A diquark–triquark model for the KN pentaquark

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Abstract

We propose a model for the recently discovered Θ^+ exotic KN resonance as a novel kind of a pentaquark with an unusual color structure: a $\bar{\mathbf{3}}_c$ ud diquark, coupled to $\mathbf{3}_c$ $ud\bar{s}$ triquark in a relative P-wave. The state has $J^P = 1/2^+$, $I = 0$ and is an antidecuplet of $SU(3)_f$. A rough mass estimate of this pentaquark is close to experiment.
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1. Introduction

1.1. Modeling the pentaquark: need both qq and $q\bar{q}$ interactions

The recent observation of the strange Θ^+ pentaquark [1–3] with a mass of 1540 MeV and a very small width ~ 20 MeV has generated a great deal of interest. Although the original prediction of an exotic KN resonance was obtained within the framework of the Skyrme model [4,5], there is an obvious and urgent need to understand what Θ^+ is in the quark language [6].

An additional nontrivial challenge for the quark interpretation [7] is that whereas the Skyrme model predicts that Θ^+ has positive parity, the "standard" pentaquark involves 5 quarks in an S-wave and therefore

has negative parity. As of now, there is no clearcut experimental information on the Θ^+ parity, but if it is positive, clearly one must have one unit of orbital angular momentum and this makes the calculation difficult.

The most straightforward interpretation of the Θ^+ in terms of quarks is that it is a $uudd\bar{s}$ pentaquark, so it has both qq and $q\bar{q}$ interactions. At present it is not possible to compute the properties of such a state from first principles, so it is necessary to use a model which is known to reliably deal with both types of interactions.

The quark model we use provides such a unified treatment of both types of interactions in mesons and baryons. Pioneered by Sakharov and Zeldovich [8], it has subsequently been extended and motivated within the framework of QCD by De Rujula, Georgi and Glashow [9], in terms of color-magnetic interaction model for the hyperfine interaction, and augmented by Jaffe's color-spin algebra [10] for multiquark systems.

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To provide a basis for the credibility for our use of the model and to prepare the tools for the analysis of the pentaquark, we now briefly review and update the successes of the model for a unified treatment of mesons and baryons of all flavors.

1.2. Summary of successful mass relations from hadrons containing no more than one strange or heavy quark

Early evidence that mesons and baryons are made of the same quarks was provided by the remarkable successes of the constituent quark model [8], in which static properties and low lying excitations of both mesons and baryons are described as simple composites of asymptotically free quasiparticles with a flavor dependent linear mass term and hyperfine interaction, yielding a unified mass formula for both mesons and baryons

$$M = \sum_i m_i + \sum_{i>j} \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i m_j} v_{ij}^{\text{hyp}}, \qquad (1.1)$$

where m_i is the effective mass of quark i, $\vec{\sigma}_i$ is a quark spin operator and v_{ij}^{hyp} is a hyperfine interaction with different strengths but the same flavor dependence and we have added the explicit flavor dependence of the hyperfine interaction [9].

The effective quark mass appears in two different terms in Eq. (1.1): as an additive term and in the denominator of the hyperfine interaction. In all the relations for masses and magnetic moments obtained in the light (uds) flavor sector, and for hadrons containing no more than one heavy or strange quark, agreement with experiment has been obtained by assuming that the values of the effective quark masses in these two terms has been the same and that the values are the same for mesons and baryons. Both the mass difference and the mass ratio between two quarks of different flavors were found to have the same values to a good approximation when they are bound to a nonstrange antiquark to make a meson and bound to a nonstrange diquark to make a baryon.

For example, the effective quark mass difference $m_s - m_u$ is found to have the same value $\pm 3\%$ and the mass ratio m_s/m_u the same value $\pm 2.5\%$, when calculated from baryon masses and from meson masses [8,11,12], with a simple recipe for removing

the hyperfine contribution. Thus the mass difference of two quarks, denoted by Q and q, can be obtained from meson masses,

$$\langle m_Q - m_q \rangle_{\text{Mes}}$$
$$= \frac{3 M_{\mathcal{V}_{Q\bar{u}}} + M_{\mathcal{P}_{Q\bar{u}}}}{4} - \frac{3 M_{\mathcal{V}_{q\bar{u}}} + M_{\mathcal{P}_{q\bar{u}}}}{4}, \qquad (1.2)$$

where $\mathcal{V}_{Q\bar{u}}$ and $\mathcal{P}_{Q\bar{u}}$ denote the vector and pseudo-scalar mesons with the constituents $Q\bar{u}$, etc. The same observable can also be obtained from baryon masses,

$$\langle m_Q - m_u \rangle_{\text{Bar}} = M_{\Lambda_Q} - M_N \qquad (1.3)$$

so that for $Q = s$ and $q = u$ one has

$$\langle m_s - m_u \rangle_{\text{Bar}} = M_\Lambda - M_N = 177 \text{ MeV},$$
$$\langle m_s - m_u \rangle_{\text{Mes}} = \frac{3 M_{K^*} + M_K}{4} - \frac{3 M_\rho - M_\pi}{4}$$
$$= 179 \text{ MeV}. \qquad (1.4)$$

The same approach has been applied to heavy flavors [13,14] with excellent results. In this context we note a new relation [14], showing the common nature of the hyperfine interactions in mesons and baryons of different flavors,

$$\frac{M_{\Sigma_c} - M_{\Lambda_c}}{M_\Sigma - M_\Lambda}$$
$$= 2.16 \approx \frac{(M_\rho - M_\pi) - (M_{D^*} - M_D)}{(M_\rho - M_\pi) - (M_{K^*} - M_K)} = 2.10. \qquad (1.5)$$

We exhibit this success in more detail, by showing that mass differences and mass ratios are fit with a single set of quark masses, chosen to give an eyeball fit to the baryon mass differences and to fit the isoscalar nonstrange baryon magnetic moment

$$\mu_p + \mu_n = 2 M_p \frac{Q_I}{M_I} = \frac{2 M_N}{M_N + M_\Delta} = 0.865 \text{ n.m.} \qquad (1.6)$$

(EXP $= 0.88$ n.m.),

where $Q_I = \frac{1}{2}(\frac{2}{3} - \frac{1}{3}) = \frac{1}{6}$ and $M_I = \frac{1}{6}(M_N + M_\Delta)$ denote the charge and mass, respectively, of an effective "isoscalar nonstrange quark".[1] The quark

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[1] Note the implicit assumption that in M_I the contribution of the hyperfine interaction is canceled between the nucleon and the Δ.

Table 1
Theoretical and experimental hadron mass differences and ratios

Hadron mass differences			
Mass difference	Theoretical from Eq. (1.7)	Experimental from mesons ($X = d$)	Experimental from baryons ($X = ud$)
$m_s - m_u = M(sX) - M(uX)$	180	179	177
$m_c - m_u = M(cX) - M(uX)$	1350	1360	1346
$m_b - m_u = M(bX) - M(uX)$	4690	4701	4685
$m_c - m_s = M(cX) - M(sX)$	1170	1180	1169
$m_b - m_s = M(bX) - M(sX)$	4510	4521	4508
$m_b - m_c = M(bX) - M(cX)$	3340	3341	3339
Quark mass ratios			
Mass ratio	Theoretical from Eq. (1.7)	Experimental from mesons ($X = d$)	Experimental from baryons ($X = ud$)
m_s/m_u	1.5	1.61	1.53
m_c/m_u	4.75	4.46	4.36
m_b/m_u	14.0	13.7	?
m_c/m_s	3.17	2.82	2.82
m_b/m_s	9.35	8.65	?
m_b/m_c	2.95	3.07	?

masses chosen for the fit were

$$m_u = 360 \text{ MeV}, \qquad m_s = 540 \text{ MeV},$$
$$m_c = 1710 \text{ MeV}, \qquad m_b = 5050 \text{ MeV}. \qquad (1.7)$$

The results are shown in Table 1.

While we await for QCD calculations to explain these striking experimental facts from first principles, we use the method to analyse the pentaquark color structure and to estimate its mass.

2. The dynamics of a diquark–triquark pentaquark

Most quark model treatments of multiquark spectroscopy use the color-magnetic short-range hyperfine interaction [9] as the dominant mechanism for possible binding. The treatment of exotic color configurations not found in normal hadrons is considerably simplified by the use of color-spin $SU(6)$ algebra [10]. The hyperfine interaction between two quarks denoted by i and j is then written as

$$V_{\text{hyp}} = -V(\vec{\lambda}_i \cdot \vec{\lambda}_j)(\vec{\sigma}_i \cdot \vec{\sigma}_j), \qquad (2.1)$$

where $\vec{\lambda}$ and $\vec{\sigma}$ denote the generators of $SU(3)_c$ and the Pauli spin operators, respectively. The sign and magnitude of the strength of the hyperfine interaction are normalized by $\Delta-N$ mass splitting. The quark–quark interaction (2.1) is seen to be attractive in states symmetric in color and spin where $(\vec{\lambda}_i \cdot \vec{\lambda}_j)$ and $(\vec{\sigma}_i \cdot \vec{\sigma}_j)$ have the same sign and repulsive in antisymmetric states whee they have opposite signs. This then leads to the "flavor-antisymmetry" principle [15]: the Pauli principle forces two identical fermions at short distances to be in a state that is antisymmetric in spin and color where the hyperfine interaction is repulsive. Thus *the hyperfine interaction is always repulsive between two quarks of the same flavor, such as the like-flavor uu and dd pairs in the nucleon or pentaquark.*

This flavor antisymmetry suggests that the bag or single-cluster models commonly used to treat normal hadrons may not be adequate for multiquark systems. In such a state, with identical pair correlations for all pairs in the system, all same-flavor quark pairs are necessarily in a higher-energy configuration, due to the repulsive nature of their hyperfine interaction. The $uudd\bar{s}$ pentaquark is really a complicated five-body system where the optimum wave function to give minimum color-magnetic energy can require flavor-dependent spatial pair correlations for different pairs in the system; e.g., that keep the like-flavor uu and dd pairs apart, while minimizing the distance and optimizing the color couplings within the other pairs.
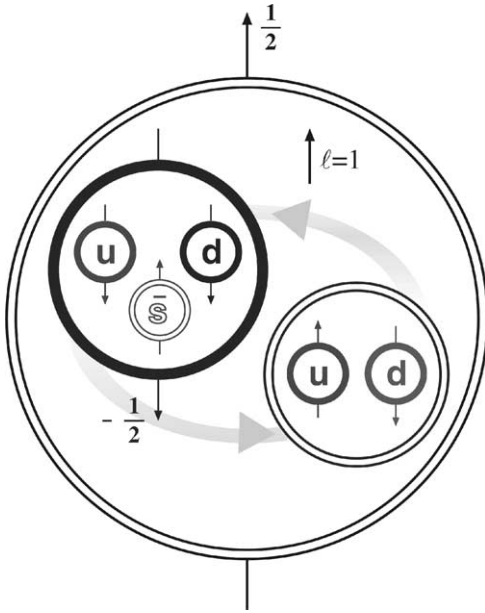
Fig. 1. The diquark–triquark configuration of the $uudd\bar{s}$ pentaquark.

We consider here a possible model for a strange pentaquark that implements these ideas by dividing the system into two color nonsinglet clusters which separate the pairs of identical flavor. The two clusters, a ud diquark and a $ud\bar{s}$ triquark, are in a relative P-wave and are separated by a distance larger than the range of the color-magnetic force and are kept together by the color electric force. Therefore the color hyperfine interaction operates only within each cluster, but is not felt between the clusters, as shown schematically in Fig. 1.

The ud diquark is in the $\bar{\mathbf{3}}$ of the color $SU(3)$ and in the $\bar{\mathbf{3}}$ of the flavor $SU(3)$ and has $I = 0$, $S = 0$, like the ud diquark in the Λ. It is in the symmetric $\mathbf{21}$ of the color-spin $SU(6)$ and is antisymmetric in both spin and color.

The $\mathbf{21}$ representation of $SU(6)$ contains a color antitriplet with spin 0 and a color sextet with spin 1.

The ud in the $ud\bar{s}$ triquark is in $\mathbf{6}$ of $SU(3)_c$, in $\bar{\mathbf{3}}$ of $SU(3)_f$ and has $I = 0$, $S = 1$. It is also in the symmetric $\mathbf{21}$ of the color-spin $SU(6)$, but is symmetric in both spin and color.

The triquark consists of the diquark and antiquark coupled to an $SU(3)_c$ triplet and has $I = 0$, $S = 1/2$. It is in the fundamental $\mathbf{6}$ representation of the color-spin $SU(6)$. It is in a $\bar{\mathbf{6}}$ of $SU(3)_f$.

We now define the classification of the diquarks with spin S, denoted by $|(2q)^S\rangle$ and the triquark, denoted by $|(2q\bar{s})^{\frac{1}{2}}\rangle$, in a conventional notation $|D_6, D_3, S, N\rangle$ [16,17] where D_6 and D_3 denote the dimensions of the color-spin $SU(6)$ and color $SU(3)$ representations in which the multiquark states are classified, S and N denote the total spin and the number of quarks in the system,

$$|(2q)^1\rangle = |21, 6, 1, 2\rangle,$$
$$|(2q)^0\rangle = |21, \bar{3}, 0, 2\rangle,$$
$$|(2q\bar{s})^{\frac{1}{2}}\rangle = |6, 3, \tfrac{1}{2}, 3\rangle. \tag{2.2}$$

A standard treatment using the $SU(6)$ color-spin algebra [16,17] gives the result in the $SU(3)$-flavor symmetry limit that the hyperfine interaction is stronger by $\frac{1}{6}(M_\Delta - M_N)$ for the diquark–triquark system than for the kaon-nucleon system,

$$\left[V\left(2q\bar{s}^{\frac{1}{2}}\right) + V\left(2q^0\right)\right] - \left[V(K) + V(N)\right]$$
$$= -\frac{1}{6}(M_\Delta - M_N) \approx -50\,\text{MeV}. \tag{2.3}$$

The physics here is simple. The spin-zero diquark is the same as the diquark in a Λ and has the same hyperfine energy as a nucleon. A triquark with one quark coupled with the $\bar{s}$ antiquark to spin zero has the same hyperfine energy as a kaon but no interaction with the other quark. The triquark coupling used here allows the $\bar{s}$ antiquark to interact with both the u and d quarks and gain hyperfine energy with respect to the case of the kaon. For an isolated triquark such a configuration is of course forbidden, since it is a color nonsinglet, but here it is OK, since the triquark color charge is neutralized by the diquark.

We see that had it not been for the cost of the P-wave excitation, the triquark–diquark system would be somewhat more bound than a kaon and a nucleon. The diquark and triquark will have a color electric interaction between them which is identical to the quark–antiquark interaction in a meson. If we neglect the finite sizes of the diquark and triquark we can compare this system with analogous mesons. We can use the effective quark masses (1.7) that fit the low-lying mass spectrum [8,14] to find a very rough estimate

$$m_{\text{diq}} = 720\,\text{MeV}, \qquad m_{\text{triq}} = 1260\,\text{MeV},$$
$$m_r(\text{di–tri}) = 458\,\text{MeV}, \tag{2.4}$$

where m_{diq} and m_{triq} denote the effective masses of the diquark and triquark, and $m_r(\text{di–tri})$ denotes the reduced mass for the relative motion of the diquark–triquark system.

A crucial observation is that the diquark–triquark system may not exist in a relative S-wave. This is because in S-wave the hyperfine interaction acts not only within the clusters but also between them. The repulsive terms may then win and the would be S-wave gets rearranged into the usual KN system. The situation is different in a P-wave, because then the diquark and the triquark are separated by an angular momentum barrier and the color-magnetic interactions operate only within the two clusters. The price is the P-wave excitation energy.

We can obtain a rough estimate of this P-wave excitation energy, using experimental information about the excited states of D_s, since the reduced mass of the $c\bar{s}$ system used to describe the internal structure of the D_s spectrum is 410 MeV, quite close to that of the diquark–triquark system.

It has been proposed that the recently discovered extremely narrow resonance $D_s(2317)$ [18–20] is a P-wave excitation [21] of the ground state 0^- $D_s(1969)$. If so, the 350 MeV excitation energy then consists of a P-wave contribution, on top of a contribution from color hyperfine splitting. We can estimate the net P-wave excitation energy $\delta E^{P-\text{wave}}$ by subtracting the c–s hyperfine splitting obtained from the mass difference between D_s^* and D_s,

$$\delta E^{P-\text{wave}} \approx 350 - (m_{D_s^*} - m_{D_s}) = 207 \text{ MeV}. \quad (2.5)$$

From Eq. (2.3) we infer that without the P-wave excitation energy the diquark–triquark mass is $m_N + m_K - \frac{1}{6}(M_\Delta - M_N) \approx 1385$ MeV, so that the total mass of the P-wave excitation of the diquark–triquark system is expected to be

$$M_{\text{di–tri}} \approx 1385 + 207 = 1592 \text{ MeV}, \quad (2.6)$$

about 3% deviation from the observed mass of the Θ^+ particle. It should be kept in mind, however, that this is only a very rough qualitative estimate and this close agreement might well be fortuitous, as there are several additional model-dependent effects which should be taken into account: the reduced mass of D_s is $\sim 12\%$ lower than $m_r(\text{di–tri})$, we do not know the spatial wave functions and we have neglected the spatial extent of the diquark and triquark

and possible molecular Van der Waals interactions spatially polarizing the two, breaking of flavor $SU(3)$, etc.

In addition to the parity and the mass, we also note that our model naturally gives a state with isospin zero because both the diquark and triquark have $I = 0$. The isospin has not yet been determined experimentally, but no isospin partners of the Θ^+ have been found and the Skyrme also predicted $I = 0$. This should be contrasted with attempts to envision the Θ^+ as a KN molecule in a P-wave [22], which have a problem in getting rid of the $I = 1$ state.

Our model also naturally predicts that the Θ^+ is in an antidecuplet of $SU(3)$ flavor. The diquark is a $\bar{\mathbf{3}}$, the triquark a $\bar{\mathbf{6}}$ and in $SU(3)$ $\bar{\mathbf{3}} \otimes \bar{\mathbf{6}} = \overline{\mathbf{10}} \oplus \mathbf{8}$ and only $\overline{\mathbf{10}}$ has the right strangeness. KN is $\mathbf{8} \otimes \mathbf{8}$ in $SU(3)_f$ and contains $\mathbf{27}$ with an isovector with the right strangeness, in addition to an antidecuplet. The antidecuplet prediction is again in agreement with the Skyrme model.

Since $M_{\text{di–tri}}$ is above the KN threshold, the system will eventually decay to KN, but the orbital angular momentum barrier and the required color rearrangement will make such a decay relatively slow, possibly explaining the observed narrow width of the Θ^+.

3. Effects of flavor symmetry breaking

The treatment above assumes flavor symmetry; i.e., that all quarks and the antiquark have the same mass. We now examine the symmetry breaking for a pentaquark $\Theta(uudd\bar{Q})$, with an antiquark of flavor Q, with a mass different from the mass of the four quarks. This applies not only to the Θ^+ with a strange antiquark but also to states with heavier antiquarks. The mass difference between the pentaquarks $\Theta(uudd\bar{Q})$ and $\Theta(uudd\bar{q})$, where the antiquark $\bar{q}$ has the same mass as the u and d, is just the sum of the differences in the masses and in the hyperfine energies of the antiquarks.

The same treatment which leads to Eq. (2.3) now gives for the total hyperfine interaction in our diquark–triquark model for Θ_Q:

$$V(\Theta_Q) = -(7 + 13\zeta)\frac{m_\Delta - m_N}{12}, \quad (3.1)$$

where $\zeta \equiv m_u/m_Q$. This should be compared with the hyperfine energy of the nucleon and the $u\overline{Q}$ meson,

$$V(N) + V(u\overline{Q}) = -(1 + 2\zeta)\frac{m_\Delta - m_N}{2} \qquad (3.2)$$

so that the difference in the hyperfine interaction between the diquark–triquark configuration and the N $u\overline{Q}$ system is

$$\begin{aligned} &V(\Theta_Q) - \left[V(N) + V(u\overline{Q})\right] \\ &= -(1 + \zeta)\frac{m_\Delta - m_N}{12}. \end{aligned} \qquad (3.3)$$

For $\zeta = 1$ we recover the result in Eq. (2.3). For a realistic m_s, we take $\zeta = 2/3$, obtaining a small correction

$$\begin{aligned} &V(\Theta^+) - \left[V(N) + V(K)\right] \\ &= -\frac{5}{36}(m_\Delta - m_N) = -42 \text{ MeV}. \end{aligned} \qquad (3.4)$$

The same approach can be used to treat pentaquarks with $\bar{c}$ and $\bar{b}$ antiquarks [23].

We now examine the $\Xi^*(I = 3/2)$, which has the quark constituents $(uuss\bar{d})$ and the same mass as the Θ^+ in the $SU(3)$ limit. For this case we set $\zeta = m_u/m_s = (2/3)$.

For the hyperfine interaction in the us diquark with spin 0 and $\zeta = (2/3)$ we obtain

$$V(us) = -\frac{\zeta}{2}(M_\Delta - M_N) = -\frac{1}{3}(M_\Delta - M_N). \qquad (3.5)$$

For the $(us\bar{d})$ triquark hyperfine interaction we obtain

$$\begin{aligned} V(us\bar{d}) &= -(13 + 15\zeta)\frac{m_\Delta - m_N}{24} \\ &= -\frac{23}{24}(M_\Delta - M_N). \end{aligned} \qquad (3.6)$$

Here the quark–quark interaction is modified by a factor ζ, while the quark–antiquark interaction is modified by a factor $(1 + \zeta)/2$, since only half of the two quarks is strange. Combining (3.5) and (3.6), we obtain the total hyperfine interaction in $\Xi^*(I = 3/2)$

$$\begin{aligned} V\left(\Xi^*(I = 3/2)\right) &= -(13 + 27\zeta)\frac{m_\Delta - m_N}{24} \\ &= -\frac{31}{24}(M_\Delta - M_N). \end{aligned} \qquad (3.7)$$

The difference between the $\Xi^*(I = 3/2)$ and Θ^+ hyperfine interactions is then

$$\begin{aligned} \delta V_{\text{hyperfine}} &\equiv V\left(\Xi^*(I = 3/2)\right) - V\left(\Theta^+\right) \\ &= (1 - \zeta)\frac{m_\Delta - m_N}{24} = \frac{M_\Delta - M_N}{72} \\ &= 4.2 \text{ MeV}. \end{aligned} \qquad (3.8)$$

The $\Xi^*(I = 3/2)$ mass is obtained from the experimentally known mass of Θ^+ by adding the quark mass difference $(m_s - m_u)$ and the hyperfine energy difference,

$$\begin{aligned} M_{\Xi^*(I=3/2)} &= M_{\Theta^+} + (m_s - m_u) + \delta V_{\text{hyperfine}} \\ &= 1540 + 178 + 4 = 1722 \text{ MeV}. \end{aligned} \qquad (3.9)$$

Since $M_\Xi + M_\pi = 1460$ MeV, the mass of the $\Xi^*(I = 3/2)$ is about 260 MeV above threshold.

4. Summary and conclusions

We propose the interpretation of the recently discovered Θ^+ exotic KN resonance as a novel kind of a pentaquark, involving a recoupling of the five quarks into a diquark–triquark system in nonstandard color representations. We estimate the Θ^+ mass using the simple generalized Sakharov–Zeldovich mass formula which holds with a single set of effective quark mass values for all ground state mesons and baryons having no more than one strange or heavy quark.

Our rough numerical estimate indicates that such a color recoupling might put the pentaquark mass in the right ballpark of the experimentally observed Θ^+ mass. Our model naturally predicts that Θ^+ has spin 1/2, positive parity, is an isosinglet and is an antidecuplet in $SU(3)_f$. We calculate the effect of $SU(3)_f$ symmetry breaking and the mass splitting between the Θ^+ and another member of the antidecuplet, the $\Xi^*(I = 3/2)$.

Regardless of the specific details of the model, we have addressed the problem what kind of a five-quark configuration can describe the Θ^+. We have shown that our new diquark–triquark model with color recoupling gives a lower mass than the simplest $uudd\bar{s}$ and it looks promising. The diquark–triquark configuration might also turn out to be useful if negative parity exotic baryons are experimentally discovered in future.

Acknowledgements

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