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Infrared regularization of baryon chiral perturbation theory reformulated

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Abstract

We formulate the infrared regularization of Becher and Leutwyler in a form analogous to our recently proposed extended on-mass-shell renormalization. In our formulation, IR regularization can be applied to multi-loop diagrams with an arbitrary number of particles with arbitrary masses.

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1. Introduction

Starting with Weinberg's fundamental work on phenomenological Lagrangians [1], it became possible to systematically calculate corrections to the soft-pion results obtained within the framework of current algebra [2]. The corresponding effective field theory (EFT)—chiral perturbation theory (ChPT)—has been very successful in describing the strong interactions at low energies (for a recent review see, e.g., Ref. [3]). In the mesonic sector, the combination of standard dimensional regularization (DR) and the modified minimal subtraction scheme of ChPT ($\widetilde{\mathrm{MS}}$) led to a straightforward correspondence between the loop expansion and the chiral expansion in terms of momenta and quark masses at a fixed ratio [4,5]. The one-baryon sector proved to be more complicated [6]. In particular, using the same combination of DR and $\widetilde{\mathrm{MS}}$ as in mesonic ChPT, higher-order loops contribute in lower chiral orders and therefore the correspondence between the loop expansion and the chiral expansion seems to be lost (see Fig. 2 of Ref. [6]). One solution to this power-counting problem was given in the framework of heavy-baryon chiral perturbation theory (HBChPT) [7], and most of the recent calculations have been performed within this approach [8,9]. While successful in many cases, HBChPT destroys the analytic structure in part of the low-energy region. Several methods have been suggested to reconcile power counting with the constraints of analyticity in a

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manifestly relativistic approach [10–15]. The one most widely used is the infrared (IR) regularization of Ref. [11] by Becher and Leutwyler. A possible generalization to multi-loop diagrams has been suggested in Ref. [16].

In the present Letter we provide a formulation of the IR regularization of Becher and Leutwyler in a form analogous to the extended on-mass-shell (EOMS) renormalization of Ref. [15]. As a result of the reformulation, IR regularization is applicable to multi-loop diagrams [17] as well as to diagrams involving several fermion lines and/or resonances.

2. Comparison of IR regularization and EOMS renormalization

In order to reformulate the infrared regularization of Becher and Leutwyler [11] in a form analogous to the EOMS renormalization of Ref. [15], we consider as an example the characteristic, dimensionally regularized, one-loop integral of the fermion self-energy,

$$I_{N\pi}(-p,0)=i\int\frac{d^nk}{(2\pi)^n}\frac{1}{[(k-p)^2-m^2+i0^+][k^2-M^2+i0^+]}, \tag{1}$$

where n denotes the number of space–time dimensions. The masses m and M refer to the nucleon mass in the chiral limit and the lowest-order pion mass, respectively. Using the standard power counting of Refs. [18,19] we assign the order Q^{n-3} to the integral $I_{N\pi}$. Here, Q collectively denotes small expansion parameters such as the pion mass or small external momenta. (Note that $I_{N\pi}$ satisfies power counting only after subtraction [11,15].)

To implement the IR regularization and to compare with the EOMS renormalization scheme we use the Feynman parametrization formula

$$\frac{1}{ab}=\int_0^1\frac{dz}{[az+b(1-z)]^2}, \tag{2}$$

with $a=(k-p)^2-m^2+i0^+$ and $b=k^2-M^2+i0^+$, interchange the order of integrations, perform the integration over loop momenta k, and obtain

$$I_{N\pi}(-p,0)=-\frac{1}{(4\pi)^{n/2}}\Gamma(2-n/2)\int_0^1 dz\,\bigl[A(z)\bigr]^{(n/2)-2}, \tag{3}$$

where

$$A(z)=-p^2(1-z)z+m^2z+M^2(1-z)-i0^+.$$

In the approach of Becher and Leutwyler, the integral $I_{N\pi}$ is divided into the IR singular part I and the remainder R, $I_{N\pi}=I+R$, defined as

$$I=-\frac{1}{(4\pi)^{n/2}}\Gamma(2-n/2)\int_0^\infty dz\,\bigl[A(z)\bigr]^{(n/2)-2}, \tag{4}$$

$$R=\frac{1}{(4\pi)^{n/2}}\Gamma(2-n/2)\int_1^\infty dz\,\bigl[A(z)\bigr]^{(n/2)-2}. \tag{5}$$

In this decomposition, for noninteger n the integral I is proportional to a noninteger power of the pion mass ($\sim M^{n-3}$) and thus satisfies the power counting. On the other hand, the remainder R does not satisfy the power counting but, for arbitrary n, contains non-negative powers of small parameters and is thus absorbed into an infinite number of counterterms. Divergent parts of I are also absorbed in an infinite number of counterterms.

In Ref. [15] we have formulated the EOMS renormalization scheme using the dimensional counting method of Ref. [20], which can be considered as an intuitive version of the more rigorous "strategy of regions" (see, e.g., Ref. [21]). In our EOMS renormalization scheme [15] we apply a conventional renormalization prescription which allows us to identify the terms which we subtract from a given integral without calculating the integral beforehand. In essence we work with a modified integrand which is obtained from the original integrand by subtracting a suitable number of counterterms. To find the subtraction terms we consider the series

$$
\begin{aligned}
&\sum_{l,j=0}^{\infty} \frac{(p^2-m^2)^l (M^2)^j}{l!j!} \left\{ \left(\frac{1}{2p^2} p_\mu \frac{\partial}{\partial p_\mu} \right)^l \left(\frac{\partial}{\partial M^2} \right)^j \frac{1}{[(k-p)^2 - m^2 + i0^+][k^2 - M^2 + i0^+]} \right\}_{p^2=m^2,\, M^2=0} \\
&\quad = \frac{1}{(k^2 - 2k\cdot p + i0^+)(k^2 + i0^+)}\bigg|_{p^2=m^2} + M^2 \frac{1}{(k^2 - 2k\cdot p + i0^+)(k^2 + i0^+)^2}\bigg|_{p^2=m^2} \\
&\qquad + \left(p^2 - m^2\right)\bigg[\frac{1}{2m^2} \frac{1}{(k^2 - 2k\cdot p + i0^+)^2} - \frac{1}{2m^2} \frac{1}{(k^2 - 2k\cdot p + i0^+)(k^2 + i0^+)} \\
&\qquad - \frac{1}{(k^2 - 2k\cdot p + i0^+)^2 (k^2 + i0^+)} \bigg]_{p^2=m^2} + \cdots,
\end{aligned} \tag{6}
$$

where $[\ldots]_{p^2=m^2}$ means that we consider the coefficients of $(p^2-m^2)^l (M^2)^j$ only for four-momenta p^μ which satisfy the on-mass-shell condition. Although the coefficients still depend on the direction of p^μ, after integration of this series with respect to the loop momenta k and evaluation of the resulting coefficients for $p^2 = m^2$, the integrated series is a function of p^2 only. We subtract from Eq. (1) those terms of the expansion of Eq. (6) which violate the power counting. These terms are analytic in the small parameters and do not contain infrared divergences. For the given example we only need to subtract the first term of the expansion of Eq. (6).

We note that integrating Eq. (6) term by term reproduces the expansion of R of Eq. (5) in M^2 and $p^2 - m^2$. This can be checked by explicitly integrating the first few coefficients of the expansion of Eq. (6); we indeed see that they coincide with the coefficients of the expansion of R of Ref. [11]:

$$
R = -\frac{m^{n-4}\Gamma(2-n/2)}{(4\pi)^{n/2}(n-3)} \left[1 - \frac{p^2-m^2}{2m^2} + \frac{(n-6)(p^2-m^2)^2}{4m^4(n-5)} + \frac{(n-3)M^2}{2m^2(n-5)} + \cdots \right]. \tag{7}
$$

A more straightforward and transparent way of obtaining the IR regular part R is to rewrite $I_{N\pi}$ using the Feynman (or Schwinger) parameterization, integrate over loop momenta, expand the resulting integrand (of the integration over parameters) in a Taylor series of Lorentz-invariant small expansion parameters (small masses and Lorentz-invariant combinations of external momenta and large masses), and, finally, interchange summation and integration: $\int dx \sum \to \sum \int dx$. As is shown in the next section, the above observation is correct in general, i.e., by expanding the integrand of any integral with an arbitrary number of nucleon and pion denominators in small parameters and interchanging summation and integration, one reproduces the expansion of the IR regular part of the integral.[2] In other words, the IR regular part corresponds to the analytic part of the dimensional counting method [20] or, equivalently, to the hard part of a given loop integral in the technique of the "strategy of regions" [21].

[2] Note the important difference with Ref. [10], where the expansion of the integrand with subsequent interchange of integration and summation reproduces the chiral expansion of the power-counting preserving part. As shown in Ref. [11] this expansion does not always converge.

3. General case

Let us consider the general one-loop scalar integral corresponding to diagrams with one fermion line and an arbitrary number of pion and fermion propagators:

$$I_{N\cdots\pi\cdots}(p_1,\ldots,q_1,\ldots)=i\int\frac{d^nk}{(2\pi)^n}\frac{1}{b_1\cdots b_l a_1\cdots a_m},\tag{8}$$

where

$$b_j=(k+p_j)^2-m^2+i0^+,\qquad a_i=(k+q_i)^2-M^2+i0^+.$$

Tensor integrals are reduced to the scalar integrals of Eq. (8) in the standard fashion [22].

Following Ref. [11] we apply the infrared regularization to the integral of Eq. (8). We start by combining all meson propagators using the formula

$$\frac{1}{a_1\cdots a_m}=\left(\frac{\partial}{\partial M^2}\right)^{(m-1)}\int_0^1 dx_1\cdots\int_0^1 dx_{m-1}\,\frac{X}{A}.\tag{9}$$

The numerator X is given by

$$X=\begin{cases}1, & \text{for } m=2,\\ x_2(x_3)^2\cdots(x_{m-1})^{m-2}, & \text{for } m>2,\end{cases}$$

and the denominator A is given by the recursive expression

$$A=A_m,$$
$$A_1=a_1,$$
$$A_{p+1}=x_pA_p+(1-x_p)a_{p+1}\quad(p=1,\ldots,m-1).$$

The result for A is of the form

$$A=(k+\bar q)^2-\bar A+i0^+,\tag{10}$$

where the constant term $\bar A$ is of order Q^2, and $\bar q$ is a linear combination of external momenta and is of order Q^1.

Analogously we combine the nucleon propagators

$$\frac{1}{b_1\cdots b_l}=\left(\frac{\partial}{\partial m^2}\right)^{(l-1)}\int_0^1 dy_1\cdots\int_0^1 dy_{l-1}\,\frac{Y}{B}.\tag{11}$$

The numerator Y is given by

$$Y=\begin{cases}1, & \text{for } l=2,\\ y_2(y_3)^2\cdots(y_{l-1})^{l-2}, & \text{for } l>2,\end{cases}$$

and the denominator B is given by the recursive expression

$$B=B_l,$$
$$B_1=b_1,$$
$$B_{p+1}=y_pB_p+(1-y_p)b_{p+1}\quad(p=1,\ldots,l-1).$$

The result for B reads

$$B=(k+\bar P)^2-\bar B+i0^+,\tag{12}$$

where $\bar P$ is a linear combination of external momenta, $\bar P^2=m^2+\mathcal{O}(Q)$ and $\bar B=m^2+\mathcal{O}(Q)$.

Next we combine the denominators A and B using

$$\frac{1}{AB}=\int_0^1 \frac{dz}{[(1-z)A+zB]^2}$$

and obtain for the integral of Eq. (8)

$$i\left(\frac{\partial}{\partial M^2}\right)^{(m-1)}\left(\frac{\partial}{\partial m^2}\right)^{(l-1)}\int_0^1 dz\int_0^1 dy_1\cdots\int_0^1 dy_{l-1}\int_0^1 dx_1\cdots\int_0^1 dx_{m-1}\,YX\int\frac{d^nk}{(2\pi)^n}\frac{1}{[(1-z)A+zB]^2}. \tag{13}$$

Substituting A and B from Eqs. (10) and (12) in Eq. (13), evaluating the derivatives, and shifting $k\to k-\bar{P}z-\bar{q}(1-z)$, we obtain

$$i(l+m-1)!\int_0^1 dz\,z^{l-1}(1-z)^{m-1}\int_0^1 dy_1\cdots\int_0^1 dx_{m-1}\,YX\int\frac{d^nk}{(2\pi)^n}\frac{1}{[k^2-f(z)]^{l+m}}, \tag{14}$$

where

$$f(z)=\bar{P}^2z^2-\left(\bar{P}^2-\bar{B}\right)z+\bar{A}(1-z)-\left(\bar{q}^2-2\bar{P}\cdot\bar{q}\right)z(1-z)-i0^+.$$

Finally, the integration of Eq. (14) over k yields

$$\frac{(-1)^{1-l-m}}{(4\pi)^{n/2}}\Gamma(l+m-n/2)\int_0^1 dz\,z^{l-1}(1-z)^{m-1}\int_0^1 dy_1\cdots\int_0^1 dx_{m-1}\,YX\big[f(z)\big]^{(n/2)-l-m}. \tag{15}$$

To apply the IR regularization we rewrite the z integration as

$$\int_0^1 dz\cdots=\int_0^\infty dz\cdots-\int_1^\infty dz\cdots.$$

The result of the first integration is identified as the IR singular part and of the second as the IR regular part. In the IR regular part one can expand the integrand in small momenta and masses and interchange summation and integration [11]. This leads to integrals over z of the type

$$I_i=\int_1^\infty dz\,z^{n+i}, \tag{16}$$

where i is an integer number. These I_i are multiplied by (further) integrals over x_j and y_k which do not depend on n. The integrals of Eq. (16) are calculated by analytic continuation from the domain of n in which they converge, i.e.,

$$I_i=\left.\frac{z^{n+i+1}}{n+i+1}\right|_1^\infty=-\frac{1}{n+i+1}. \tag{17}$$

On the other hand, if we expand the integrand in Eq. (15) in small momenta and masses and interchange summation and integration, we obtain exactly the same expansion as for the IR regular part of the IR regularization with the only difference that instead of the integrals I_i of Eq. (16) we now have[3]

$$J_i = -\int_0^1 dz\, z^{n+i}. \tag{18}$$

Calculating these integrals by analytical continuation from the domain of n in which they converge, we obtain:

$$J_i = -\frac{z^{n+i+1}}{n+i+1}\bigg|_0^1 = -\frac{1}{n+i+1}. \tag{19}$$

Comparing Eqs. (17) and (19) we see that the expansion of the integrand in Eq. (15) with subsequent interchange of summation and integration exactly reproduces the result of the IR regular part of the loop integral. Next we observe that, if we expand the integrand of Eq. (14) in small parameters and interchange summation and integration over k, we obtain exactly the same result as by expanding the integrand in Eq. (15) in small parameters with subsequent interchange of summation and integration over Feynman parameters. We further note that the result of the expansion of the integrand of Eq. (14) in small parameters with subsequent interchange of summation and integration over k coincides with the series which is obtained when we *formally* expand the integrand of the original integral in small parameters, using a formula analogous to Eq. (6), interchange summation and integration, and rewrite the integrals of the obtained series in Feynman parametrization. We thus conclude that the IR regular part of the original integral can be obtained by expanding the integrand in small parameters and interchanging summation and integration over loop momenta. In practical calculations of the IR regular parts of loop integrals it is convenient to reduce the loop integrals to integrals over (Feynman/Schwinger) parameters, expand the integrand in Lorentz-invariant small expansion parameters (small masses and Lorentz-invariant combinations of external momenta and large masses), and interchange integration and summation.

4. Applications

As a check and application of our formulation of the IR regularization we have explicitly verified for all integrals of pion–nucleon scattering of Ref. [23] (to the order which is needed for the accuracy of calculations of that work) that, by expanding the integrands in small parameters and changing the order of summation and loop integration, one reproduces the IR regular parts of these integrals. The IR regular parts as well as the IR singular parts separately contain additional divergences which are not present in the original integral. (In our expansion of the IR regular parts these divergences occur as IR divergences.[4]) In the approach of Becher and Leutwyler these divergences of both parts are absorbed in counterterms. (In fact they exactly cancel each other and hence do not give any contributions in counterterms.) In our formulation the IR regularized integrals are obtained by subtracting the IR regular parts, from which the IR divergences are removed beforehand, from the full expressions of the integrals. Clearly our expressions of the IR regularized integrals coincide with the results of the Becher–Leutwyler approach.

It is straightforward to apply our formulation of IR regularization to diagrams with multiple nucleon lines. We have checked that our approach reproduces the results of Ref. [24] for diagrams with two nucleon propagators. As an illustration let us consider the following integral:[5]

$$I_{NN\pi}(P_1, -P_2, 0) = i\int \frac{d^n k}{(2\pi)^n} \frac{1}{[(k+P_1)^2 - m^2 + i0^+][(k-P_2)^2 - m^2 + i0^+][k^2 - M^2 + i0^+]}. \tag{20}$$

[3] The minus sign relative to Eq. (16) stems from the definition of R as $-\int_1^\infty dz \cdots$.

[4] Note that, using dimensional regularization, IR divergences are also parametrized as $1/(n-4)$ poles.

[5] Our notations differ from those of Ref. [24].

Using Feynman parametrization and performing loop momenta integration one can write $I_{NN\pi}$ as [24]

$$I_{NN\pi}(P_1,-P_2,0)=\frac{1}{(4\pi)^{n/2}}\frac{\Gamma(3-n/2)}{2}\int_0^1 dz\,z\int_{-1}^1 dw\,\big[C(w,z)-i0^+\big]^{(n/2)-3}, \tag{21}$$

where

$$C(w,z)=(1-z)M^2+zm^2-z^2\big(1-w^2\big)\frac{(P_1+P_2)^2}{4}-\frac{z(1-z)(P_1^2+P_2^2)}{2}-\frac{wz(1-z)(P_1^2-P_2^2)}{2}.$$

Following Ref. [24] we define the IR regular part of the integral $I_{NN\pi}(P_1,-P_2,0)$ as

$$R_{NN\pi}(P_1,-P_2,0)=\frac{1}{(4\pi)^{n/2}}\frac{\Gamma(3-n/2)}{2}\int_1^\infty dz\,z\left(\int_{-\infty}^{-1} dw+\int_1^\infty dw\right)\big[C(w,z)-i0^+\big]^{(n/2)-3}. \tag{22}$$

To calculate $R_{NN\pi}$ we expand the integrand of Eq. (22) in powers of M^2, $4m^2-(P_1+P_2)^2$, $P_1^2-m^2$, and $P_2^2-m^2$ and interchange integration and summation [24]. Doing so we obtain a series, the coefficients of which are proportional to the integrals

$$I_{ij}=\int_1^\infty dz\,\big(z^2\big)^{(n/2)-3}z^{1+i}\left(\int_{-\infty}^{-1} dw+\int_1^\infty dw\right)\big(w^2\big)^{(n/2)-3}w^j,$$

where i and j are integers. Again, the integrals I_{ij} are calculated by analytical continuation from the domain of n in which they converge, leading to

$$I_{ij}=\frac{1+(-1)^j}{(n-4+i)(n-5+j)}. \tag{23}$$

On the other hand, in our approach we identify the IR regular part of $I_{NN\pi}$ by expanding the integrand in Eq. (21) in powers of small parameters (M^2, $4m^2-(P_1+P_2)^2$, $P_1^2-m^2$, and $P_2^2-m^2$) and interchanging summation and integration over Feynman parameters. This leads to exactly the same expansion that we obtained above for $R_{NN\pi}(P_1,-P_2,0)$, but instead of the integrals I_{ij} we now have

$$J_{ij}=\int_0^1 dz\,\big(z^2\big)^{(n/2)-3}z^{1+i}\int_{-1}^1 dw\,\big(w^2\big)^{(n/2)-3}w^j,$$

which we calculate by analytically continuing from the domain of n in which they converge:

$$J_{ij}=\frac{1+(-1)^j}{(n-4+i)(n-5+j)}. \tag{24}$$

Clearly, in analogy to the one-nucleon sector our formulation of IR regularization reproduces the results of Ref. [24] for diagrams involving two nucleon lines.

Recently, we have shown [25] that, within the EOMS renormalization scheme, one can set up a consistent power counting in the effective field theory with (axial) vector mesons included explicitly. Analogously we could apply the IR regularization in our formulation. When treating vector mesons in the antisymmetric tensor field representation and analyzing the diagrams contributing to the electromagnetic form factors of the nucleon up to and including $\mathcal{O}(q^4)$, we observe that in Ref. [26] all relevant loop diagrams have actually been taken into account. This is due to the fact that the integrals involving only vector meson and nucleon propagators vanish in IR regularization.

Finally, we have also applied our formulation of the IR regularization to the integral considered in Ref. [27] in the context of including the Δ resonance:

$$I_{0\pi}(-p,0) = i\int \frac{d^n k}{(2\pi)^n}\frac{1}{[(k-p)^2+i0^+][k^2-M^2+i0^+]}. \tag{25}$$

For $p^2 \gg M^2$ the chiral dimension of this integral should be Q^{n-2} [27]. An explicit calculation of the integral $I_{0\pi}$ results in

$$I_{0\pi}(-p,0) = -\frac{M^{n-4}}{(4\pi)^{n/2}}\frac{\Gamma(2-n/2)\Gamma(n/2-1)}{\Gamma(n/2)}F\left(1,2-n/2;n/2;\frac{p^2+i0^+}{M^2}\right), \tag{26}$$

where $F(a,b;c;z)$ is the hypergeometric function [28]. For $p^2 > M^2$ we rewrite Eq. (26) as

$$\begin{aligned} I_{0\pi}(-p,0) = {} & \frac{M^{n-2}}{(4\pi)^{n/2}}\frac{\Gamma(1-n/2)}{p^2}F\left(1,2-n/2;n/2;\frac{M^2}{p^2+i0^+}\right) \\ & - \frac{(-p^2-i0^+)^{(n/2)-2}}{(4\pi)^{n/2}}\frac{\Gamma(2-n/2)\Gamma(n/2-1)\Gamma(n/2-1)}{\Gamma(n-2)}\left(1-\frac{M^2}{p^2+i0^+}\right)^{n-3}. \end{aligned} \tag{27}$$

Analogously to Ref. [11] we identify the first term in Eq. (27), which, for noninteger values of n, is proportional to a noninteger power of M, as the IR singular part and the second term as the IR regular part. The IR singular part satisfies the power counting and would generate an infinite number of terms if the function multiplying M^{n-2} were expanded in powers of M^2. This differs from the result of Ref. [27], where only the first term of such an expansion was identified as the IR singular part of $I_{0\pi}(-p,0)$.

In analogy to the self-energy integral considered above, it is straightforward to check explicitly that, if one expands the integrand of Eq. (25) in powers of M^2 and interchanges integration and summation, one exactly reproduces the expansion of the second (regular) term of Eq. (27) in powers of M^2/p^2.

The integral $I_{0\pi}(-p,0)$ has an imaginary part for $p^2 > M^2$ which in both definitions, ours and that of Ref. [27], is included in the IR regular part.[6] This imaginary part is given by

$$-\frac{1}{16\pi}\left(1-M^2/p^2\right)$$

in $n=4$ dimensions and violates the power counting. Therefore, although the regular part is analytic in M^2 and consequently its real part can be absorbed by counterterms of the Lagrangian, the imaginary part cannot be altered. As a result there exists no subtraction scheme within which the renormalized version of $I_{0\pi}(-p,0)$ would satisfy the power counting. However, from this observation one should *not* draw the conclusion that there is no consistent power counting in a manifestly Lorentz-invariant formulation of BChPT with spin $3/2$ particles included explicitly. Rather, as already pointed out in Ref. [27], the integral $I_{0\pi}(-p,0)$ occurs when the spin $3/2$ particle propagator is decomposed using projection operators and the apparent puzzle disappears once the results for this decomposition are put together.

5. Summary and conclusions

We have reformulated the IR regularization of Becher and Leutwyler [11] in a form analogous to our EOMS renormalization scheme of Ref. [15]. Within this (new) formulation the subtraction terms are found by expanding the integrands of loop integrals in powers of small parameters (small masses and Lorentz-invariant combinations of external momenta and large masses) and subsequently exchanging the order of integration and summation.

[6] In Ref. [27] the same boundary condition as in Eq. (25) has been assumed [29].

Isolating the infrared divergences from these terms and subtracting them from the original relativistic loop integral one obtains the IR regularized expression of the integral. One advantage of the new formulation of IR regularization is that it can be applied to diagrams with an arbitrary number of propagators with various masses (e.g., resonances) and/or diagrams with several fermion lines as well as to multi-loop diagrams [17].

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