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Determining critical links in a road network: vulnerability and congestion indicators

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Abstract

This article focuses on the performance attributes *congestion* and *vulnerability*, proposing a conceptual framework based on recent research. We present a case study that allows observing the differences obtained in the ranking of importance of the links of a road network according to each of the two attributes. The differences observed suggest that the exclusive use of congestion indicators to determine performance bottlenecks can lead to misleading solutions.

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Keywords: congestion; vulnerability; road network; performance indicators.

1. Introduction

Various problems can impair the efficiency of urban road networks, with the most visible symptom being traffic jams. With the worldwide growth of cities, the performance of urban road systems is a question of increasing importance. Congestion, however, is not the only attribute by which to measure the performance of road networks. In this sense, various authors have studied the theme and proposed indicators to assess the performance of road systems and their components, from a variety of angles. The need to optimize performance and the frequent limitation of budget resources makes it indispensable to use tools that allow prioritizing investments based on objective and consistent criteria. It is therefore important to discuss the main performance attributes, seeking criteria

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to determine the relative importance of the links making up the network, to enable more rational ordering of priorities.

Here we emphasize the performance attributes congestion and vulnerability, proposing a concept that is coherent with the recent bibliography. Our objective is to investigate the similarities and differences between hierarchies or rankings resulting from the use of indicators of these two attributes, with the help of a case study.

2. Congestion and vulnerability: concepts and indicators

2.1. Definitions of congestion and vulnerability

Although congestion is an intuitive phenomenon, the literature presents various definitions of congestion, more complementary than distinct. Stopher (2004), for example, proposes that congestion happens when the input volume (demand) exceeds the output capacity (supply). In turn, Zhang & Lomax (2007) consider that a road segment is congested when the distance between vehicles is below a minimum level, i.e., when the traffic density has reached its maximum. The HCM (TRB, 2010) presents a method to determine the level of service of a road or set of roads, where the service levels “F” and possibly “E” characterize a situation of congestion. In summary, a segment of a road system is considered to be congested when the traffic demand approaches or exceeds the stable flow capacity of the road.

The vulnerability of road networks has been the topic of many studies, particularly in the past decade. Normally, the concept of vulnerability is associated with the ability of a network to maintain its level of performance after an event that impairs one or more of its links (Jenelius *et al*., 2006; Jenelius & Mattsson; 2011). Scott *et al.* (2006) and Sullivan *et al.* (2010) prefer the term *robustness* when proposing indicators. However, robustness is the reciprocal of vulnerability, so a robustness indicator is also a vulnerability indicator.

The concept of vulnerability is often confused with that of reliability. Chen *et al.* (2002), for example, make no distinction between the concepts in proposing their capacity reliability index. Li (2008) discriminates the two concepts, presenting reliability as the probability that a network will perform at an adequate service level in a determined period of time, while robustness (inverse of vulnerability) is the ability of the network to continue operating at an adequate service level even in the presence of incidents. By this framework, reliability is linked to daily events while vulnerability is related to occasional incidents. Dehghanisanij *et al*. (2013) include the probability of the occurrence of disturbing events in the determination of their vulnerability indicator, associating the risk attribute to vulnerability. Finally, Taylor *et al*. (2006) argue for the distinction of vulnerability, reliability and risk. For this article, we consider the concept proposed by Oliveira *et al*. (2013), resulting from a critical analysis of the existing studies in the literature: “*Vulnerability is the performance attribute related to the impact of non-recurring or infrequent random events in a road network able to impair the capacity of a link or group of links.”*

2.2. Congestion indicators

The literature contains a large number of indicators associated with the congestion attribute. They can be classified into three main groups, in function of the principal variables involved in their determination:

- Balance between supply and demand – indicators that assess congestion based on the impairment of the traffic flow capacity of the link under analysis;
- Velocity and time – indicators that evaluate the trip flow in terms of average speed (or its variation) or average time (or its variation); and
- Others – indicators that cannot be classified in the above two groups.

Table 1 presents examples of indicators and their respective bibliographical references, classified in the above groups.

Among the congestion indicators presented, here we adopt V/C, from the Supply x Demand group, and CI, from the Velocity/Time group. We chose V/C since it is the most widely used indicator of its group and is easy to obtain. Among the indicators of the Velocity/Time group, which involve the difference or ratio between times under congested and uncongested periods, the TTI is perhaps the most widely used. It is defined as the ratio between the trip time during a peak traffic period and under free-flow conditions. In turn, the CI is defined as the ratio between a trip under congested conditions (peak period or not) and under free-flow conditions. We chose the CI, since it is a more general form of the most widespread index.

Table 1. Congestion indicators

Group	Indicator
Supply x Demand	V/C – Volume/capacity ratio (TRB, 2010)
	LOS – Level of service (TRB, 2010)
	K – Average density (Elefteriadou *et al*., 2012)
	Reserve capacity (Chen *et al*., 2002)
Velocity/Time	CI – Congestion index(Zhang & Lomax, 2007)
	TTI – Travel time index (Schrank *et al*., 2011)
	Average delay (Quiroga, 2000)
Others	Congestion length (Litman, 2012)
	Percentage of local sub-area network congested (Portugal & Araújo, 2008)
	Throughput (Morán, 2010)

2.2.1. Determining the V/C ratio

The capacity measure, the denominator of the V/C ratio, is determined following the recommendation of the Highway Capacity Manual – HCM (TRB, 2010). It is the capacity estimated for level of service “E”. The numerator, the traffic volume, is obtained by direct traffic countings or from simulations involving a road network model.

2.2.2. Determining the Congestion Index

The congestion index (CI) (Zhang & Lomax, 2007) aims to measure the level of congestion of a road from the ratio between the trip time under congested traffic conditions (t_c) and the time under free-flow conditions (t_f), as described in equation 1. It can be determined directly, in the field, or from simulations using a road network model.

$$CI = \frac{t_c}{t_f} \tag{1}$$

2.3. Vulnerability indicators

To determine the vulnerability indicators, the network performance is measured in two situations: a) with the network under normal conditions, before the occurrence of a disturbance; and b) with the network subject to disturbance of one or more of its links (interruption or impaired capacity). The difference between the two performance indicators (generally total trip cost or time) is the value of the vulnerability index.

The main methodological difference for determination of different vulnerability indicators involves the characteristics of the trip assignment model used, whether dynamic or static (Oliveira *et a*l., 2013).

Table 2 presents some of the indicators found in the literature and the respective reference, indicating the approach for the assignment model (static, dynamic or mixed).

Since the available calibrated network is a model parameterized for TransCAD, with static allocation, we dropped the indicators that require dynamic or mixed assignment. Also, since our aim is to compare the results obtained from analysis of congestion and vulnerability, we sought indicators not affected by other attributes. The NRI-m, NRI and Importance index fit in this category. In reality, the indicator proposed by Chen *et al*. (2002) is also affected by the reliability attribute while that proposed by Dehghanisanij *et a*l. (2013) is affected by the risk attribute. The NRI-m differs from the NRI and importance index by the way it treats road capacity. While the latter two deal with the impact of total interruption of a link, the first one allows partial reduction of capacity. To consider the impact of the two effects (interruption and reduction of capacity), we chose an indicator of each characteristic, NRI and NRI-m, for the case study.

Table 2 – Vulnerability indicators

Assignment	Indicator
Static	Vulnerabilty and reliability index (Chen *et al.*, 2002)
	Importance index (Jenelius *et al.*, 2006)
	Network robustness index – NRI (Scott *et al.*, 2006)
	Modified network robustness index – NRI-m (Sullivan *et al.*, 2010)
	Vulnerability index (Dehghanisanij *et al.*, 2013)
Dynamic	Vulnerability indices (Knoop *et al.*, 2012)
	Vulnerability indices (Snelder *et al.*, 2012)
Mixed	Vulnerability indices (Tampère *et al.*, 2007)

2.3.1. Determining the NRI

The NRI measures the importance of a link in a road network by simulating the effect on the network's performance of blocking that link (Scott et al., 2006). For that purpose, the total trip time or cost of the network is compared in the situations with and without the link under analysis. The NRI represents the difference in time (or cost) between the two situations, according to the formulation presented in Equation 2.

$$NRI_k = \sum_i t'_i \times v'_i - \sum_i t_i \times v_i \tag{2}$$

Where: NRI_k = Network robustness Index for link k; t'_i = time (or cost) of link i in the situation with link k blocked; v'_i = traffic volume of link i in the situation with link k blocked; t_i = time (or cost) of link i without any blocked links; v_i = traffic volume of link i without any blocked links.

2.3.2. Determining the NRI-m

The modified network robustness index – NRI-m (Sullivan *et al*., 2010) is similar to the NRI, differing only in that instead of completely eliminating the link under analysis, the performance of the network is evaluated with this link's capacity reduced. Hence, NRI-m indices can be calculated for various reductions of capacity. In the present study, we considered a reduction of 77.5%, the midpoint of the range of reduction that produced the most stable results in the experiments carried out by Sullivan *et al.* in Chittenden County, Vermont, USA in 2010. The mathematical formulation is identical to the NRI, with t' and v' (Equation 2) representing the situation of reduced capacity of link k.

3. Case study: methodology

The base for this study was the road network representing the Rio de Janeiro Metropolitan Region (RJMR), prepared for the Urban Transportation Master Plan (PDTU, 2005). It is a complex network, composed of some 22,000 nodes and 60,000 links. The software with GIS characteristics used for the simulations was TransCAD, from Caliper Corporation, the same used during preparation of the PDTU.

For trip assignment, we employed the user's equilibrium (UE) method, available in TransCAD, which is based on the Frank-Wolf algorithm (Frank & Wolf, 1956, cited in Caliper Corporation, 1996). Again, this same model was used during preparation of the PDTU.

Since this is an iterative method, one must consider a stopping criterion. Possible criteria for this purpose are discussed in Rose *et al.* (1988) and summarized in Slavin *et al*. (2006), where the authors compare different methods of allocating trips by the user equilibrium framework. Slavin *et al.* (2006) recommend using the *relative gap*, which can be defined as the ratio of the difference between the total cost (or time) by the UE method and the total cost (or time) by the "all or nothing" (AON) method over the total cost (or time) by the UE method, as in Equation 3. For the present case, we considered a relative gap value of 0.005 as the stopping criterion (limit of the convergence test) of the iterative process. In fact, the use of smaller relative gaps did not produce appreciable gains in precision, while the processing costs increased considerably. For example, when we simulated the entire network, the difference in the results obtained from relative gaps of 0.005 and 0.0001 was around 0.38%, for a processing time nearly 25 times longer.

$$\frac{\sum x_{UE} \times c(x_{UE}) - x_{AON} \times c(x_{UE})}{\sum x_{UE} \times c(x_{UE})} \tag{3}$$

Where: x_{UE} = flow over each link as a result of a UE solution; x_{AON} = flow over each link as a result of a AON solution; $c(x_{UE})$ = volume delay function – cost or time at each link.

Fig. 1 – Links selected for the comparative study

The trip matrix considered was that of automobile traffic in 2003, for peak hour, obtained by transforming the trip matrices of the PDTU (see Oliveira, 2012 for details of the process). We assumed that buses have fixed routes, so they would not be altered. The traffic of cargo vehicles is not substantial during rush hour, so it is possible to consider only passenger cars, which have freedom to choose a route, satisfying the assumption of the method of assignment by equilibrium.

The determination of the congestion V/C and CI indicators is direct after the assignment process. On the other hand, the determination of the NRI and NRI-m requires individual modification of each link of the road network, in a process that is computationally demanding when dealing with a complex urban system, as here. Therefore, for determination of the vulnerability indicators and subsequent comparative analysis, we chose a group of links belonging to a predefined sub-area network, composed of 1,266 links. This sub-area network was selected to allow analysis of the connection of one area of the RJMR (Copacabana, Ipanema, Leblon, Gávea and Jardim Botânico) with the other areas of the city. This region has peculiar geographic features that determine the topology of the road network, with many steep hills, squeezed between the coastline and a lagoon just inland. To select the links for analysis, we considered two main rankings as references: a) the ranking of the links of the sub-area network in function of the congestion indicators; and b) the ranking of the links of the sub-area network in function of the number or routes between the centroids they serve. It is important to mention that although the analysis was concentrated in a sub-area network the indicators were those calculated for the entire RJMR.

The final criteria for selecting the links for comparative analysis were:

- Each link had to be among the first 50 of each of the rankings – in other words, each of the 50 most congested links according to the V/C or CI indicators or one of the 50 links that serve the largest number of minimum routes between centroids of the entire network representing the RJMR (rather than just the delimited sub-area network); and
- A single corridor could only have more than one link if there were intersections with not strictly local roads.

With this procedure, we selected 24 links, shown in Figure 1, which depicts the sub-area network studied. Figure 2 presents the RJMR network, with the sub-area network demarcated in the rectangular area.

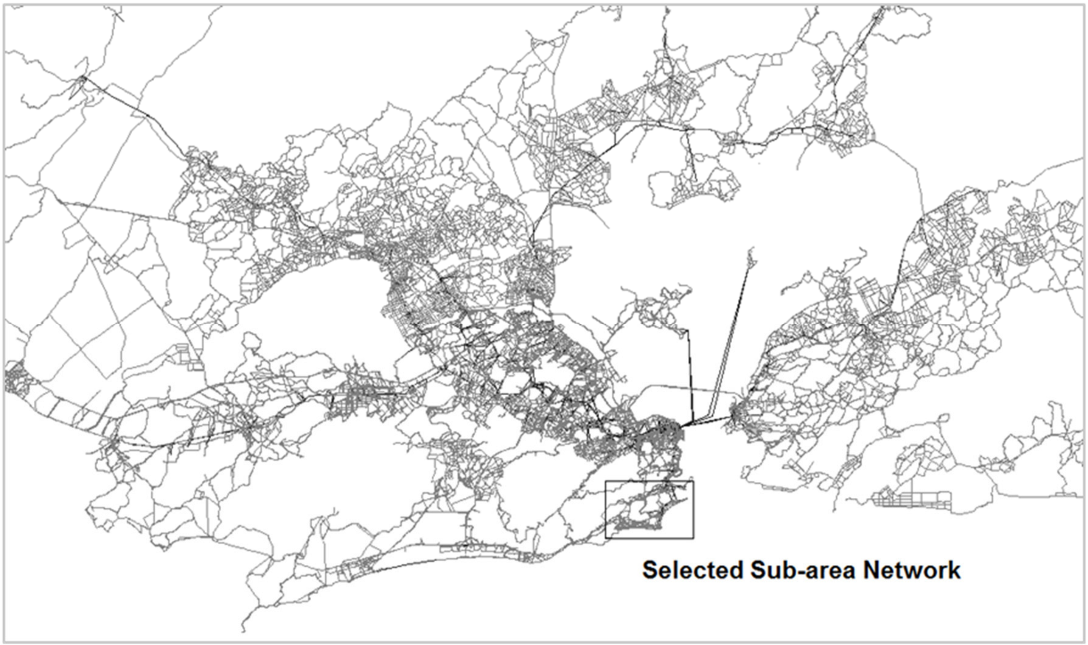

Fig. 2 – RJMR road network, with the sub-area network studied demarcated

The NRI and NRI-m were then calculated for each of the selected links by an iterative process in which as many assignments are carried out as there are links to analyze, 24 in the present case.

Figure 3 shows a diagram of the procedure to determine the NRI and NRI-m.

4. Case study: results

The values found for the congestion and vulnerability indicators for the selected links, as well as their position in the ranking, are presented in Table 3. Besides the real value of the indicators, they are also presented in normalized form, on a scale from 0 to1, to facilitate comparison. Based on an equal weight for the attributes and respective indicators, a hierarchy is derived from the mean of these indicators, which are ordered in the table. The column with the means is also normalized on a scale from 0 to 1.

Figure 4 shows the comparison between the V/C and CI values. A strong correlation can be seen between the two indicators, when considering an exponential function. Likewise, there is a correlation between the NRI and NRI-m indicators, with an approximately linear relation, although with lower coefficient of determination, as can be observed in Figure 5.

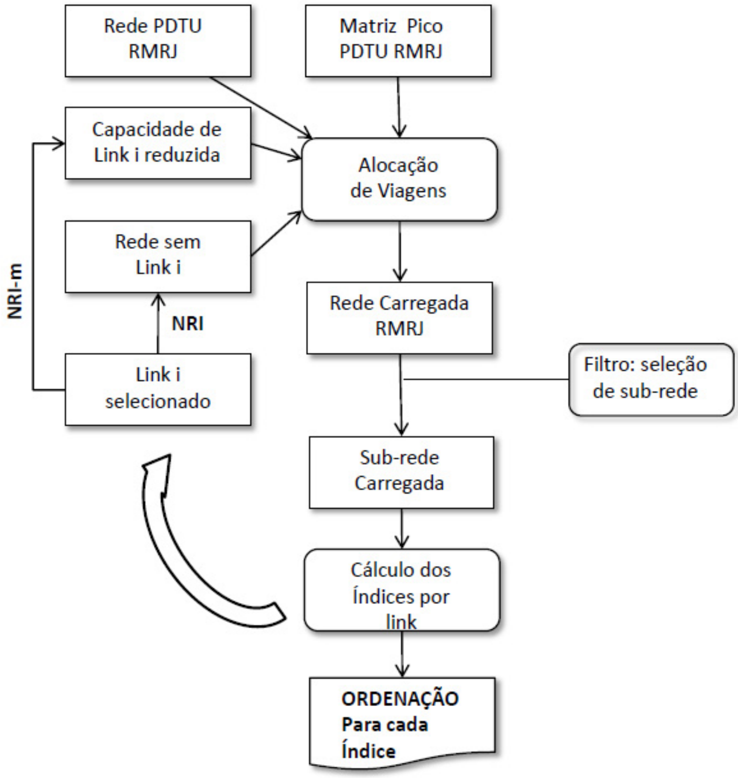

Fig. 3 – Process for determining the NRI and NRI-m

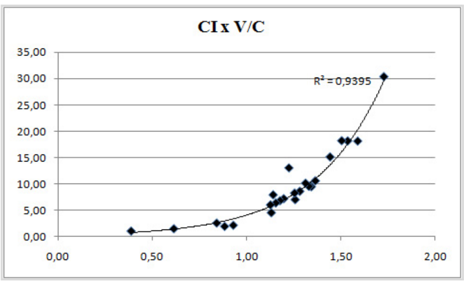

Fig. 4 – CI x V/C Correlation

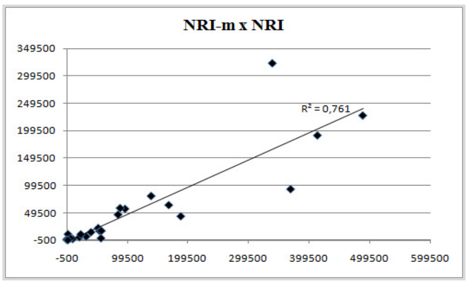

Fig. 5 – NRI-m x NRI Correlation

Comparison of the NRI and NRI-m indicators with each of the congestion indicators does not show any correlation (R2 lower than 0.05 for all the curves tested), as can be visualized in Figures 6 and 7, which exemplify the comparison with the V/C. Therefore, as expected, there was a correlation between indicators of the same attribute, but not between indicators of different attributes.

Another point worth noting is the behavior of links 20 and 21, chosen for belonging to the minimum routes between a large number of origin-destination pairs. These links are among the four most significant in terms of vulnerability indicators, but have little relevance when considering congestion, for which they are among the five last ones in the hierarchy. Apparently bottlenecks before those links act as flow regulators, preventing their congestion and masking their importance. In this case, links 20 and 21 are critical for the performance of the road network, although they do not stand out regarding the level of congestion observed.

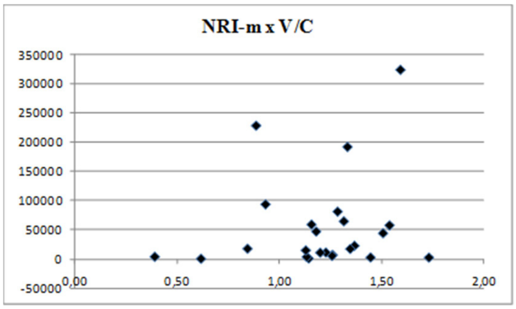

Fig. 6 – NRI x V/C Correlation

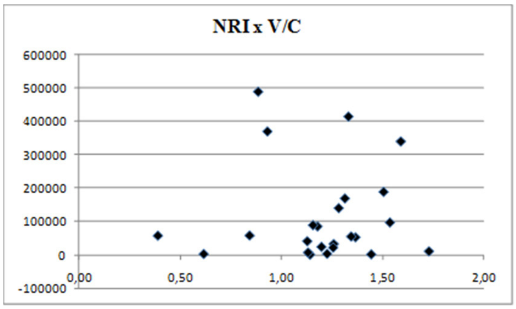

Fig. 7 – NRI-m x V/C Correlation

Table 3 – Indicators by link

	Congestion						Vulnerability						Cong.+Vuln.			
	V/C			CI			NRI			NRI-m			Normalized			
Link	Abs.	Norm.	Hier.	Abs.	Norm.	Hier.	Abs.	Norm.	Hier.	Abs.	Norm.	Hier.	Avg.	Norm.	Hier.	Group
2	1,59	0,90	2	18,11	0,58	4	339417	0,69	4	323332	1,00	1	0,79	1,00	1	I
8	1,33	0,70	8	9,53	0,29	9	413836	0,85	2	191339	0,59	3	0,61	0,76	2	
21	0,89	0,37	21	1,94	0,03	22	488660	1,00	1	227783	0,70	2	0,53	0,65	3	II
1	1,73	1,00	1	30,33	1,00	1	9831	0,02	19	1845	0,01	22	0,51	0,62	4	
4	1,51	0,83	4	18,19	0,59	2	188260	0,39	5	43346	0,13	10	0,48	0,59	5	
3	1,54	0,86	3	18,14	0,58	3	96160	0,20	8	56985	0,18	8	0,45	0,55	6	
9	1,31	0,69	9	10,14	0,31	8	168357	0,34	6	63826	0,20	6	0,39	0,46	7	III
20	0,93	0,40	20	2,15	0,04	21	369505	0,76	3	92917	0,29	4	0,37	0,45	8	
10	1,28	0,67	10	8,59	0,26	11	139207	0,28	7	80482	0,25	5	0,36	0,44	9	
5	1,44	0,79	5	15,12	0,48	5	534	0,00	23	1916	0,01	21	0,32	0,38	10	
6	1,37	0,73	6	10,58	0,33	7	51751	0,11	14	21879	0,07	11	0,31	0,36	11	
7	1,35	0,71	7	9,46	0,29	10	54062	0,11	13	16436	0,05	13	0,29	0,34	12	
16	1,16	0,57	16	6,38	0,18	17	88034	0,18	9	58286	0,18	7	0,28	0,32	13	
15	1,18	0,59	15	6,81	0,20	16	84566	0,17	10	46398	0,14	9	0,28	0,32	14	
13	1,23	0,62	13	13,03	0,41	6	2071	0,00	21	10540	0,03	15	0,27	0,31	15	
12	1,26	0,65	12	8,27	0,25	12	20908	0,04	18	4985	0,02	18	0,24	0,27	16	
11	1,26	0,65	11	7,01	0,20	15	31751	0,07	16	6348	0,02	17	0,23	0,27	17	
14	1,20	0,60	14	7,19	0,21	14	23030	0,05	17	10229	0,03	16	0,22	0,25	18	
19	1,13	0,55	19	6,02	0,17	18	40023	0,08	15	14237	0,04	14	0,21	0,24	19	IV
17	1,14	0,56	17	7,92	0,23	13	-50	0,00	24	237	0,00	23	0,20	0,22	20	
18	1,13	0,55	18	4,53	0,12	19	6173	0,01	20	2930	0,01	20	0,17	0,19	21	
24	0,84	0,34	22	2,57	0,05	20	57061	0,12	11	16863	0,05	12	0,14	0,14	22	
23	0,62	0,17	23	1,49	0,02	23	1881	0,00	22	-200	0,00	24	0,05	0,02	23	
22	0,39	0,00	24	1,05	0,00	24	56401	0,12	12	3195	0,01	19	0,03	0,00	24	

V/C e CI [adimensional]; NRI and NRI-m [minutes]. Columns: Abs. (absolute values); Norm. (normalized values); Hier. (hierarchy); Avg. (average normalized values).

Finally, the normalization of the means obtained from the different indicators allows distributing the links into four groups with amplitude of 0.25, where the most critical regarding congestion and vulnerability are those belonging to group I, with a normalized mean greater than 0.75. In fact, the links of group I are among the five most critical regarding vulnerability (NRI and NRI-m rankings) and among the ten most critical regarding congestion (V/C and CI rankings). On the other hand, the six links of group IV (normalized mean less than or equal to 0.25) are all among the lowest half of the rankings of all the indicators, except for link 24, which appears in 11th place in the NRI ranking and 12th in the NRI-m hierarchy.

5. Conclusion

The concept of performance of a road network covers multiple angles, in function of the attributes considered. Congestion, for example, is the main symptom of the poor performance of a network. Therefore, the importance of the congestion indicators, among them V/C and CI, cannot be disregarded, despite their limitations regarding detection of critical links, which do not necessarily make them visible for a given spatial traffic distribution. In fact, these indicators act as "thermometers" of performance, revealing local problems and allowing formulation of a hierarchy according to the gravity of this congestion. Vulnerability indicators tend to detect critical links better, taking into account the configuration of the network's structure, and appear to be complementary indicators for prioritizing investments. Besides this, vulnerability indicators are mainly adequate for ranking links in regions subject to extreme climatic events, such as snowstorms and flooding, among others, for the purpose of mitigating the effects on traffic. In urban regions, anthropogenic events cannot be overlooked, such as popular manifestations or even terrorist attacks.

The results obtained in this case study suggest that hierarchies resulting from indicators based on the same performance attribute are more similar than hierarchies resulting from indicators based on different performance attributes. This suggests that the decision on the indicator used should be preceded by a decision on which performance perspective to adopt, which will determine the attribute(s) to be emphasized.

A possible solution is the combined analysis of various attributes and their respective indicators, in which case normalization, to produce a common scale, facilitates the comparative approaches for their ranking. In the case study presented here, we assigned identical weights to the indicators tested, but the careful use of differentiated weights can be used to help solving specific performance problems. Naturally, a deeper study is necessary to identify the best procedure and the weights to be used. This would allow obtaining multi-criteria hierarchies for use in planning and setting investment priorities. In the case study presented here, for example, the links of group I should be priority candidates for funding to improve performance from the standpoint of reducing congestion and vulnerability.

A limitation of this study is that the criterion for pre-selection of a group of links that are more congested or more present in minimum routes added a bias that does not permit, for example, guaranteeing that the most significant links in terms of vulnerability were all present in the analysis. Nevertheless, the heavy computational demand and time constraints for determining the vulnerability indicators of a complex network required us to circumscribe this study to a group of selected links. The main objective of this article, to present the different rankings derived from the two attributes analyzed, was met, independent of the possible bias.

Despite the rapid advances in computation technology, analysis of the complex networks of a great metropolis is still a daunting task. The different levels of demand for resources can lead to greater use of indicators that are simpler to obtain, at times leading to misleading results. Indeed, the prioritization of investments in road networks based on vulnerability indicators is still a relatively underexplored approach. The combination of indicators in successive steps, as presented in this article in simplified form, can be a more effective alternative method for ranking critical links based on multiple criteria.

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