Interacting cosmic fluids in power-law Friedmann–Robertson–Walker cosmological models

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Abstract

We provide a detailed description for power-law scaling Friedmann–Robertson–Walker cosmological scenarios dominated by two interacting perfect fluid components during the expansion. As a consequence of the mutual interaction between the two fluids, neither component is conserved separately and the energy densities are proportional to $1/t^2$. It is shown that in flat FRW cosmological models there can exist interacting superpositions of two perfect fluids (each of them having a positive energy density) which accelerate the expansion of the universe. In this family there also exist flat power-law cosmological scenarios where one of the fluids may have a "cosmological constant" or "vacuum energy" equation of state ($p = -\rho$) interacting with the other component; this scenario exactly mimics the behavior of the standard flat Friedmann solution for a single fluid with a barotropic equation of state. These possibilities of combining interacting perfect fluids do not exist for the non-interacting mixtures of two perfect cosmic fluids, where the general solution for the scale factor is not described by power-law expressions and has a more complicated behavior. In this study is considered also the associated single fluid model interpretation for the interaction between two fluids.
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1. Introduction

Recent observational data give a strong motivation to study general properties of Friedmann–Robertson–Walker (FRW) cosmological models containing more than one fluid. The standard modern cosmology considers the total energy density of the Universe dominated today by the densities of two components: the dark matter (which has an attractive gravitational effect like usual matter), and the dark energy (a kind of vacuum energy with a negative pressure) [1].

Usually the universe is modeled with perfect fluids and with mixtures of non-interacting perfect fluids [2]. This means that it is assumed that there is no conversion (energy transfer) among

the components and that each of them evolves separately according to standard conservation laws. However, there are no observational data confirming that this be the only possible scenario. This means that we can consider plausible cosmological models containing fluids which interact with each other. In this case the transfers of energy among these fluids play an important role. Thanks to these energy exchanges, in some cosmological models it is possible, for example, to give a reasonable explanation for the observed late acceleration of the universe [3] and for the coincidence problem [4,5], since some mechanisms could exist for converting one fluid into another. There are many other cosmological situations where this exchange of energy was considered. For example, the interaction between dust-like matter and radiation was first considered by Tolman [6] and Davidson [7]. It is interesting to note that Davidson considered only positive pressures since at that time there was no observational evidence for negative stresses in intergalactic space. Also were considered cosmological models with decay of massive

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particles into radiation, or with matter creation [8]. For more examples see Barrow and Clifton [9], and the cites contained therein.

On the other hand, for a long time cosmologists have used the most simple solutions of the Einstein field equations, applying them to cosmology, and developing the so-called standard model. In this sense, the main aim of this Letter is to consider the most simple non-trivial cosmological scenarios for an interacting mixture of two cosmic fluids described by power-law scale factors, i.e., the expansion (contraction) as a power-law in time. In a general context the power-law cosmologies are defined by their growth of the cosmological scale factor as $a(t) \propto t^{\alpha}$. The observed expanding stage of the universe is described by $\alpha > 0$; for $\alpha < 0$ we have a contracting universe ($t > 0$). The behavior of the universe in power-law cosmologies is completely described by the Hubble parameter $H = \dot{a}/a$ and the deceleration parameter $q(t) = -\ddot{a}a/\dot{a}^2$. For $a(t) = t^{\alpha}$ it takes the form $q_0 = -(\alpha - 1)/\alpha$ implying that the universe expands with a constant velocity for $\alpha = 1$ and with an accelerated expansion for $\alpha > 1$ since, if the expansion is speeding up, the deceleration parameter must be negative.

The interest in power-law FRW cosmologies is not new. The motivation for studying this kind of cosmological scenarios comes for example from the following aspects. There is good evidence for such a power-law expansion during the radiation and matter dominated epochs, for which $\alpha = 1/2$ and $\alpha = 2/3$, respectively, so in both cases we have $\alpha < 1$, implying that these epochs had a decelerated expansion.

One may also consider a simple inflationary model characterized by a period in which the scale factor is a power-law in time with $\alpha > 1$, which is called power-law inflation [10]. This occurs when the state parameter ω in the barotropic equation of state $p = \omega \rho$ is constant and $\omega < -1/3$. Power-law inflationary models allow us to solve the horizon and flatness problems, among others; however the main theoretical problem which arises from these models is that inflation never comes to an end because its slow-roll parameter is proportional to $1/\alpha$ and then is constant [11]. Nevertheless its advantage lies in the possibility of analytically computing the solutions of the perturbation equations and the corresponding power spectra [12].

On the other hand, it is interesting to note that there exist a class of cosmological models, that attempt to dynamically solve the Cosmological constant problem, in which the scale factor grows as a power law in time, regardless of the matter content or cosmological epochs [13]. Such power-law cosmologies, with $\alpha \approx 1$, satisfy the observational constraints on the present age of the Universe, the magnitude-redshift relation of the type Ia supernova and the angular size for a large sample of millisecond compact radio sources; however there are some inconsistencies with the requirement that primordial nucleosynthesis produces light elements in abundances consistent with those inferred from observational data [14].

Lastly, although there is no clear evidence for a pure power-law expansion today, maybe the Universe has entered an epoch of accelerated power-law expansion, or perhaps in the future it could enter such an expansion, and this could imply that the Universe will expand forever and never will exit from this stage. In this case only cosmological scenarios with $\alpha > 1$ may present a physical interest to us.

Another remarkable property of a power-law scale factor is that in our study the mutual exchange of energy between two perfect fluids can be described by energy densities which are proportional to $1/t^2$ and the interaction term proportional to $1/t^3$. The advantage of considering this kind of interacting fluids is that the energy densities evolve at the same rate, so their ratio is a constant quantity, thus satisfying the so-called cosmological coincidence problem, namely: Why the matter and dark energy densities are of the same order today?

2. Field equations for two interacting fluids

For an open, closed or flat FRW universe filled with two fluids ρ_1 and ρ_2, the Friedmann equation is given by

$$3H^2 + \frac{3k}{a^2} = \kappa(\rho_1 + \rho_2), \tag{1}$$

where $k = -1, 0, 1$ (from now on we shall set $\kappa = 8\pi G = 1$). We postulate that the two components ρ_1 and ρ_2 interact through the interaction term Q according to

$$\dot{\rho}_1 + 3H(\rho_1 + p_1) = Q, \tag{2}$$
$$\dot{\rho}_2 + 3H(\rho_2 + p_2) = -Q. \tag{3}$$

Note that if $Q > 0$ we have that there exists a transfer of energy from the fluid ρ_2 to the fluid ρ_1. The nature of the Q term is not clear at all. It may arise in principle from some microscopic mechanisms [5]. For solving these equations different forms for the interaction term Q have been considered.

If $Q = 0$ we have two non-interacting fluids, and then each fluid satisfies the standard conservation equation separately. Let us consider the flat case, i.e., $k = 0$. Putting $Q = 0$ into Eqs. (2) and (3) we have for each conserved component that $\rho_1 = C_1 a(t)^{-3(1+\omega_1)}$, $\rho_2 = C_2 a(t)^{-3(1+\omega_2)}$. Since we are interested in power law scenarios, the above energy densities take the following form: $\rho_1 = \rho_{10} t^{-3\alpha(1+\omega_1)}$, $\rho_2 = \rho_{20} t^{-3\alpha(1+\omega_2)}$, and from Friedmann equation (1) we obtain that $\omega_1 = \omega_2$. This implies that always both fluids have the same equation of state and then the non-interacting superposition of two fluids is really a trivial case in power-law cosmologies. However, as we shall see, the description of a superposition of two interacting fluids is not at all trivial.

2.1. Closed and open power-law interacting cosmologies

Let us now consider FRW cosmologies with $k = -1, 1$ filled with interacting matter sources which satisfy a barotropic equation of state, i.e.,

$$p_1 = \omega_1 \rho_1, \qquad p_2 = \omega_2 \rho_2, \tag{4}$$

where ω_1, ω_2 are constant state parameters. We shall define the scale factor as $a(t) = t^{\alpha}$, where α is a constant parameter. This implies that $H = \alpha/t$ and, taking into account the curvature term $3k/a^2$ of Eq. (1), we conclude that $\alpha = 1$, in order to obtain energy density scales in the same manner as the curvature

term. Since $a = t$ we have no acceleration and the universe will either expand with constant velocity or collapse with constant velocity.

This strictly linear evolution of the scale factor has been considered before in the literature. For example in Ref. [15] it is shown that an open FRW cosmology with a linear evolution of the scale factor is consistent with the latest SNe Ia observations and constraints arising from age of old quasars.

Now, from Eq. (1) and the resultant equation from the addition of Eqs. (2) and (3), we obtain that

$$\rho_{k1}(t) = \frac{(1+k)(1+3\omega_2)}{(\omega_2-\omega_1)t^2}, \tag{5}$$

$$\rho_{k2}(t) = -\frac{(1+k)(1+3\omega_1)}{(\omega_2-\omega_1)t^2}, \tag{6}$$

and then the interacting term is given by

$$Q(t) = \frac{(3+k)(1+3\omega_1)(1+3\omega_2)}{3(\omega_2-\omega_1)t^3}. \tag{7}$$

This term may be rewritten as

$$Q(t) = (1+3\omega_1)H\rho_{k1} = -(1+3\omega_2)H\rho_{k2}, \tag{8}$$

which implies that the interacting term is proportional to the expansion rate of the universe and to one of the individual densities, so $Q \sim t^{-3}$.

2.2. Flat power-law interacting cosmologies

Let us now consider interacting matter sources in flat FRW universes satisfying the barotropic equations of state (4). This means that we must put $k = 0$ and $a(t) = t^\alpha$ into Eqs. (1)–(3). Taking into account the Friedmann equation (1) and the resultant equation from the addition of Eqs. (2) and (3) we conclude that the general solution is given by

$$\rho_1(t) = \frac{\rho_{10}}{t^2} = \frac{\alpha(-2+3\alpha(1+\omega_2))}{(\omega_2-\omega_1)t^2}, \tag{9}$$

and

$$\rho_2(t) = \frac{\rho_{20}}{t^2} = \frac{\alpha(2-3\alpha(1+\omega_1))}{(\omega_2-\omega_1)t^2}, \tag{10}$$

where the Q-term takes the form

$$Q = \frac{\alpha(3\alpha(1+\omega_1)-2)(3\alpha(1+\omega_2)-2)}{(\omega_2-\omega_1)t^3}. \tag{11}$$

This implies that the interaction term may be rewritten as

$$Q = \frac{(3\alpha(1+\omega_1)-2)}{\alpha}H\rho_1 = -\frac{(3\alpha(1+\omega_2)-2)}{\alpha}H\rho_2. \tag{12}$$

From this equation we conclude that, as before, the Q-term is proportional to one of the individual densities and to the expansion rate of the universe, so $Q \sim t^{-3}$.

3. Specific two-fluid interactions

Since we are primarily interested in a characterization of a cosmological interaction between two fluids, we shall mainly consider special assumptions in order to have some classification schemes and detailed relationships between the power-law scale factor and equations of state of the interacting cosmic fluids. In this sense we shall consider that the weak energy condition (WEC) holds and then we shall require the simultaneous fulfillment of the conditions $\rho_1 \geqslant 0$, $\rho_2 \geqslant 0$, which implies that $\rho_{\text{eff}} = \rho_1 + \rho_2 \geqslant 0$. These conditions will imply some constraints on the state parameters ω_1 and ω_2.

3.1. Open and closed FRW universes

In this section we first consider the case $k \neq 0$. It is clear from Eqs. (5) and (6) that for $k = -1$ the energy densities ρ_{k1} and ρ_{k2} vanish so, in this case of an open FRW, it is not possible to have a cosmological evolution with two interacting fluids. This kind of evolution is possible only for a closed FRW Universe. Putting $k = 1$ into Eqs. (5) and (6) and requiring the fulfillment of the conditions $\rho_1 \geqslant 0$, $\rho_2 \geqslant 0$, we obtain the following constraints on the state parameters:

$$\omega_1 \leqslant -1/3, \qquad \omega_2 \geqslant -1/3, \tag{13}$$

for $\omega_2 > \omega_1$ or, equivalently $\omega_2 \leqslant -1/3$, $\omega_1 \geqslant -1/3$, for $\omega_1 > \omega_2$. From these expressions we conclude that always one of the interacting fluids must be either a dark or a phantom fluid. The constraints (13) on the state parameters imply that $Q < 0$ (see Eq. (8)), so the energy is transferred from a dark ($-1 \leqslant \omega_1 \leqslant -1/3$) or a phantom ($\omega_1 < -1$) fluid to the other matter component whose state parameter $\omega_2 > -1/3$.

Another aspect to be considered is the behavior of the constant ratio of energies $r_1 = \rho_{12}/\rho_{11} = -\frac{1+3\omega_1}{1+3\omega_2}$ as a function of the model parameters ω_1 and ω_2. For cosmological scenarios which satisfy the requirement $\omega_1 + \omega_2 < -2/3$, the matter component whose state parameter $\omega_2 > -1/3$ dominates over the dark or phantom fluid ($\omega_2 > \omega_1$).

As here we have always a constant ratio for energy densities since they are proportional to $1/t^2$, we shall use hereafter the word "dominate" in the sense of "the largest matter component of the universe is". Note that this same "dominant = predominant" component will always continue to be the larger one throughout the cosmic evolution of the cosmological model.

As examples of the behavior of energy densities in some interacting closed FRW cosmologies, Fig. 1 is plotted. The interaction of dust with dark or with phantom fluid is considered. We see that both energy densities are positive for $\omega_1 < -1/3$ and the dark fluid dominates over dust for $-2/3 < \omega_1 < -1/3$. For $-1 < \omega_1 < -2/3$ and for $\omega_1 < -1$ the dust distribution dominates over the dark and the phantom fluids respectively. On the other hand, also is considered the interaction of radiation with dark or with phantom fluid. The energy densities are positive for $\omega_1 < -1/3$. The dark fluid dominates over radiation for $-1 < \omega_1 < -1/3$ and, for $\omega_1 < -1$ the radiation distribution dominates over the phantom fluid.

3.2. Two-fluid interactions in flat FRW universes

Now we shall study interacting fluids in flat FRW cosmologies, i.e., $k = 0$. In order to make the characterization of the
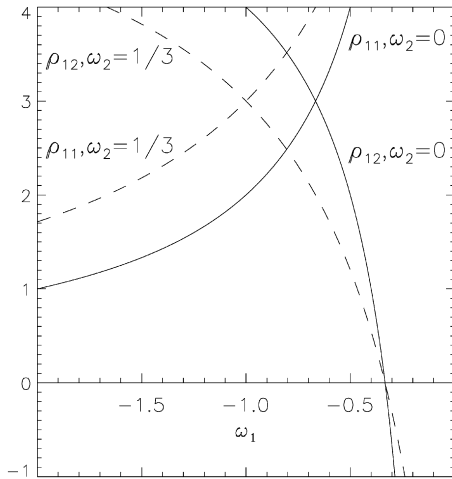
Fig. 1. The behaviors of energy densities $\rho_{k1}t^2$ and $\rho_{k2}t^2$ from Eqs. (5) and (6) are plotted, for closed ($k = 1$) interacting FRW models, as functions of ω_1. In one case (solid lines) we have taken $\omega_2 = 0$ so that one of interacting fluids is a dust distribution. There are dust dominated cosmological scenarios for $\omega_1 < -2/3$. Note that both energy densities are positive for $\omega_1 < -1/3$. In another case (dashed lines) we have taken $\omega_2 = 1/3$ so that one of interacting fluids is a radiation distribution. There are radiation dominated cosmological scenarios for $\omega_1 < -1$. Note that both energy densities are positive also for $\omega_1 < -1/3$.

interaction between the two fluids we first consider the parameter α to be a free one in Eqs. (9) and (10). This means that we shall seek all possible cosmic expansion rates for cosmological scenarios with fixed equations of state for the two interacting fluids. Note that $\rho_1 = 0$ if $\alpha = 0$ and if $\alpha = 2/(3(1 + \omega_2))$; $\rho_2 = 0$ if $\alpha = 0$ and if $\alpha = 2/(3(1 + \omega_1))$. So if we require simultaneously $\rho_{10} \geqslant 0$ and $\rho_{20} \geqslant 0$, we obtain the following possible combinations. For $\omega_2 > \omega_1$:

$$\frac{2}{3(1+\omega_2)} < \alpha < \frac{2}{3(1+\omega_1)} \quad (\omega_2 > \omega_1 > -1), \tag{14}$$

$$-\infty < \alpha < \frac{2}{3(1+\omega_1)}, \qquad \frac{2}{3(1+\omega_2)} < \alpha < +\infty$$

$$(\omega_1 < -1 < \omega_2), \tag{15}$$

$$\frac{2}{3(1+\omega_2)} < \alpha < \frac{2}{3(1+\omega_1)} \quad (\omega_1 < \omega_2 < -1). \tag{16}$$

For $\omega_2 < \omega_1$:

$$\frac{2}{3(1+\omega_1)} < \alpha < \frac{2}{3(1+\omega_2)} \quad (-1 < \omega_2 < \omega_1), \tag{17}$$

$$-\infty < \alpha < \frac{2}{3(1+\omega_2)}, \qquad \frac{2}{3(1+\omega_1)} < \alpha < +\infty$$

$$(\omega_2 < -1 < \omega_1), \tag{18}$$

$$\frac{2}{3(1+\omega_1)} < \alpha < \frac{2}{3(1+\omega_2)} \quad (\omega_2 < \omega_1 < -1). \tag{19}$$

Notice that Eqs. (14) and (17) are valid for configurations which include two interacting fluids obeying the dominant energy condition (DEC), Eqs. (15) and (18) are valid for configurations where one interacting fluid obeys DEC and the other is a phantom fluid, and Eqs. (16) and (19) are valid for the description of two interacting phantom fluids.

Now we shall consider specific two-fluid interactions. It must be noted that the relations (14)–(19) are valid for $\omega_1 \neq -1$ (or $\omega_2 \neq -1$). At the end of this section we will study configurations for which $\omega_1 = -1$ (or $\omega_2 = -1$).

3.2.1. Dust–perfect fluid interaction ($\omega_1 = 0$, $\omega_2 \neq 0$)

We shall begin with the consideration of the interaction of dust with any other perfect fluid configuration. This means that we must put $\omega_1 = 0$ into Eqs. (9) and (10), while ω_2 is still a free parameter. Thus we have for a dust (d) and a perfect fluid (pf) interacting configurations

$$\rho_{\mathrm{d}} = \frac{\alpha(-2 + 3\alpha(1 + \omega_2))}{\omega_2 t^2}, \tag{20}$$

and

$$\rho_{\mathrm{pf}} = \frac{\alpha(2 - 3\alpha)}{\omega_2 t^2}, \tag{21}$$

with the equations of state $p_{\mathrm{d}} = 0$, $p_{\mathrm{pf}} = \omega_2 \rho_{\mathrm{pf}}$. For the requirement of simultaneous fulfillment of the conditions $\rho_1 \geqslant 0$ and $\rho_2 \geqslant 0$ we obtain from Eqs. (14)–(19) that the following constraints must be satisfied:

$$\frac{2}{3(\omega_2 + 1)} < \alpha < \frac{2}{3} \quad (\omega_2 > 0), \tag{22}$$

$$\frac{2}{3} < \alpha < \frac{2}{3(\omega_2 + 1)} \quad (-1 < \omega_2 < 0), \tag{23}$$

$$\frac{2}{3} < \alpha < +\infty, \qquad -\infty < \alpha < \frac{2}{3(\omega_2 + 1)}$$

$$(-\infty < \omega_2 < -1). \tag{24}$$

Note that, for the above constraints, the specified values of ω_2 imply that really $0 < \alpha < 2/3$ (for $\omega_2 > 0$), $2/3 < \alpha < \infty$ (for $-1 < \omega_2 < 0$), and $2/3 < \alpha < \infty$ or $-\infty < \alpha < 0$ (for $-\infty < \omega_2 < -1$).

As a specific example we shall now consider in some detail the dust-radiation interaction ($\omega_1 = 0$, $\omega_2 = 1/3$). In this case we have

$$\rho_1 = \frac{\rho_{10}}{t^2} = \frac{6\alpha(2\alpha - 1)}{t^2}, \quad p_1 = 0,$$
$$\rho_2 = \frac{\rho_{20}}{t^2} = \frac{3\alpha(2 - 3\alpha)}{t^2}, \quad p_2 = \frac{1}{3}\rho_2. \tag{25}$$

In order to have simultaneously positive energy densities we must require that $1/2 < \alpha < 2/3$. The interaction term is given by $Q = \frac{3\alpha(3\alpha-2)(4\alpha-2)}{t^3}$. For the interval $1/2 < \alpha < 2/3$ the Q-term is positive and this means that we have a transfer of energy from radiation to the dust. In this scenario, for the value $1/2 < \alpha = 7/11 < 2/3$, both densities are equal during all evolution. For the interval $1/2 < \alpha < 7/11$ we have a radiation dominated universe, and for $7/11 < \alpha < 2/3$ we have a dust dominated universe (see Fig. 2). In other words there exist dust-radiation interacting cosmological scenarios dominated by radiation or dust throughout all their evolution.

We want to remark that, in the case of non-interacting dust and radiation, the expansion rate speeds up from $a(t) = t^{1/2}$ to the $a(t) = t^{2/3}$ law. For the interacting dust-radiation case we have a single expansion rate given by the $a(t) = t^\alpha$ law,
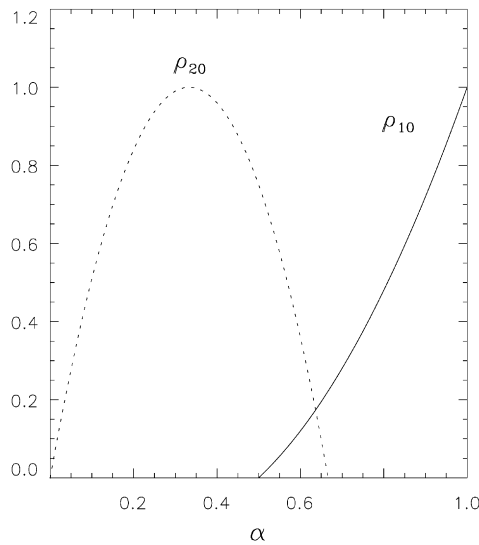
Fig. 2. The behaviors of interacting dust distribution ($\rho_1 t^2 = \rho_{10}$) and radiation distribution ($\rho_2 t^2 = \rho_{20}$) are plotted as functions of α. Both densities are positives in the interval $\frac{1}{2} < \alpha < \frac{2}{3}$, and are equal at $\alpha = \frac{7}{11}$. In the dust-radiation interacting case all possible scenarios have a decelerated expansion and the whole evolution is dominated by radiation if $\frac{1}{2} < \alpha < \frac{7}{11}$, and is dominated by dust if $\frac{7}{11} < \alpha < \frac{2}{3}$.
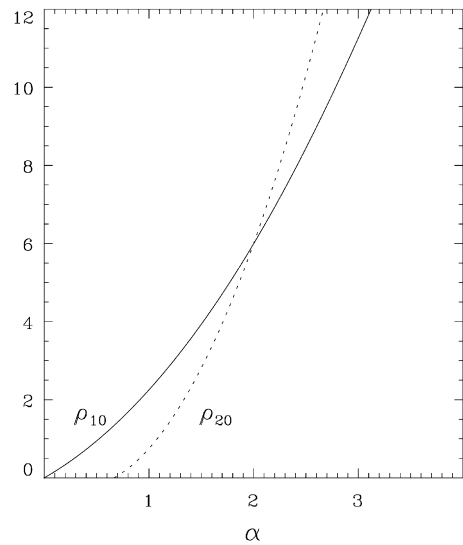

Fig. 3. The behaviors of interacting dust distribution ($\rho_1 t^2 = \rho_{10}$) and phantom matter distribution ($\rho_2 t^2 = \rho_{20}$) are plotted as functions of α. In this case the interaction is consistent with an expanding universe, and we have a non-accelerated expansion for $2/3 < \alpha < 1$, and an accelerated one for $1 < \alpha < \infty$. For $\alpha = 2$ we have $\rho_{10} = \rho_{20}$, so if $2/3 < \alpha < 2$ the universe is dominated by dust, and for $2 < \alpha < \infty$ the universe is dominated by the phantom matter component.

where $1/2 < \alpha < 2/3$. Thus in both cases the expansion of the universe is decelerated.

3.2.2. Phantom fluid–perfect fluid interaction ($\omega_1 = -4/3$, $\omega_2 \neq 0$)

On the other hand we shall now consider the interaction of a phantom fluid with any other perfect fluid configuration. We choose as a representative cosmic fluid of phantom matter the perfect fluid given by the equation of state $p = -4/3\rho$. This kind of perfect fluid was considered for example by the authors of Ref. [16]. This means that we must put $\omega_1 = -4/3$ into the Eqs. (9) and (10), while ω_2 is still a free parameter. Thus we have for a phantom fluid (ph) and a perfect fluid (pf) interacting configurations

$$\rho_{\mathrm{ph}} = \frac{3\alpha(-2 + 3\alpha(1 + \omega_2))}{(3\omega_2 + 4)t^2}, \quad (26)$$

$$\rho_{\mathrm{pf}} = \frac{3\alpha(2 + \alpha)}{(3\omega_2 + 4)t^2}, \quad (27)$$

with the equations of state $p_{\mathrm{ph}} = -4/3\rho_{\mathrm{ph}}$, $p_{\mathrm{pf}} = \omega_2 \rho_{\mathrm{pf}}$. In order to have $\rho_{\mathrm{ph}} \geqslant 0$ and $\rho_{\mathrm{pf}} \geqslant 0$ we obtain from Eqs. (14)–(19) that the following constraints must be satisfied:

$$-\infty < \alpha < -2, \qquad \frac{2}{3(\omega_2 + 1)} < \alpha < \infty \quad (\omega_2 > -1), \quad (28)$$

$$\frac{2}{3(\omega_2 + 1)} < \alpha < -2 \quad (-4/3 < \omega_2 < -1), \quad (29)$$

$$-2 < \alpha < \frac{2}{3(\omega_2 + 2)} \quad (\omega_2 < -4/3). \quad (30)$$

Note that, for the above constraints, the specified values of ω_2 imply that really $-\infty < \alpha < -2$ or $0 < \alpha < \infty$ (for $\omega_2 > -1$),

$-\infty < \alpha < -2$ (for $-4/3 < \omega_2 < -1$), and $-2 < \alpha < 0$ ($\omega_2 < -4/3$).

So for this cosmological scenario with a phantom fluid (given by the state parameter $\omega_1 = -4/3$) interacting with a perfect fluid ($p_{\mathrm{pf}} = \omega_2 \rho_{\mathrm{pf}}$) the universe expands only if $\omega_2 > -1$. In this case we can have accelerated and non-accelerated expanding cosmologies.

As a specific example we shall consider the interaction of this kind of phantom matter with a dust distribution. In this case we have that $\omega_1 = -4/3$, $\omega_2 = 0$ and then $\rho_1 = \frac{\rho_{10}}{t^2} = \frac{3\alpha(3\alpha-2)}{4t^2}$, $p_1 = -\frac{4}{3}\rho_1$, $\rho_2 = \frac{\rho_{20}}{t^2} = \frac{3\alpha(\alpha+2)}{4t^2}$, $p_2 = 0$. In order to have simultaneously positive energy densities, we must require that $\alpha < -2$ or $\alpha > 2/3$. The interaction term is given by $Q = \frac{3\alpha(\alpha+2)(2-3\alpha)}{4t^3}$. For an interacting expansion, i.e., $\alpha > 2/3$, the Q-term is positive and this means that we have a transfer of energy from dust to the phantom matter. The interaction is consistent with an expanding universe, and we have a non-accelerated expansion for $2/3 < \alpha < 1$, and an accelerated one for $1 < \alpha < \infty$. It is interesting to note that, in this scenario, for $\alpha = 2$ both densities are equal and this implies that for $2/3 < \alpha < 2$ we have scenarios where the universe is dominated by dust and, for $2 < \alpha < \infty$ we have cosmologies dominated by the phantom matter component (see Fig. 3). In other words there exist dust–phantom matter interacting cosmological scenarios dominated by dust or by the phantom matter component throughout all their evolution.

In conclusion, for the interacting dust–phantom matter case we have a single expansion rate given by the $a(t) = t^\alpha$ law, where $\alpha < -2$ or $\alpha > 2/3$, and the expansion of the universe may be decelerated or accelerated. Notice that this result implies that we can have scenarios with $\alpha \gtrsim 1$ where the universe

has an accelerated expansion but dust is dominating over phantom matter.

3.3. Interaction between effective "vacuum energy" and a perfect fluid

As we stated above, Eqs. (14)–(19) are valid for $\omega_1 \neq -1$ and $\omega_2 \neq -1$. This means that these equations cannot be applied to interacting fluids with an equation of state of the form $p_1 = -\rho_1$ and $p_2 = -\rho_2$. However it is easy to see from Eqs. (9) and (10) that the state parameters ω_1 and ω_2 may take, although not simultaneously, the value minus one.

Consider from now on in this section, that $\omega_1 = -1$. Putting this value into Eqs. (9) and (10) we obtain

$$\rho_v = \frac{\rho_{0v}}{t^2} = \frac{\alpha(-2 + 3\alpha(1 + \omega_2))}{(1 + \omega_2)t^2}, \tag{31}$$

and

$$\rho_{\text{pf}} = \frac{\rho_{0\text{pf}}}{t^2} = \frac{2\alpha}{(1 + \omega_2)t^2}, \tag{32}$$

where the first fluid has a "cosmological constant" or "vacuum energy" equation of state $p_v = -\rho_v$ and the second one is a standard perfect fluid with an equation of state $p_{\text{pf}} = \omega_2 \rho_{\text{pf}}$. The requirements that $\rho_v \geqslant 0$ and $\rho_{\text{pf}} \geqslant 0$ imply that

$$\alpha > \frac{2}{3(1 + \omega_2)} > 0 \quad (\omega_2 > -1), \tag{33}$$

$$\alpha < \frac{2}{3(1 + \omega_2)} < 0 \quad (\omega_2 < -1). \tag{34}$$

It is interesting to note that the interaction of a perfect fluid with a fluid with a "cosmological constant" or "vacuum energy" equation of state exactly mimics the behavior of the standard Friedmann solution for a single fluid with a barotropic equation of state since for $-1 < \omega_2 < -1/3$ the expansion is accelerated ($\alpha > 1$), for $\omega_2 > -1/3$ we have decelerated expansion ($\alpha < 1$), and for $\omega_2 < -1$ we have that $-\infty < \alpha < 0$.

In this case the interacting term is given by

$$Q = \frac{2\alpha(2 - 3\alpha(1 + \omega_2))}{(1 + \omega_2)t^3}, \tag{35}$$

and we conclude that the interacting term is positive for

$$0 < \alpha < \frac{2}{3(1 + \omega_2)} \quad (\omega_2 > -1), \tag{36}$$

$$\frac{2}{3(1 + \omega_2)} < \alpha < 0 \quad (\omega_2 < -1). \tag{37}$$

From Eqs. (33) and (36) we obtain for fluids which satisfy the DEC (i.e., $\omega_2 > -1$) that the interacting term $Q < 0$, so that in the here considered interacting cosmological scenarios always the energy is transferred from the effective "vacuum energy" to the perfect fluid obeying the DEC.

Another aspect to be considered is the behavior of the constant ratio of energies $r = \rho_{\text{pf}}/\rho_v = \frac{2}{3\alpha(1+\omega_2)-2}$ as function of the model parameters ω_2 and α. It is easy to see that $r(\alpha, \omega_2) > 1$ if

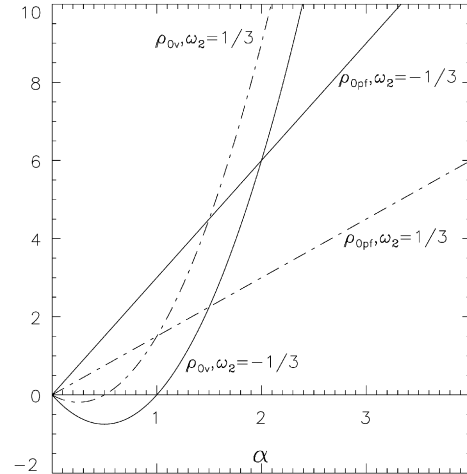$$\frac{2}{3(1 + \omega_2)} < \alpha < \frac{4}{3(1 + \omega_2)}. \tag{38}$$

Fig. 4. In the figure is shown the behavior of the energy densities of the interacting effective "vacuum energy" $\rho_v t^2$ and perfect fluid $\rho_{\text{pf}} t^2$ for the extreme cases $\omega_2 = -1/3$ (solid lines) and $\omega_2 = 1/3$ (dashed lines), see Eq. (38). In the case of interaction between effective "vacuum energy" and radiation we see that the radiation dominates only at stages with decelerated expansion ($1/2 < \alpha < 1$). At $\alpha = 1$ we have that $\rho_v = \rho_{\text{pf}}$. For the case $\omega_2 = -1/3$ we have that the perfect fluid dominates over the effective "vacuum energy" at the range $1 < \alpha < 2$ so we have accelerating expansion. For $\alpha > 2$ the effective "vacuum energy" dominates over the perfect fluid.

So the perfect fluid dominates over the effective "vacuum energy" if, for a given ω_2, the dimensionless parameter α varies in the specified above range. From Eq. (38) we see that, if $-1/3 < \omega_2 < 1/3$, there are cosmological scenarios where the universe has accelerated and non-accelerated expansions and is dominated by the perfect fluid. For $-1 < \omega_2 < -1/3$ the Eq. (38) implies that we have only accelerated scenarios where the dark component dominates over the effective "vacuum energy" (see Figs. 4 and 5). Note that for $1/3 < \omega_2 < 1$ we can have decelerated expansion where the effective "vacuum energy" dominates over the perfect fluid component.

4. The effective fluid interpretation

The main idea of this section is to study the conditions under which these two interacting sources are equivalent to an effective fluid filling the universe. In other words we want to associate an effective fluid interpretation with the interaction of the two-fluid mixture. This can be made by associating with the sum of pressures p_1 and p_2 an effective pressure p, i.e.,

$$p_1 + p_2 = \omega_1 \rho_1 + \omega_2 \rho_2 = p, \tag{39}$$

which has an equation of state given by

$$p = \gamma \rho = \gamma(\rho_1 + \rho_2), \tag{40}$$

where γ is a constant effective state parameter. Note that the equation of state of the associated effective fluid is not produced by physical particles and their interaction [17].

In this sense for example, in the above discussed case of closed FRW interacting cosmologies this single interpretation implies that the effective fluid has an equation of state given by $p = -1/3\rho$, and for the interaction between dust and radiation
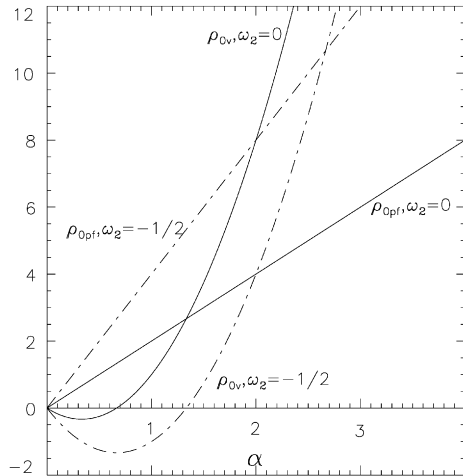
Fig. 5. In the figure is shown the behavior of the energy densities of the interacting effective "vacuum energy" $\rho_v t^2$ and perfect fluid $\rho_{pf} t^2$ for the cases $\omega_2 = -1/2$ (dashed lines) and $\omega_2 = 0$ (solid lines). In the case of dust-effective "vacuum energy" interaction we see that dust dominates only in the range $2/3 < \alpha < 4/3$. It is clear that for $1 < \alpha < 4/3$ there is an accelerated expansion dominated by dust. For the case $\omega_2 = -1/2$ we have that the perfect fluid dominates over the effective "vacuum energy" at the range $4/3 < \alpha < 8/3$ so we have accelerating expansion. For $\alpha > 8/3$ the effective "vacuum energy" dominates over the perfect fluid.

(see Eqs. (25)) this single fluid interpretation implies that the effective fluid has an equation of state given by $p = \gamma \rho$, where $0 < \gamma < 1/3$. This means that the dust-radiation interacting universe behaves as a FRW universe filled with a single fluid with a state parameter varying in the range $0 < \gamma < 1/3$, preserving DEC.

Making some algebraic manipulations with Eqs. (39) and (40) we find that the effective state parameter γ is related to the parameter α by

$$\gamma = \frac{2 - 3\alpha}{3\alpha}. \tag{41}$$

From this expression we see that the effective state parameter γ behaves as $\gamma \to -1$ for $\alpha \to \pm\infty$. For $\alpha < 0$ we have the phantom sector, since $\gamma < -1$.

Now we shall explore in more detail the effective interpretation of the interacting two perfect fluids.

4.1. Effective radiation and effective dust

As we mentioned above we can associate a single fluid model with the interaction between two perfect fluids. In this section we want to find all possible interacting superpositions for a given α-parameter. In order to do this we shall consider α to be a given parameter, and ω_1 and ω_2 to be as free ones. This means that, for a fixed scale factor (or Hubble parameter H), we shall find all possible state equation configurations for each of the two interacting fluids. From Eqs. (9) and (10) we shall obtain the constraints on the free parameters ω_1 and ω_2.

If we now require that $\rho_1 \geqslant 0$ and $\rho_2 \geqslant 0$ simultaneously, we obtain that

$$\omega_1 < \frac{2 - 3\alpha}{3\alpha} < \omega_2 \quad (\alpha > 0, \ \omega_2 > \omega_1),$$

$$\omega_1 < \frac{-2 - 3|\alpha|}{3|\alpha|} < \omega_2 \quad (\alpha < 0, \ \omega_2 > \omega_1), \tag{42}$$

or equivalently $\omega_2 < \frac{2-3\alpha}{3\alpha} < \omega_1$ ($\alpha > 0$) and $\omega_2 < \frac{-2-3|\alpha|}{3|\alpha|} < \omega_1$ ($\alpha < 0$); for $\omega_2 < \omega_1$.

Here we have excluded the value $\omega_1 = 2/3\alpha - 1$ (or $\omega_2 = 2/3\alpha - 1$) since this case gives a FRW universe filled with a single fluid. From the above equations we see that, for a physically plausible two-fluid interacting model associated with a single effective model with equation of state $p = \gamma\rho$ (see Eq. (41)) the whole ranges of validity of the parameters ω_1 and ω_2 do not intersect each other. If we want to have interacting perfect fluids which obey the DEC, we constrain the parameters to the inequalities $-1 \leqslant \omega_1 \leqslant 1$ and $-1 \leqslant \omega_2 \leqslant 1$. In this case one component (or both) may be a dark perfect fluid ($-1 \leqslant \omega_i \leqslant -1/3$, $i = 1, 2$). For the single effective model the state parameter γ also may obey the DEC $-1 \leqslant \gamma \leqslant 1$. But we can consider more general situations. There are interacting configurations where one fluid obeys DEC and the other component does not (phantom fluid), but its interaction behaves like a perfect fluid which obeys DEC. Note that this picture completely excludes the possibility of having two interacting phantom perfect fluids behaving like a fluid which obeys DEC.

As explicit examples we shall consider two interacting perfect fluids which behave like either radiation, or dust or a kind of phantom matter.

4.1.1. Effective radiation fluid

If the effective fluid behaves like radiation ($\alpha = 1/2$, $\gamma = 1/3$), then the free parameters ($\omega_2 > \omega_1$) vary in the ranges $-\infty < \omega_1 < 1/3$ and $1/3 < \omega_2 < \infty$. If we require that the second fluid satisfies the DEC (i.e. $1/3 < \omega_2 \leqslant 1$), then we can consider its interaction with a standard perfect fluid ($-1/3 < \omega_1 < 1/3$), or with a dark fluid ($-1 \leqslant \omega_1 < -1/3$), or with a phantom fluid ($-\infty < \omega_1 < -1$). This model has a decelerated expansion.

4.1.2. Effective dust

If the effective fluid behaves like dust ($\alpha = 2/3$, $\gamma = 0$), then the free parameters ($\omega_2 > \omega_1$) vary in the ranges $-\infty < \omega_1 < 0$ and $0 < \omega_2 < \infty$. If we require that the second fluid satisfies the DEC (i.e., $0 < \omega_2 \leqslant 1$), then we can consider its interaction with a standard perfect fluid ($-1/3 < \omega_1 < 0$), or with a dark fluid ($-1 \leqslant \omega_1 < -1/3$), or with a phantom fluid ($-\infty < \omega_1 < -1$). This model has a decelerated expansion.

4.1.3. An effective phantom fluid

If the effective fluid behaves like a phantom one with state parameter $\gamma = -4/3$ ($\alpha = -2$), then the free parameters ($\omega_2 > \omega_1$) vary according to the ranges $-\infty < \omega_1 < -4/3$ and $-4/3 < \omega_2 < \infty$. In this case clearly we have the possibility of having an interacting superposition of two phantom fluids. If we require that the second fluid satisfies the DEC (i.e., $-1 < \omega_2 \leqslant 1$), then we can consider its interaction with only a phantom fluid with state parameter $-\infty < \omega_1 < -4/3$. In this case we always have a contracting universe.

5. Discussion

In this Letter we have provided a detailed description for power-law scaling cosmological models in the case of a FRW universe dominated by two interacting perfect fluid components during the expansion. We have shown that in this mathematical description it is possible for each fluid component to require that the conditions $\rho_1 \geqslant 0$ and $\rho_2 \geqslant 0$ may be simultaneously fulfilled in order to have reasonable physical values of state parameters ω_1 and ω_2 (we mean either DEC, i.e., $-1 \leqslant \omega_1, \omega_2 \leqslant 1$; or else $\omega_1, \omega_2 < -1$). So from the required conditions we may gain some insights for understanding essential features of two-fluid interactions in power-law FRW cosmologies. For example, in the case of flat FRW universes, if we have a dust universe (i.e., $a = t^{2/3}$) or a radiation universe ($a = t^{1/2}$), the interacting fluids cannot both be dark (or phantom) fluids. In other words, "dust" or "radiation" effective universes cannot be filled with two interacting dark (or phantom) fluids.

On the other hand, we may apply our results to flat inflationary cosmological models involving power-law evolution for the scale factor. This means that the parameter α must be constrained to the range $\alpha > 1$, thus implying that any power-law inflationary model can be filled by two interacting fluids with state parameters given by $\omega_1 < -1/3 < \omega_2$, so always one of the interacting fluids must be either a dark fluid or a phantom one.

One consequence of our results is that one may consider accelerated cosmological models where one of the fluids is described with the help of a minimally coupled scalar field which interacts with a perfect fluid. Specifically, an exponential potential may be used for the dark energy interacting component which has a constant state parameter constrained to the range $-1 < \omega_1 < -1/3$ provided that $\alpha > 1$ [18]. In this case the scalar field evolves as $\Phi \propto \ln t$ and the perfect fluid has an equation of state of the form $p = \omega_2 \rho$. Another possibility to be considered is that the interacting dark energy component also may be modelled as a rolling tachyon field. In general a rolling tachyon condensate may be described by an effective fluid with energy density and pressure given by $\rho = V(T)/\sqrt{1 - \dot{T}^2}$ and by $p = -V(T)\sqrt{1 - \dot{T}^2}$, where T is the tachyon field and $V(T)$ is the tachyon potential [19,20]. It is possible to obtain power-law inflationary cosmological models by assuming that the potential is an inverse square in terms of the tachyon field, i.e., $V(T) = \beta T^{-2}$, where $\beta > 0$ [20]. The same fields may be considered for describing the present accelerating stage of the universe. Notice that in this case it is also possible to consider the interaction of a perfect fluid with phantom energy ($\omega_1 < -1$) in the form of an imaginary tachyon field [21], which may be obtained by simply Wick rotating the tachyon field [22]. A detailed analysis of the here discussed ideas is in progress and will be published elsewhere.

The here described variety of possibilities for combining interacting perfect fluids with energy densities $\propto t^{-2}$ does not exist for the non-interacting mixtures of two perfect cosmic fluids, where the general solution for the scale factor is not described by power-law expressions and has a more complicated behavior.

Note that the considered power-law cosmologies may describe satisfactorily the interaction of dark matter (which is generally assumed to be collisionless, i.e., described by a pressureless fluid [23]) with any other perfect fluid configuration. So the relations obtained in Section 3 for dust-perfect fluid interaction may be applied to interacting dark matter.

It is interesting to observe that the here considered variety of flat power-law scaling cosmological models is related to the study made by Barrow and Clifton for cosmological models with a mutual exchange of energy between two fluids at rates which are proportional to a linear combination of their individual densities and to the expansion rate of the universe [9]. An advantage of considering this type of interacting fluids is that the energy densities at late times evolve at the same rate, so their ratio is a constant quantity in agreement with the so-called cosmological coincidence problem.

Specifically, for the kind of interaction studied by Barrow and Clifton, the power-law solutions behave at late times as attractors of the general solution for the field equations (1)–(3) of Ref. [9]. In particular, those authors provided a simple mathematical description of the two interacting fluids in an expanding flat FRW universe and showed that the evolution can be reduced to a single nonlinear master differential equation for the Hubble parameter H of the form $\ddot{H} + AH\dot{H} + BH^3 = 0$, where A and B are constants. This equation can be solved in physically relevant cases and the authors provide an analysis of all possible evolutions. Particular power-law solutions exist for the expansion scale factor and are attractors at late times under particular conditions. Note that the power-law scale factors are solutions (self-similar solutions) for the master equation $\ddot{H} + AH\dot{H} + BH^3 = 0$ with the parameters A and B constrained.

For the interacting flat cosmological scenarios discussed in our Letter, we see that Eq. (12) implies that the here considered power-law cosmologies are the attractors for the particular solution with $\alpha_{\mathrm{BC}} = \frac{(3\alpha(1+\omega_1)-2)}{\alpha}$, $\beta_{\mathrm{BC}} = 0$ (or $\alpha_{\mathrm{BC}} = 0$, $\beta_{\mathrm{BC}} = \frac{(3\alpha(1+\omega_2)-2)}{\alpha}$) of the above mentioned general Barrow–Clifton solution, so all relations discussed in this work may be applied to the late time behavior of this particular solution and could help us clarify which kind of specific interacting matter configurations are physically plausible today.

Lastly, today the observational data of Type Ia supernovae are suggesting that our universe is undergoing accelerated expansion [3], so accelerated interacting superpositions may play an important role in the study of two interacting fluids at rates that are proportional to a linear combination of their individual densities and to the expansion rate of the universe. On the other hand, although there is no clear evidence for a pure power-law expansion today, maybe the Universe has entered an epoch of accelerated power-law expansion, or perhaps in the future it could enter such an expansion, and this could imply that the Universe will expand forever and never will exit from this stage. From this point of view all found parameter constraints may shed light on the possible cosmological scenarios to be considered. So in this sense all interacting configurations with

$0 < \alpha < 1$ could not represent interest today due to observational data.

Acknowledgements

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