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Thermal analysis of porous pin fin used for electronic cooling

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Abstract

The present work investigates the temperature distribution, performance parameters and heat transfer rate through a porous pin fin in natural convection condition. This study is based on finite-length fin with insulated tip. To formulate the heat transfer equation for the porous fin, the energy balance and Darcy's model are used. An analytical technique called Adomian decomposition method (ADM) is proposed for the solution methodology. To validate the analytical results, a numeric scheme, namely, finite difference method is adopted. The results indicate that the numerical data and analytical approach are in agreement with each other. The effects of various geometric and thermophysical parameters on the dimensionless temperature distribution and fin performance are studied that may help in optimum design analysis of a porous pin fin. Finally, the increase in heat transfer is noticed by selecting porous medium condition in the fin.

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Keywords: porous surface; convection; performance; electronic cooling; pin fin

1. Introduction

Most of the engineering applications mainly in electronic devices require highly efficient components in the perspective of heat dissipation with progressively less weights, volumes, accommodating shapes and costs. Under the conditions of high clock speeds and shrinking package size, the heat discharge per unit volume from these devices has increased radically over the past decades. Thus, the effective cooling technology becomes essential for reliable operation of such electronic components. Although various cooling mechanisms have been continuously

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employed to remove heat from heat sinks but fins or extended surfaces play an important role to augment the rate of heat transfer. Extended surface is used specially to enhance the heat transfer rate between a solid and an adjoining fluid. It is well known that the rate of heat transfer from the fin decreases with the increase of fin length and hence, the entire heat transfer surface of a fin may not be equally utilized. For this reason, there is a continuous effort by the researchers as well the designers to determine the optimum shape of fin that will maximize the rate of heat transfer for a given heat transfer rate.

Nomenclature	
А	Adomian polynomials, i=0.1.2
c _p	specific heat of the fluid passing through porous fin(J/kg-K)
Ďа	Darcy number
g	gravity constant (m/s^2)
h	heat transfer coefficient over the fin surface(W/m ² K)
k	thermal conductivity(W/mK)
k _r	thermal conductivity ratio
К	permeability of the porous fin(m ²)
m	mass flow rate of fluid passing through porous fin(kg/s)
NU D	Nusselt number
P	perimeter of the lin(m) setup here transfer rate per unit area of the percent fin(W/m^2)
Ч О	dimensionless actual heat transfer rate per unit area (W/m^2)
Q a:	ideal heat transfer rate per unit area (W/m^2)
Q_1	dimensionless ideal heat transfer rate per unit area (W/m^2)
	heat transfer rate per unit area through the same base area in un-finned situation(W/m^2)
Q _w	dimensionless heat transfer rate per unit area in un-finned situation(W/m^2)
Ra	Rayleigh number
R	radius of the fin(m)
Та	ambient temperature(K)
T _b	base temperature(K)
Т	fin surface temperature(K)
v	average velocity of fluid passing through porous fin(m/s)
X	axial length measured from fin tip(m)
Х	dimensionless axial distance
Greek symbols	
β	coefficient of thermal expansion(K^{-1})
ε	effectiveness of the fin
η	efficiency of the fin
ξ	porosity or void ratio
ψ	radius to length ratio
θ	dimensionless temperature
θ_0	dimensionless tip temperature
γ	kinematic viscosity(m^2/s)
ρ	density of the fluid(kg/m ³)
Subscripts	
S	solid properties
f - CC	fluid properties
ejj	porous properties

An overview of the fin optimum shaping issue has been depicted by Snider and Kraus [1]. Mever [2] in his famous book with a simple manner describes the temperature distribution in conventional fins. But in the recent years, porous fins have become an excellent passive means to provide high heat transfer rate for electronic components in a small, light weight, low maintenance and energy free package. A good insight into the subject are given by Pop and Ingham [3], Nield and Bejan [4]. A simple method has developed by Kiwan [5] to analyze the performance of porous fins in a natural convection environment. Bassam and Hijleh [6] investigated a problem of natural convection from a horizontal cylinder with multiple equally spaced high conductivity permeable fins on its outer surface. They concluded that porous fins provide much higher heat transfer rate than traditional fins. Gorla [7] and Kiwan [8] investigate thermal analysis of natural convection and radiation in porous fins of rectangular shape. Varol et al. [9] performed a theoretical study of buoyancy-driven flow and heat transfer in an inclined trapezoidal enclosure filled with a fluid-saturated porous medium heated and cooled from inclined walls. On the other hand natural convection heat transfer and fluid flow in porous triangular enclosures with vertical solid adiabatic thin fin have been numerically analyzed by Varol et al. [10]. Kiwan and Al-Nimr [11] numerically investigated the effect of using porous fins to enhance the heat transfer from a given surface. The thermal performance of porous fins is also estimated and compared with that of the conventional solid fins. Kundu and Bhanja [12] developed an analytical model for determination of the performance and optimum dimensions of porous fin of rectangular shape. On the other hand, Bhanja and Kundu [13] established an analytical model to determine the temperature distribution and thermal performance parameters of a constructal T-shaped porous fin. Recently, Kundu et al. [14] worked on the performance and optimum design analysis of porous fin of various profiles operating in convection environment. For the analytical solution they have used Adomian decomposition method. Yu and Chen [15] performed a study on optimization of circular fin with variable thermal parameter. Saedodin and Olank [16] investigated the temperature distribution over fin surface and compared the results with conventional fins. For the analysis they have selected a pin fin subject to heat transfer in natural convection condition.

From the through literature survey summarized above, it is apparent that a very few work is present about porous pin fin. In this study, an analytical methodology followed by Adomian decomposition method [17] is applied to solve the nonlinear class of governing energy equations of a porous pin fin attached to a vertical isothermal wall. The present approximate analytical technique is a very useful and practical method for solving any class of nonlinear governing equations without adopting linearization or perturbation technique. It provides an analytical solution in the form of power series where the temperature on the fin surface can be expressed explicitly as a function of position along the length of the fin. Thus, the temperature distribution and its performances are easily being determined for a wide range of design variables of porous fins.

2. Description of the problem

Fig. 1 shows a straight porous pin fin of uniform cross-section having length L and diameter D. Fin is attached to a vertical isothermal wall from which heat has to be dissipated through natural convection. As the fin is porous, it allows fluid to penetrate through it. The porous fin increases the effective surface area of the fin through which the fin convects heat to the working fluid. In order to simplify the solution, the following assumptions are considered:

- · Porous medium is homogeneous, isotropic and saturated with a single phase fluid
- Physical properties of solid as well as fluid are considered as constant except density variation of liquid, which may affect the buoyancy term where Boussinesq approximation is employed
- · Darcy formulation is used to simulate the interaction between the porous medium and fluid
- The temperature inside the fin is only function of x
- There are no heat sources in the fin itself and no contact resistance at the fin base
- The fin tip is adiabatic type



Fig. 1: Schematic diagram for the problem under consideration

In this study, total convective heat transfer from the porous fin is calculated as the summation of convection due to motion of the fluid passing through the fin pores and that of from the solid surface. By applying an energy balance to the differential segment of the porous fin with considering only convection, mathematically it yields

$$q(x) - q(x + \Delta x) = \dot{m}c_p \left(T - T_a\right) + hP\Delta x \left(1 - \xi\right) \left(T - T_a\right)$$
⁽¹⁾

where \dot{m} is the mass flow rate of the fluid passing through the porous material, $\dot{m} = \rho v \Delta x P$. The fluid velocity can be estimated from Darcy model, $v = gK\beta(T - T_a)/\gamma$ [4]. Considering these expressions, one-dimensional energy equation of a straight porous pin fin can be written by using Fourier's law of heat conduction as

$$\frac{d^2 T}{dx^2} - \frac{4\rho c_p g K \beta (T - T_a)^2}{\gamma D k_{eff}} - \frac{4h(1 - \xi) (T - T_a)}{D k_{eff}} = 0$$
(2)

With the consideration of some conventional parameters and in order to express Eq. (2) in non-dimensional form, the following dimensionless parameters are defined as

$$(X;\psi;k_r;\theta) = \left(\frac{x}{L};\frac{R}{L};\frac{k_s}{k_f};\frac{T-T_a}{T_b-T_a}\right); \quad (Nu;Ra;Da) = \left(\frac{hD}{k_f};\frac{\rho c_p g \beta (T_b-T_a)D^3}{k_f \gamma};\frac{K}{D^2}\right);$$

$$(\omega_1;\omega_2) = \left(\frac{Ra Da}{\Omega \psi^2};\frac{Nu(1-\xi)}{\Omega \psi^2}\right); \quad \Omega = \frac{k_{eff}}{k_f} = \xi + (1-\xi)k_r$$

$$(3)$$

Eq.(2) can be written in dimensionless form by using Eq.(3) as follows:

$$\frac{d^2\theta}{dX^2} = \omega_1 \theta^2 + \omega_2 \theta \tag{4}$$

Eq.(4) is a second order differential equation. On the fin surface, there is a linear location, at X = 0, i.e., at the fin tip from where heat transfer does not take place. Thus, the boundary conditions for determining the dimensionless temperature distribution in a fin can be expressed mathematically as follows:

$$\frac{d\theta}{dX} = 0, \quad at \ X = 0 \tag{5}$$

$$\theta = 1, at X = 1 \tag{6}$$

3. Solution with Adomian decomposition method

Eq.(6) may not be solved by a usual analytical technique. In this work, Adomian decomposition method [17] is employed to obtain temperature distribution along the fin length. Eq.(6) can be written in operator form as

$$L_x \theta = \omega_1 \theta^2 + \omega_2 \theta \tag{7}$$

where, L_x is the linear second order differential operator $(L_x = d^2\theta/dX^2)$ which is invertible. Assuming that the inverse operator L_x^{-1} exists and can conveniently be taken as the two-fold indefinite integral with respect to X from 0 to X

$$L_x^{-1}(\bullet) = \int_0^X \int_0^X (\bullet) dX dX$$
(8)

Applying inverse operator to Eq.(7), it yields using Eq.(8) as

$$\theta = \theta\left(0\right) + X \frac{d\theta\left(0\right)}{dX} + \omega_1 L_x^{-1} \left(\theta^2\right) + \omega_2 L_x^{-1} \left(\theta\right)$$
(9)

where, $\theta(0)$ is the dimensionless tip temperature of the fin which can be denoted as θ_0 . Now the unknown function θ_i , $i \ge 1$ can be decomposed into a sum of components defined by the decomposition series as

$$\theta = \sum_{i=0}^{\infty} \theta_i \tag{10}$$

Therefore, Eq. (10) can be expressed as

$$\sum_{i=0}^{\infty} \theta_i = \theta_0 + \omega_1 L_x^{-1} \left[\sum_{i=0}^{\infty} A_i \right] + \omega_2 L_x^{-1} \left[\sum_{i=0}^{\infty} \theta_i \right]$$
(11)

where, A_i is the so-called Adomian polynomial corresponding to the nonlinear term θ^2 . In order to obtain the higher order terms Eq. (11) can be written with a recursive relationship as

$$\theta_{i} = \omega_{1} L_{x}^{-1} \left(A_{i-1} \right) + \omega_{2} L_{x}^{-1} \left(\theta_{i-1} \right)$$
(12)

The Adomian polynomials for the nonlinear term θ^2 can be expressed in a matrix form as

$$(A_0; A_1; A_2; A_3;) = (\theta_0^2; 2\theta_1 \theta_0; 2\theta_2 \theta_0 + \theta_1^2; 2\theta_3 \theta_0 + 2\theta_2 \theta_1;)$$
(13)

Using Eqs.(10)-(13), the following temperature expression are obtained after integration:

$$\theta = \theta_0 + \left(\omega_1 \theta_0^2 + \omega_2 \theta_0\right) \frac{X^2}{2!} + \left(2\omega_1^2 \theta_0^3 + 3\omega_1 \omega_2 \theta_0^2 + \omega_2^2 \theta_0\right) \frac{X^4}{4!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_1 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_1 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_1 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_1 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_1 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_1 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_1 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_1 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_1 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_1 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_1 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_2 \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_1 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_2 \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_1 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_1 \omega_2^3 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 10\omega_1^2 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 10\omega_1^2 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 10\omega_1^2 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 10\omega_1^2 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 10\omega_1^2 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 10\omega_1^2 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 10\omega_1^2 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 10\omega_1^2 \omega_2^2 + \omega_2^3 \theta_0\right) \frac{X^4}{6!} + \left(10\omega_1^3 \theta_0^4 + 10\omega_1^2 \psi_0^2 + \omega_2^3 \psi_0^2 + \omega_2^3$$

$$+ \left(80\omega_{1}^{4}\theta_{0}^{5} + 200\omega_{1}^{3}\omega_{2}\theta_{0}^{4} + 162\omega_{1}^{2}\omega_{2}^{2}\theta_{0}^{3} + 43\omega_{1}\omega_{2}^{3}\theta_{0}^{2} + \omega_{2}^{4}\theta_{0}\right)\frac{X^{8}}{8!} + \dots$$
(14)

It can be highlighted that the temperature distribution in the fin as obtained in Eq. (14) is a function of unknown tip temperature θ_0 . This dimensionless tip temperature θ_0 can be determined by applying boundary condition expressed in Eq. (6). A transcendental algebraic equation as a function of θ_0 is obtained as

$$1 - \theta_0 - \left(\omega_1 \theta_0^2 + \omega_2 \theta_0\right) \frac{1}{2!} - \left(2\omega_1^2 \theta_0^3 + 3\omega_1 \omega_2 \theta_0^2 + \omega_2^2 \theta_0\right) \frac{1}{4!} - \left(10\omega_1^3 \theta_0^4 + 20\omega_1^2 \omega_2 \theta_0^3 + 11\omega_1 \omega_2^2 \theta_0^2 + \omega_2^3 \theta_0\right) \frac{1}{6!} - \left(80\omega_1^4 \theta_0^5 + 200\omega_1^3 \omega_2 \theta_0^4 + 162\omega_1^2 \omega_2^2 \theta_0^3 + 43\omega_1 \omega_2^3 \theta_0^2 + \omega_2^4 \theta_0\right) \frac{1}{8!} - \dots = 0$$
(15)

After obtaining temperature distribution as a spatial function, actual heat transfer rate per unit area of the pin fin is determined by applying the Fourier's law of heat conduction at the base and can be expressed in dimensionless form as

$$Q = \frac{q}{k_f \left(T_b - T_a\right)/R} = \Omega \psi \left(\frac{d\theta}{dX}\right)_{X=1}$$
(16)

Similarly dimensionless heat transfer rate per unit area in ideal and un-finned condition can be expressed as

$$\begin{pmatrix} Q_i \\ Q_w \end{pmatrix} = \begin{pmatrix} (Ra \, Da + Nu \, (1 - \xi)) / \psi \\ 0.5Nu \end{pmatrix}$$
(17)

Thus, from definition fin performance parameters, efficiency η and effectiveness, ε can be expressed as

$$\begin{pmatrix} \eta \\ \varepsilon \end{pmatrix} = \begin{pmatrix} Q/Q_i \\ Q/Q_w \end{pmatrix}$$
 (18)

4. Results and discussion

There has been a common belief that reliable electronics can be achieved by lowering the temperature. Typically, statements such as "lower the temperature by 10°C and double the reliability" are used to characterize the effect of temperature on reliability. The maximum functional temperature to ensure operation and safe performance for an electronic device is typically under 100°C. For example, the Intel 850 chipset allowed the operating temperature to 50°C, with a recommended airflow over the package for 150 LFM (linear feet per minute). The thermal solution, which is applied to this Intel 850 chipset, is a passive simple extruded heat sink with thermal and mechanical interfaces. Additionally to these kinds of cooling methods, to improve the system cooling characteristics, a redesign of fans, vents and ducts can be considered. But because of a vast development in the IC technology, electrical devices appears in smaller scale with a higher performance ability, which come along with a general aspect that, the more heat is generated as the size of electronic device is getting smaller.

Keeping temperature limit for electronic components in mind and based on the above analysis, results mainly temperature distribution, heat transfer rate and fin performance parameters are generated with a wide range of thermo-physical and thermo-geometric parameters. To furnish the present analytical results, it is important to validate with the published model. Unfortunately, it can not be done directly due to unavailability of similar type in

the existing literature. Thus a numeric scheme based on the finite difference method has been used for this validation. A difference equation is derived by discretization of governing equation of the present problem using Taylor series central difference scheme. Finally, it is solved by Gauss-Seidal iterative method by satisfying desired convergence criteria. Fig. 2 is drawn for the validation of the present analytical model. Figure shows that both analytical and numerical results are exactly matching. Fig. 2 also shows that variation of temperature increases with the increase in Darcy parameter Da.



Fig. 2. Comparison of results predicted by proposed analytical and numerical methods.



Fig. 3. Variation of dimensionless temperature with the variation of (a) $Ra \& \xi$; (b) $k_r \& Nu$.

The variation of dimensionless temperature distribution of the fin surface with the variation of different thermophysical parameters is depicted in Fig. 3. An increase in Ra improves the effective convective heat transfer coefficient between the fin and the working fluid which enhances the heat transfer rate by convection and thus dimensionless temperature declines as predicted in Fig. 3a. The same observation has also been noticed for the

porosity parameter ξ . Actually, a high porosity decreases the effective thermal conductivity of the porous fin due to the removal of solid material and thus maintains lower temperature at the fin tip. Fig. 3b shows that dimensionless temperature increases with the increase of thermal conductivity ratio k_r . It is an obvious result as high value of k_r always indicates a good thermal conductive material which can maintain tip temperature very high and augments the rates of heat transfer. On the other hand the variation with Nu predicts a reverse trend.



Fig. 4. Fin performances with the variation of ψ , ξ and Nu: (a) fin efficiency; (b) fin effectiveness.



Fig. 5. Fin performances as a function of ξ and Da: (a) efficiency; (b) effectiveness.

The effect of porosity parameter and Nusselt number on fin performance as a function of geometric parameter ψ is envisioned in Fig. 4. Both the fin performances are dropped with the increase of these parameters. High value of porosity parameter not only decreases the effective thermal conductivity but also reduces ideal heat transfer as defined in Eq. (18). But the impact of this parameter is not so influential with low Nu. That's why both fin efficiency and effectiveness decline with increasing ξ . Moreover, ideal heat transfer rate also decreased with the increase of Nu as defined in Eq. (18) and thus performance is decremented. On the other hand both fin efficiency

and effectiveness enhance with the geometric parameter ψ . The value of parameter ψ can be increased either by increasing the radius or by decreasing length of the fin. As the entire fin length can not be effectively utilized due to conductive resistance of the fin material thus the fin having smaller length always transfers better heat and thus fin performance become high.

Fin performances as a function of porosity parameter and Darcy number is shown in Fig. 5. The main viewpoint behind using porous fins is to increase the effective surface area through which heat is convected to the surrounding fluid. When the value of ξ approaches to a unit value fin performance parameters become zero as effective thermal conductivity is reduced to a very less in magnitude. As the permeability of the porous fin increases, i.e., increasing *Da* number, the working fluid ability to penetrate through the fin pores and to convect heat increases but side by side it increases the ideal heat transfer rate also as defined in Eq. (18). Thus a reduction in fin efficiency is noticed. On the other hand there is no impact of this parameter in calculating heat transfer rate in un-finned condition and thus fin effectiveness is remarkably increased.



Fig. 6. (a) variation of temperature gradient at fin base as a function of Da, ψ and Nu; (b) heat transfer rate with the variation of k_r , ξ and Ra

Fig. 6a shows the effect of Da and Nu on temperature gradient at fin base as a function of ψ . As mentioned earlier that a high *Da* number indicates mainly high permeability of the porous fin which means more working fluid can pass through it and thus creates a higher temperature gradient at the fin base. Porous fins having small Da number behave as solid fins due to their small permeability. The effect of Nu number also shows the same trend because it increases the heat transfer coefficient over the fin surface. On the other hand, the dimensionless actual heat transfer rate through the porous pin fin surface as a function of ξ , Ra and k_r is depicted in Fig. 6b. It is clear from this figure that actual heat transfer rate enhances with the increase of these parameters. For a particular fluid, with increasing the parameter k_r , thermal conductivity of the fin material is also increased that reduces the conductive resistance in the fin surface and thus heat transfer rate is enhanced. Actually the basic philosophy behind using porous fins is to increase the effective surface area through which heat is convected to the ambient fluid. Although, effective thermal conductivity of the porous fin decreases, due to removal of solid material but this reduction is compensated with the increase in effective surface area. As predicted, increasing Ra number increases the heat transfer rate. Increasing Ra number improves the convective heat transfer coefficient between the fin and the working fluid. This improves not only performance but also heat transfer rate. However, in porous fins the convective heat transfer coefficient interacts with the fin through a volumetric surface area which is much larger than the conventional fin surface area. As a result, any improvement in the convective heat transfer coefficient causes much more improvement in the porous fin performance as compared to the improvement in the conventional solid fin performance. Thus actual heat transfer rate through the porous fin can be augmented than the solid fin with proper selected values of these parameters.

5. Concluding remarks

An effort has been made to determine the temperature distribution, fin performance and heat transfer rate over a straight porous pin fin that may help in optimum design analysis. The fin dissipates heat to the environment through natural convection. For the aforementioned conditions, an approximate analytical technique, namely, Adomian decomposition method (ADM) has been proposed for the solution of governing fin equation. This method provides solution in the form of infinite power series and it has high accuracy and fast convergence. Thus, fin performance parameters and heat transfer rate can easily be obtained from the explicit form of the temperature distribution. The following concluding remarks can be drawn from the present study:

- Temperature distribution in the porous pin fin is highly dependent upon the related parameters. A higher tip temperature is maintained with the low value of the parameters Da, ξ, Ra and Nu whereas the parameter k_r shows a reverse trend.
- Fin performance is increased with the decrease of ξ and Nu whereas performance is better with increasing geometric parameter ψ . Moreover, fin efficiency is decreased with the increase of Darcy number whereas fin effectiveness shows an opposite nature.
- Temperature gradient at fin base is increased with the increase of *Da and Nu* whereas it decreases with the increase of geometric parameter ψ.
- Actual heat transfer rate is enhanced with the increase of the parameters ξ , Ra and k_r .

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