigated aquifer and the information content of the data. The calibrated model can then be used as a support tool for the design and the management of the new flood control structure.

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Keywords: Groundwater, bgaPEST, MODFLOW, parameter estimation

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1. Introduction

During the last decades, a large number of flood control reservoirs were developed in Northern Italy (for instance on the Enza River with a volume of about 12 Mm^3, Panaro River 26 Mm^3, Parma River 12 Mm^3 and Secchia River 15 Mm^3) in order to mitigate the flood risk in urban areas. The detention reservoirs are structural measures for flood protection where a dam allows to temporary collect a portion of the inflow water then released at a flow rate that the channel downstream the structure can accommodate.

The city of Parma (Northern Italy) is located at the junction of the Parma River and its tributary Baganza River. Since 2004, a flood control reservoir is operational on the Parma River (located about 7 km upstream the city boundaries). Recently, in order to increase the downtown flood protection, a detention reservoir has been planned on the Baganza River. This new structure is particularly useful when flood events occur on the two rivers at the same time. Preliminary studies have been carried out to identify the reservoir location, its geometry, the needed storage area (1200 m x 700 m) and volume (about 4 Mm^3) and consequently the maximum head stage reached inside the basin for design flood events. The detention basin consists of a dam, with moving gates, that regulates the flow rate, a system of levees about 1700 m in length and three check dams located upstream the floodable area. In addition, in order to avoid seepage related problems, cutoff walls below the dam and the levees have been planned.

A numerical model of the aquifer beneath the reservoir area has been developed, making use of MODFLOW_2005 [1], to reproduce the groundwater system at the current conditions (no reservoir) and to forecast the groundwater levels at the design conditions (presence of detention reservoir and design flood events). A comprehensive characterization of the spatial variability of the aquifer hydraulic parameters is fundamental to correctly represent the present state and to design the structures that influence the groundwater levels, such as the cutoff walls beneath the reservoir. The hydraulic conductivity distribution was estimated by means of a Bayesian Geostatistical Approach (BGA) [2, 3]. The inverse methodology estimates the hydraulic conductivity field using both field observations (observed head levels) and prior information on the structure of the unknown parameters. This prior, making use of geostatistical functions and tools, is related with the degree of continuity and smoothness of the parameter field and regularizes the solution. The inverse procedure requires the calculation of the sensitivity of each observation to each of the estimated parameters that were efficiently evaluated making use of an adjoint state formulation of the forward model MODFLOW_2005-Adjoint [4]. The inverse code is the freeware bgaPEST [5], a highly parameterized inversion software package developed according to the PEST [6] software concept.

The aim of this work is to estimate the hydraulic conductivity field of the study area reproducing the state of the art.

2. Inverse method: Bayesian Geostatistical Approach

In this work the Quasi-Linear Bayesian Geostatistical Approach [2, 3] represents the core of the inverse problem. The Bayesian geostatistical method allows the estimation of a set of parameters that gives the best reproduction of observations and that is constrained using prior information, characterized by geostatistical functions, on the structure of the parameters themselves.

Bayes theorem, in terms of random variables and their probability function, states:

$$p(\mathbf{s} \mid \mathbf{z}) = \frac{p(\mathbf{z} \mid \mathbf{s})p(\mathbf{s})}{p(\mathbf{z})} \tag{1}$$

where $\mathbf{s}$ and $\mathbf{z}$ are the state (unknown quantities) and the data (measured quantities) vectors, respectively. $p(\mathbf{s}\,|\,\mathbf{z})$ is the posterior pdf evaluated as the product of the likelihood function $p(\mathbf{z}\,|\,\mathbf{s})$ and the prior pdf $p(\mathbf{s})$ (which represents knowledge about the unknown quantities a priori) divided by the total probability $p(\mathbf{z})$ that is a normalizing constant. Accordingly, equation (1) can be simplified to:

$$p(\mathbf{s}\,|\,\mathbf{z}) \propto p(\mathbf{z}\,|\,\mathbf{s})\,p(\mathbf{s}) \tag{2}$$

Assuming the prior pdf and the likelihood pdf Gaussian, also the posterior pdf is Gaussian.
In this work the prior has been modeled as an exponential geostatistical covariance function with two structural parameters: the variance and the integral scale.

The covariance structural parameters are estimated by means of a restricted maximum likelihood approach. For more details about the quasi-linear geostatistical approach applied to the identification of aquifer parameters see [2, 3]. In this work the Bayesian geostatistical approach has been applied as implemented in the bgaPEST [5] software package.

2.1. Sensitivity computation

In this work the MODFLOW_2005-Adjoint code [4] was used to calculate the sensitivity of each observation to each unknown parameter with the aim at creating the Jacobian matrix. MODFLOW_2005-Adjoint was developed on the basis of the MODFLOW_2005 version 1.1 05/18/2006. In order to keep into account all the new features mainly related with the cell wet/dry procedure, MODFLOW_2005-Adjoint was updated obtaining a new version based on the last MODFLOW_2005 (version 1.11.0 08/08/2013). The suggestions of [7] were used to speed up the computation. In order to reduce the number of the estimable parameters and overcome memory problems, we considered 9 adjacent flow cells with the same hydraulic conductivity obtaining lumped parameters (covering an area of 30×30 m^2) and a total number of unknowns equal to 18,141. As reported in [4], the sensitivity of the lumped unknowns is the sum of the sensitivities to each flow cell involved. The centroid of each lumped parameter was computed and used to calculate their distances and consequently their correlations.

3. Morphology and hydrology of study area

The hydrological and morphological peculiarities of the Baganza basin, characterized by a rainfall sublittoral-Apennines regime, give rise to a stream of torrential features. In particular, the fluvial shelf on the right bank of the Baganza River, consists of gravel flood soils with lenses of sandy clay soil mainly in the high brown plain, and low gravelly soils sandy cultivated and gravelly flood of the riverbed. The most recent deposits in the study area are alluvial sediments in evolution and recent alluvial sediments. The study area is an alluvial unit, consisting of gravels prevailing at the main river (alluvial deposits), and sand and gravel interbedded with silt. The area it is part of the Holocene valley of Po River and it extends between the Baganza River, which is the most representative hydraulic element, and Cinghio Creek, that are rather meandering.

4. Conceptual model

The study area (Fig. 1) was largely studied in order to first develop a conceptual model and then a numerical one. A large number of investigations (such as: boreholes, geoelectrical and geophysical surveys and pumping tests) were carried out during the spring 2012 - summer 2013 period over all the study area. In particular 28 boreholes (Fig. 1) were drilled at different depth (30-60 m).

Outside the study area the conceptual model was completed by means of large scale geological cross sections developed by the Emilia Romagna Region.

The drilling cores (Fig. 1) were carefully planned along the perimeter of the designed levees and in proximity of the dam, in order to investigate and obtain accurate information on the stratigraphy of the whole study area. Several of the boreholes were equipped with screening filters in order to allow the monitoring of the groundwater level. These filters were placed at different depths (Fig. 2) with the aim at investigating the different aquifers. Water levels were monthly measured since July 2012 and used to provide head contour maps which allow describing the flow direction, the gradient and the seasonal variation of the aquifers.

The borehole logs show variable lithology. The investigations identify two aquifer systems: an unconfined shallow aquifer (further called main aquifer) consisting of sand and gravel with a succession of clayey-silt lenses, and a deep confined one consisting of sand and gravel. The two aquifers (see Fig. 2) are separated by means of a clay layer, which was found in each borehole, with a variable thickness (from 2 to 7 m). The clay layer is at a depth of about 28 m in proximity of the dam and 15 m at the most upstream portion of the floodable area.

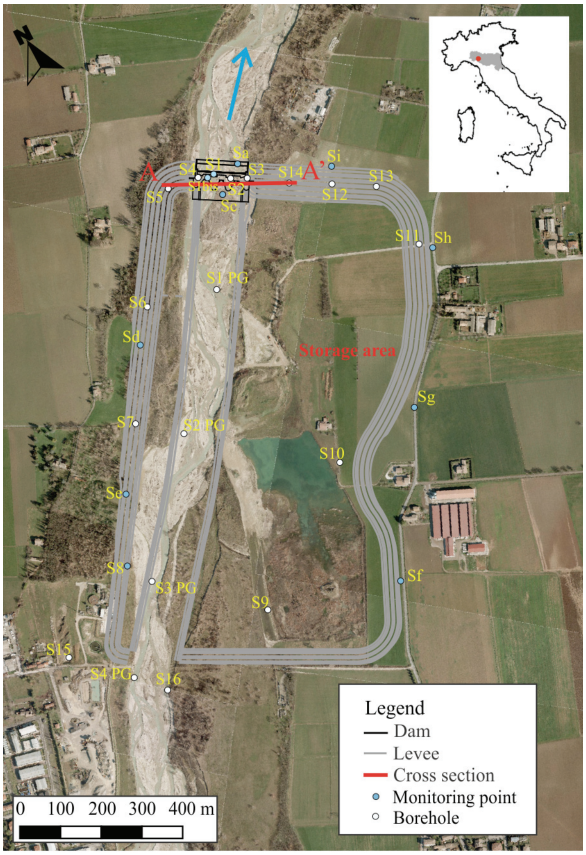

Fig. 1. Location of the study area.

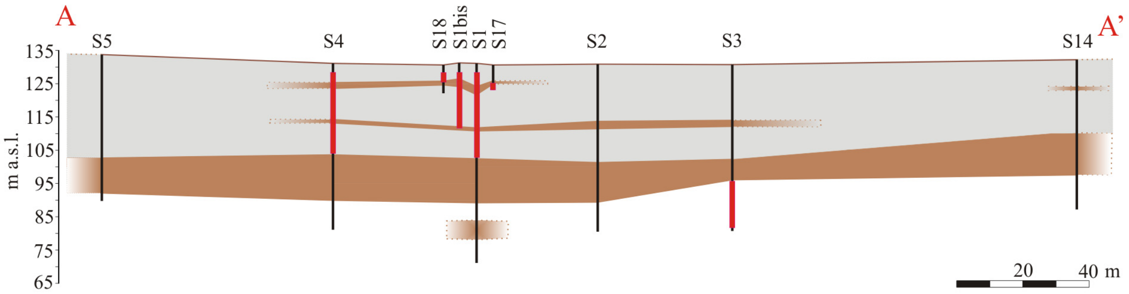

Fig. 2. Sketch view of the cross section AA' of Fig. 1. The black lines represent the borehole depth; the red lines represent the window of the piezometers; the brown area represents the low hydraulic conductivity materials and in gray the main aquifer is shown.

The main aquifer is strictly connected to the river system. Figure 2 shows the stratigraphy beneath the dam, it is possible to identify the main aquifer and the deeper one below the thick clay layer. It is important to highlight the presence of two thin clayey lenses at 10 and 15 m depth.

The first one was considered of very low importance, in fact it presents a relatively thin thickness (less than 1 m) and a very small spatial extension (see Fig. 2); while the second one extend in a quite large area even if it tends to thin out and disappear going upstream the river.

The average annual precipitation over the area is 811 mm; this value was inferred from the analysis of rain gauges, covering the study area, in the period between 1960 and 2012.

5. Numerical model

The objective of the aquifer numerical model is to simulate the groundwater flow at the present conditions, understanding the aquifer dynamics, and to predict, after calibration, the groundwater/surface water interactions under different conditions of the detention reservoir. The forward problem was solved by means of MODFLOW_2005 [1], which simulates saturated porous media under steady state and transient conditions. The conceptual model was translated into a numerical one through a three-dimensional finite differences grid. The study was focused on modeling the main aquifer, so the thick clay layer described in the previous section was considered as the impervious bottom of the model. The stratigraphy of Fig. 2 was reproduced using 3 layers: the first and the third are the main aquifer (in gray), while the second represents the clay layer at 15 m depth. The aquifer is characterized by alluvial gravel, sand and silt with a high hydraulic conductivity. The study area was represented by 300 rows, 200 columns with dimension 10×10 m^2 each, but due to the boundary conditions only 160,000 cells were active. The boundary conditions (Fig. 3) of the model were extrapolated from the head level contours estimated considering the observations collected on March the 5th, 2013. These observations were considered to be unaffected by noises such as withdrawals for irrigation. A constant head level (156 m a.s.l.) was considered as upstream boundary condition; while downstream, a general head was chosen from 110 to 114 m a.s.l. The general head takes into account the hydraulic properties of the porous medium and the observed water levels. It was decided to use this type of boundary condition since the flows involved are highly dependent on the upstream conditions. In fact, a condition of constant head downstream would be independent of the presence of structures (such as cutoff walls) and would lead to overestimate the exiting fluxes. The model is subjected to a meteoric recharge that was estimated in 19% of the mean annual value (811 mm/year).

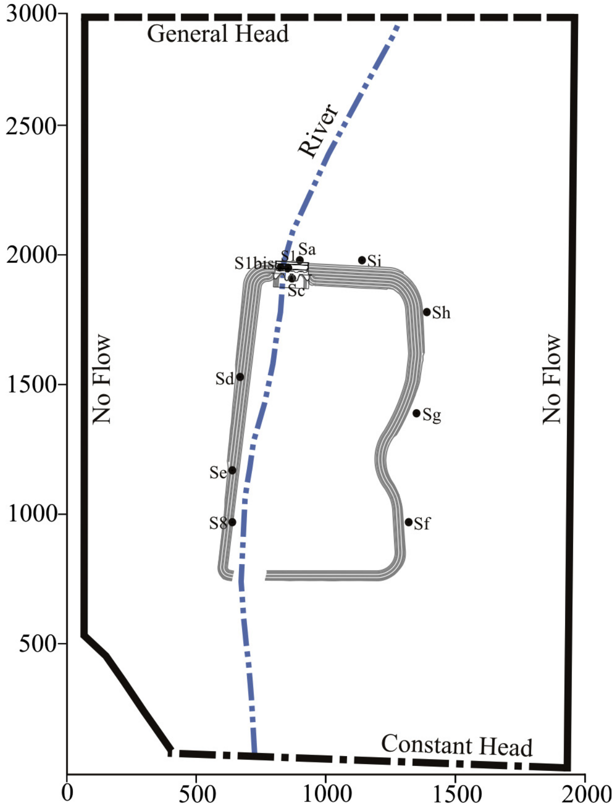

Fig. 3. Sketch view of the study area. The black lines are the boundary conditions, the grey lines are the levees, the black dots are the monitoring points, the blue dash-dot line is the river. Measures in meters.

The Baganza River is strictly connected to the groundwater and it was modeled as a river condition that has the capability to feed or drain the aquifer, according to the respective water levels. The river was described considering the conductance parameter that evaluates the infiltration capacity of the riverbed. The initial hydraulic conductivity value of the aquifer ($1.39\cdot10^{-4}$ m/s) was chosen on the basis of the performed pumping tests; while for the clay lens ($5\cdot^{10-7}$ m/s) a literature value was considered. A vertical anisotropy of 10 was considered.

6. Model Calibration and Results

The model calibration, by means of a Bayesian Geostatistical Approach, was performed in steady-state conditions, based on 11 observed water heads collected on March the 5th, 2013. It allows estimating the distribution of the hydraulic conductivities of the investigated aquifer. The resulting hydraulic conductivity field of the first layer is shown in Fig. 4 in logarithmic scale. This layer presents a lower hydraulic conductivity in the center of the reservoir and a mean hydraulic conductivity of $3.28\cdot10^{-4}$ m/s and variance of $1.42\cdot10^{-4}$ m^2/s^2. These values are congruent with the values observed through field tests. The range goes from $1.02\cdot10^{-4}$ m/s to $7.73\cdot10^{-4}$ m/s. Greater accurate estimates are certainly found near the observations; less realistic are the hydraulic conductivity values estimated along the model contours; it is important to note that the dimensions of the model are much larger of the reservoir area.

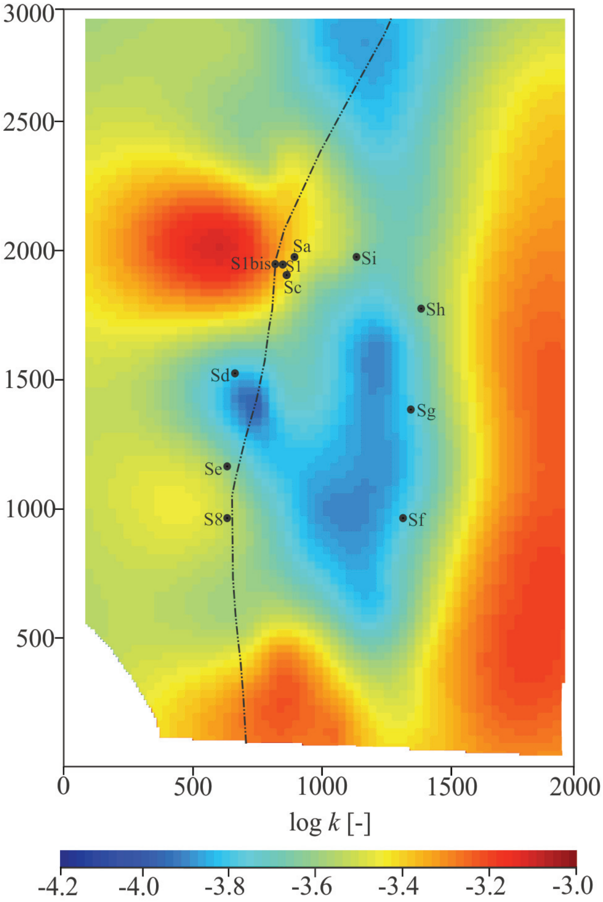

Fig. 4. Hydraulic conductivity of the first layer field in log scale. Black dots are monitoring points.

In regard to the model flow budget, the aquifer is fed by: the upstream boundary condition (South) with about 26 l/s, an average meteoric recharge of 26 l/s and the river (South of the area) with about 29 l/s. The river drains from the aquifer about 22 l/s (Northern portion of the area) and the general head condition (North) release about 59 l/s.

7. Conclusions

The objective of the present work was the estimation of the hydraulic parameters of an unconfined aquifer by means of a Bayesian Geostatistical Approach using the bgaPEST inverse code [5]. The Bayesian Geostatistical Approach resulted to be able to estimate the hydraulic conductivity field with a large number of parameters and just few observations. The results of the inversion are consistent with the alluvial nature of the investigated aquifer and the information content of the data. The calibrated model can then be used as a support tool for the design (depth of cutoff walls) and the management of the new structure. To this end, it is worth noting that the reservoir can be used not only with flood control aims but also multiple purposes such as agricultural and recreational ones.

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