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Relevance of two-boson exchange effect in quasi-elastic charged current neutrino–nucleon interaction



Krzysztof M. Graczyk

Institute of Theoretical Physics, University of Wrocław, pl. M. Borna 9, 50-204, Wrocław, Poland

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ABSTRACT

Two-boson exchange (TBE) correction in $\nu n \rightarrow l^- p$ and $\bar{\nu} p \rightarrow l^+ n$ reactions is estimated. The TBE contribution is given by $W\gamma$ box diagrams. The calculations are performed for 1 GeV neutrinos and for the MiniBooNE and the T2K energy spectra. The TBE correction to the total cross section is of the order of 2–4% (with respect to the Born contribution) in the case of ν_e and $\bar{\nu}_e$ and 1–2% in the case of ν_μ and $\bar{\nu}_\mu$.

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1. Introduction

One of the goals of the particle physics is to understand the fundamental properties of neutrinos. Remarkable effort has been made to measure the θ_{13} parameter. The next step is to investigate the CP violation. It can be done by analyzing the $\nu_\mu \rightarrow \nu_e$, $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, $\nu_e \rightarrow \nu_\mu$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ oscillation processes. Therefore a detailed estimate of the differences between the cross sections for ν_μ - and ν_e -nucleon and ν_μ - and ν_e -nuclei interactions is of importance [1]. The electrons, because $m_e \ll m_\mu$, have tendency to radiate more than the muons. Hence one may expect that the radiative corrections (RCs) for the charged current (CC) ν_e -nucleon scattering are significantly larger than those for the ν_μ -nucleon interactions.

In the long baseline experiments, like T2K [2] or NO ν A [3], with the accelerator source of ν_μ , the charged current quasi-elastic (CCQE) neutrino–nucleus scattering, $\nu + A[N, Z] \rightarrow l^- + (A-1)[N-1, Z] + p$ is the dominant process ($A = N + Z$, N and Z denotes number of neutrons and protons in the nucleus respectively). The typical neutrino energy of long baseline experiments is about 1 GeV, while the relevant four-momentum transfer $Q^2 < 1 \text{ GeV}^2$.

The RCs has been not accounted for in any Monte Carlo generator used to perform the data analysis [1]. In the case of muon neutrinos the RCs are expected to be a small and they stand negligible fraction of other systematic uncertainties characterizing the

cross section measurement (reconstruction of the neutrino energy, imprecision of the theoretical models for a ν -nuclei scattering, detector effects).

There is no systematic discussion and calculations of the RCs for the CCQE processes for neutrinos of the energy of about 1 GeV. In this paper we present the estimate of the two-boson exchange (TBE) correction. It is a part of the RCs. We neglect the nuclear effects and consider the interaction between neutrino (antineutrino) and the free nucleon. Hence in the rest of the paper the acronym CCQE will refer to the reactions

$$\nu_l(k) + n(p) \rightarrow l^-(k') + p(p'), \quad (1)$$

$$\bar{\nu}_l(k) + p(p) \rightarrow l^+(k') + n(p'), \quad l = e, \mu. \quad (2)$$

Recently the TBE effect has been extensively investigated in the elastic ep scattering. Activity in this topic was induced by observing the discrepancy between the measurements of the form factor ratio G_E^p/G_M^p ($G_{E,M}^p$ is the electric, magnetic proton form factor) obtained within two different experimental methods. The first method is based on the Rosenbluth separation. The other is based on the so-called polarization transfer (PT) measurements [4]. In the elastic ep scattering the leading TBE contribution (called letter two-photon exchange (TPE)) is given by the interference of the Born amplitude with the $\gamma\gamma$ box diagrams, which describe an exchange of two virtual photons between the electron and the target. In the standard treatment of the RCs to the ep scattering [5] the TPE correction was computed in an approximation, in which the

E-mail address: krzysztof.graczyk@ift.uni.wroc.pl.

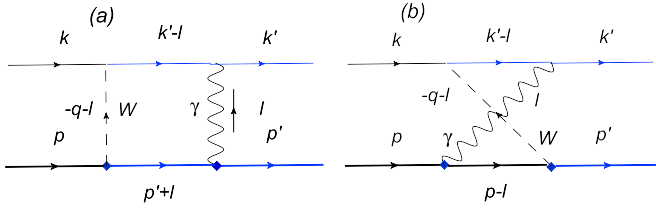


Fig. 1. Two-boson exchange $W\gamma$ box diagrams.

hard photon contribution, induced by the internal proton structure, was neglected. Inclusion this additional correction into the Rosenbluth analysis allows to solve partially the problem of inconsistency between Rosenbluth and PT G_E/G_M ratio data [6,7].

In the case of neutrino interactions the RCs have been studied for β and μ decays and also for the CCQE neutrino–deuteron interaction but only near the threshold [8,9]. The RCs were also estimated for the deep inelastic (DIS) ν –nucleon scattering [10–13].

The difference between ν_e and ν_μ CCQE cross sections induced by the RCs was discussed in Ref. [1]. In this paper the lepton leg correction formula, obtained within the leading log approximation [13], was adopted. It describes the soft and hard photon emission by the charged lepton leg and it does not include the TBE contribution.

The higher order corrections to electroweak neutrino–quark interaction can be computed within the Standard Model (SM) [11]. We recall that the electroweak interactions are described by the $SU(2) \times U(1)$ gauge theory (for pedagogical introduction see Chapters 9, 11 and 12 of [14]). However, in the discussed case the neutrinos interact with quarks confined in the nucleon in the low Q^2 range. Hence to take into consideration the nucleon structure effects, important in the case of TBE effect, an effective hadronic model must be used. It introduces into the calculations a model-dependence.

In the simplest approach the TBE effect is described by the contributions from two box $W\gamma$ diagrams (Fig. 1). They describe the corrections coming from the interaction of the charged lepton with the proton. We assume that the hadronic intermediate state in the box diagrams is the off-shell nucleon. Off-shell electroweak hadronic vertices are replaced by the on-shell ones. Their electroweak structure is motivated by the symmetries as well as it is constrained by the conserved vector current (CVC) and partially conserved axial current (PCAC) hypotheses [15]. Similar approach was used to calculate the TBE effect in elastic ep scattering (given by $\gamma\gamma$ [16] and γZ^0 [17] box contributions). A part of the inner radiative correction to the β decay was obtained in the similar approximation [9,18] as well. This approach works reasonably in ep scattering in the low and the intermediate Q^2 range [19,21]. It is the kinematical domain that is relevant for the CCQE scattering with about 1 GeV neutrinos.

In our previous paper [21] we calculated the TPE correction for elastic ep scattering. In this paper we consider the same methodology (algebraical and numerical algorithms) to compute $W\gamma$ box contribution for the CCQE scattering. We will show that TBE corrections for ν –nucleon interactions are sizeable and they are comparable with TPE effect in elastic ep scattering.

2. Formalism

A Born amplitude for the reaction (1) has a form

$$i\mathcal{M}_{\text{Born}} = i \frac{g^2 \cos\theta_C}{8(q^2 - M_W^2)} j_\mu h^\mu, \quad (3)$$

where $\theta_C = 13.04^\circ$ is Cabbibo angle, $g = e/\sin\theta_W$ is the weak coupling constant, $\sin^2\theta_W = 0.2312$, $e^2 = 4\pi\alpha$, $\alpha = 1/137$, $M_W =$

80.3 GeV is the boson W^\pm mass. The four-momentum transfer is defined as $q^\mu = p'^\mu - p^\mu$, then, $Q^2 \equiv -q^2$. By E we denote the neutrino energy.

We introduce the leptonic $j_\mu = \bar{u}(k')\gamma_\mu(1 - \gamma_5)u(k)$ and the hadronic $h^\mu(q) = \bar{u}(p')\Gamma_{cc}^\mu u(p)$ one-body currents. The electroweak nucleon on-shell vertex depends on four form factors [15]

$$\Gamma_{cc}^\mu(q) = \Gamma_V^\mu(q) - \gamma_\mu \gamma_5 F_A(q) - \frac{q^\mu \gamma_5}{2M} F_P(q), \quad (4)$$

where the vector vertex, after applying the CVC hypothesis, reads $\Gamma_V^\mu = \Gamma_p^\mu(q) - \Gamma_n^\mu(q)$; $\Gamma_{p,n}^\mu(q)$ is the electromagnetic proton, neutron vertex. M is the averaged nucleon mass, $M_{p,(n)}$ is the proton (neutron) mass. F_A and F_P are the axial form factors, which can be related to each other (PCAC hypothesis), namely $F_P(q) = 4M^2 F_A(q)/(m_\pi^2 - q^2)$, where m_π is the pion mass. We assume that $F_A(q) = g_A/(1 + Q^2/M_A^2)^2$, where $g_A = 1.267$, M_A is an axial mass, and as a default value we take $M_A = 1$ GeV.

The proton (neutron) electromagnetic vertex is

$$\Gamma_{p,n}^\mu(q) = \gamma^\mu F_1^{p,n}(q) + \frac{i\sigma^{\mu\nu}q_\nu}{2M_{p,n}} F_2^{p,n}(q), \quad (5)$$

where $F_{1,2}^{p(n)}$ is the proton (neutron) electromagnetic form factor. It is convenient to use the electric and the magnetic proton (neutron) form factors,

$$F_1^{p,n} = \frac{1}{(1 + \tau_{p,n})} (G_E^{p,n} + \tau_{p,n} G_M^{p,n}),$$

$$F_2^{p,n} = \frac{1}{(1 + \tau_{p,n})} (G_M^{p,n} - G_E^{p,n}), \quad \tau_{p,n} = \frac{Q^2}{4M_{p,n}^2}.$$

We consider a dipole parametrization of the electric and the magnetic form factors, namely, $G_E^p(Q^2) = G_M^{p,n}(Q^2)/\mu_{p,n} = \Lambda^4/(Q^2 + \Lambda^2)^2$, where $\Lambda = 0.84$ GeV is the cut-off parameter. The electric neutron form factor is assumed to be zero ($G_E^n(Q^2) = 0$).

The TBE correction is given by the interference of the Born with two $W\gamma$ amplitudes. The hadronic intermediate state is the off-shell nucleon. In this paper we do not discuss the inelastic TBE contribution given by the diagrams with resonances as the hadronic intermediate states. In the first approximation this correction can be neglected. It was demonstrated that in the elastic ep scattering this contribution is small in the Q^2 range relevant for neutrino reactions [20] (see also Fig. 8 of Ref. [21]).

The TBE box amplitudes are denoted by $i\Box^\parallel = -\cos\theta_C e^2 g^2 \times I_{W+\gamma}^\parallel/8$ (Fig. 1a) and $i\Box^\times = -\cos\theta_C e^2 g^2 I_{W+\gamma}^\times/8$ (Fig. 1b), where

$$I^\parallel = \int \frac{d^4l}{(2\pi)^4} \frac{\mu^{\nu\alpha\beta} h_{\parallel}^{\alpha\beta} S_{\mu\alpha}^\gamma(l) S_{\nu\beta}^W(l+q)}{D(p', M_p)}, \quad (6)$$

$$I^\times = \int \frac{d^4l}{(2\pi)^4} \frac{\mu^{\nu\alpha\beta} h_{\times}^{\alpha\beta} S_{\mu\alpha}^\gamma(l) S_{\nu\beta}^W(l+q)}{D(-p, M_n)}, \quad (7)$$

$$\mu^{\mu\nu} = \bar{u}(k')\gamma^\mu(\hat{k}' - \hat{l} + m)\gamma^\nu(1 - \gamma_5)u(k),$$

$$h_{\mu\nu}^\parallel = \bar{u}(p')\Gamma_{\mu}^p(-l)(\hat{p}' + \hat{l} + M_p)\Gamma_{\nu}^{cc}(q+l)u(p),$$

$$h_{\mu\nu}^\times = \bar{u}(p')\Gamma_{\nu}^{cc}(q+l)(\hat{p} - \hat{l} + M_n)\Gamma_{\mu}^n(-l)u(p),$$

$\hat{p} \equiv p^\mu \gamma_\mu$; $D(x, M_x) = [(q+l)^2 - M_W^2 + i\epsilon][l^2 + i\epsilon][(k'-l)^2 - m^2 + i\epsilon][(x+l)^2 - M_x^2 + i\epsilon]$, m denotes the lepton mass. The photon and W -boson propagators (in $SU(2) \times U(1)$ gauge model [14]) take the form

$$S_{\mu\nu}^\gamma(l) = \frac{g_{\mu\nu} + (\xi_\gamma - 1)l_\mu l_\nu/l^2}{l^2 + i\epsilon} \quad (8)$$

and

$$S_{\mu\nu}^W(l) = \frac{g_{\mu\nu} + l_\mu l_\nu / M_W^2}{l^2 - M_W^2 + i\epsilon} \quad (9)$$

$$+ \frac{l_\mu l_\nu / M_W^2}{l^2 - \xi_W M_W^2}. \quad (10)$$

The ξ_γ and ξ_W are the gauge fixing parameters. The calculations have been done in the physical gauge ($\xi_\gamma = 1$, $\xi_W \rightarrow \infty$). It is a combination of the Feynman and unitary gauges [22].

The TBE correction to spin-averaged cross section is given by the interference of the Born and TBE box amplitudes, namely,

$$\Delta_{TBE} = \text{Re} \sum_{spin} \mathcal{M}_{Born}^* (\square^{\parallel} + \square^{\times}), \quad (11)$$

and it can be written as

$$\Delta_{TBE} = \frac{g^4 e^2 \mathcal{K} \cos^2 \theta_C}{64(M_W^2 - q^2)}, \quad (12)$$

where

$$\mathcal{K} = \text{Im} \sum_{spin} (j_\alpha h^\alpha)^* (I^{\parallel} + I^{\times}) \sim \int d^4l \frac{N}{D}. \quad (13)$$

The above expression is the one-loop integral with complicated structure. The numerator N is a polynomial, which depends on four independent Lorentz scalars: l^2 , $l \cdot p'$, $l \cdot k'$ and $l \cdot q$. Denominator D can be of the order of at most l^8 (because of the form factors). In practice \mathcal{K} has been expressed as a combination of scalar one-loop integrals [23] and then calculated numerically (see Appendix A of Ref. [21]). The calculations are performed with the help of FeynCalc package [24] and LoopTool library [25].

The presence of the form factors in the nucleon vertices makes both box amplitudes ultraviolet finite (UV). But the amplitude $i\square^{\parallel}$ is infrared (IR) divergent. To extract the hard photon contribution from this amplitude we subtract its IR divergent part calculated in the soft photon approximation. The soft photon contribution is obtained by setting $l = 0$ in the numerator of \square^{\parallel} and keeping the leading divergent terms in denominator. This contribution does not depend on the form factors. Indeed,

$$\Delta_{TBE}(\square^{\parallel}, soft) = -4k' \cdot p' e^2 |\mathcal{M}_{Born}|^2 \text{Im} C_0,$$

where

$$C_0 = \int \frac{d^4l}{(2\pi)^4} \frac{1}{l^2 ((k' - l)^2 - m^2) ((p' + l)^2 - M_p^2)}. \quad (14)$$

To deal with the IR divergences, the photon mass μ is introduced $1/l^2 \rightarrow 1/(l^2 - \mu^2)$.

Eventually we define the TBE correction as the fraction,

$$\delta_{TBE} = \frac{\Delta_{TBE} - \Delta_{TBE}(\square^{\parallel}, soft)}{\frac{1}{2} \sum_{spin} |\mathcal{M}_{Born}|^2}, \quad (15)$$

where $d\sigma_{Born+TBE} = d\sigma_{Born}(1 + \delta_{TBE})$.

3. Results and discussion

3.1. Model-dependence of TBE

There are several sources of the model-dependence of the estimate of the TBE effect.

First is the choice of the regularization scheme used to extract the hard-photon contribution. We consider one of the simplest and straightforward procedures. Indeed the TBE correction is given by the difference between “full” and soft photon contributions. The latter has relatively simple analytical form [23]. Therefore one can

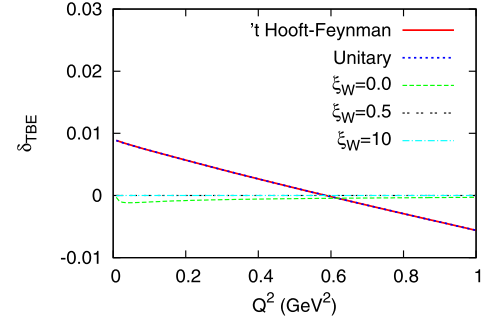


Fig. 2. TBE effect for $\nu_\mu n \rightarrow \mu^- p$, $E = 1$ GeV computed in 't Hooft-Feynman and physical gauges (the curves coincide). The gauge corrections to the TBE contribution generated by gauge-dependent part of the W propagator (Eq. (10)), calculated for several values of ξ_W are also plotted.

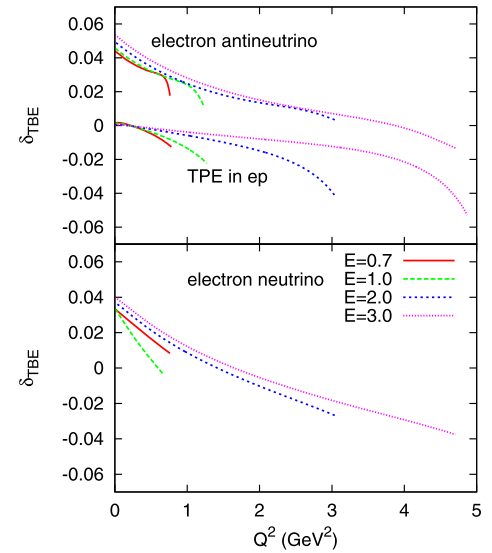


Fig. 3. δ_{TBE} (Eq. (15)) computed for $\nu_e n \rightarrow e^- p$ (bottom), $\bar{\nu}_e p \rightarrow e^+ n$ (top) reactions. In the top panel the TPE corrections (lower set of curves) for $e^- p \rightarrow e^- p$ scattering are also presented. The values of neutrino antineutrino and electron energies are in units of GeV.

easily compare the results of this paper with the other, which will be obtained within different regularization scenarios. For the reference, in Fig. 3 (top panel), we plot the TBE (Eq. (15)) and the TPE corrections, calculated in the same regularization scheme. It is interesting to notice that the Q^2 dependence of the TPE and TBE corrections is very similar but in the case of the ep scattering it is negative in the wide Q^2 range.

Another problem, which must be mentioned is the fact that gauge-invariance of the TBE amplitudes is not explicitly obeyed. In the case of the TPE effect in the elastic ep scattering it is easy to show that the sum of direct and box diagrams is gauge invariant. In the CCQE reactions the electric charge flows from the leptonic to the hadronic lines. As the result the TBE correction (as defined in Eq. (15)) depends on the gauge choice. However, this dependence turns out to be small. Indeed the gauge-dependence induced by the photon propagator is suppressed by the soft photon contribution (see Eq. (15)), while the gauge-dependence of TBE amplitude caused by the boson W propagator (contribution (10)) is also negligible in the discussed kinematical region. It is illustrated in Fig. 2, where we plot the δ_{TBE} calculated in the physical and 't Hooft-Feynman gauges (the δ_{TBE} corrections are the same in both cases). Additionally we plot the corrections to the δ_{TBE} generated by the gauge-dependent part of the W propagator (Eq. (10)) calculated

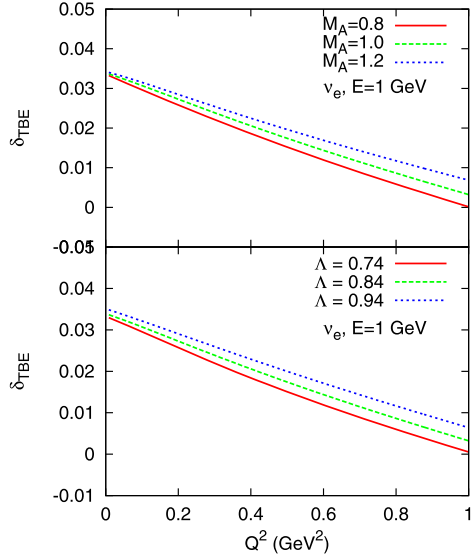


Fig. 4. δ_{TBE} dependence on the hadronic model parameters: axial mass M_A (top panel) and the vector mass Λ (bottom panel). Plots obtained for $\nu_e n \rightarrow e^- p$ reaction. The values of neutrino energies and M_A and Λ are in units of GeV.

for several values of ξ_W . We see that these corrections are negligible.

Although the TBE effect depends weakly on the gauge choice to remove this dependence the contribution from the other contributions allowed by the gauge symmetry should be considered. But in the latter case the gauge-dependence does not necessarily have to be small.

Eventually we remark that, similarly as in the case of the TPE effect [6], the δ_{TBE} depends weakly on the hadronic model parameters. It is shown in Figs. 4 and 5, where we plot the TBE contribution computed for several values of M_A and Λ as well as for the electromagnetic form factors with the G_{Ep} modified in order to agree with the PT measurements [26],

$$G_E^p(Q^2) = (-0.130 Q^2 + 1.002) G_M^p(Q^2) / \mu_p. \quad (16)$$

3.2. Numerical results and summary

In Figs. 3 and 6 we plot the δ_{TBE} calculated for several neutrino (antineutrino) energies. It is a monotonic function of Q^2 and it is the largest for low values of Q^2 . It can be seen that for $\bar{\nu}$ -proton reaction the effect is systematically larger than for ν -neutron scattering.

As mentioned in the introduction, the precise estimate of the relative difference between ν_e and $\bar{\nu}_e$ cross sections is important to take into account in the future neutrino oscillation data analysis. The difference induced by the TBE effect is shown in Fig. 5, where we plot the quantity,

$$\mathcal{R} = \frac{d\sigma_{Born}^{\nu_e} + d\sigma_{TBE}^{\nu_e}}{d\sigma_{Born}^{\nu_\mu} + d\sigma_{TBE}^{\nu_\mu}} \frac{d\sigma_{Born}^{\nu_\mu}}{d\sigma_{Born}^{\nu_e}} - 1. \quad (17)$$

The TBE correction to the total cross section is of the order of 2% and 4% for ν_e and $\bar{\nu}_e$ respectively. In the case of the ν_μ the TBE effect is negligible, while for $\bar{\nu}_\mu$ it increases the total cross section by about 2% (see Fig. 7).

In the typical long baseline experiments the energy of neutrinos in the beam is characterized by wide energy spectrum $\Phi(E)$. In Fig. 8 we plot the TBE correction computed using the energy spectra of the MiniBooNE [27] and T2K experiments. It turns out that the TBE effects for both experiments are very similar. Hence

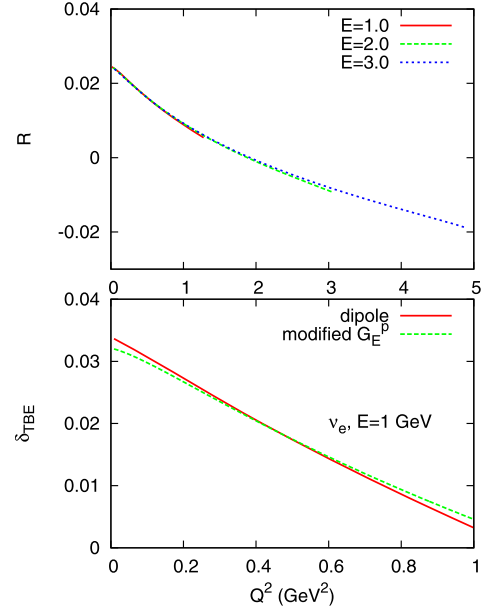


Fig. 5. Top panel: ratio (17) calculated for $E = 1$ GeV. Bottom panel: δ_{TBE} computed for $\nu_e n \rightarrow e^- p$ reaction with taking into account the dipole electromagnetic form factors and the G_E^p modified (Eq. (16)) to agree with the PT data. The neutrino energies are in units of GeV.

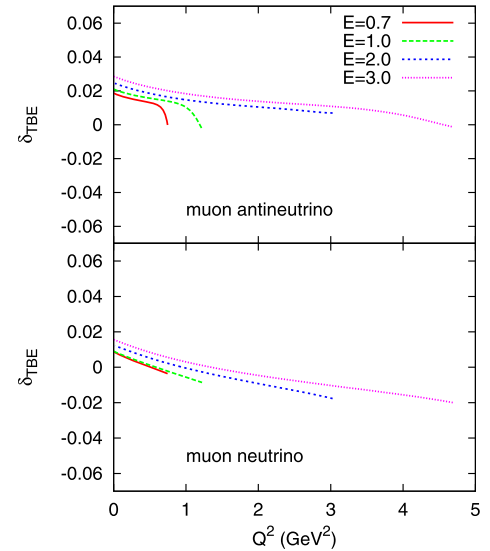


Fig. 6. δ_{TBE} (Eq. (15)) computed for $\nu_\mu n \rightarrow \mu^- p$ (bottom), $\bar{\nu}_\mu p \rightarrow \mu^+ n$ (top) reactions. The values of neutrino (antineutrino) energies are in units of GeV.

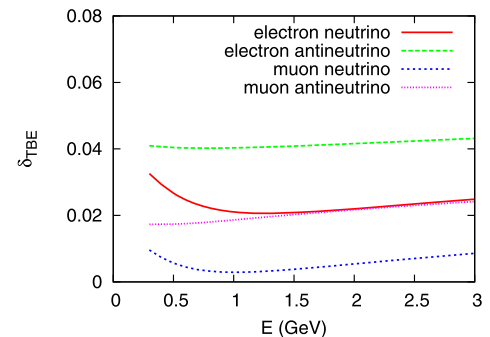


Fig. 7. The δ_{TBE} correction to the total cross section.

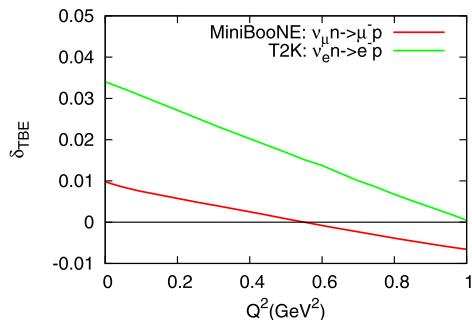


Fig. 8. δ_{TBE} computed for the MiniBooNE and the T2K ν_μ energy spectra.

for the MiniBooNE we plot the δ_{TBE} computed for ν_μ , while in the case of T2K experiment for ν_e . But in later the ν_μ energy spectrum is also used.

As it has been mentioned our estimate of the TBE effect depends weakly on the gauge choice. It complicates the direct application of the results of this paper to the experimental analysis. The TBE contribution should be supplemented by the full set of diagrams, allowed by the gauge invariance. Notice also that the RCs renormalize the tree-level values of the parameters of the SM (such as weak mixing angle) [12,28].

To summarize, we have discussed the TBE effect in the CCQE ν -nucleon scattering. The TBE correction is two times larger for ν_e than for ν_μ interactions. The relative difference between the TBE correction to ν_e and ν_μ cross sections is of the order of 2%.

Eventually let us notice that the systematic differences between ν_e and ν_μ cross sections will be critically verified by NuSTORM experiment [29], dedicated to the precise studies of the ν_e and ν_μ scattering off nucleon and nuclei.

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