Contribution of asymmetric strange–antistrange sea to the Paschos–Wolfenstein relation

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Abstract

The NuTeV Collaboration reported a value of \( \sin^2 \theta_w \) measured in neutrino–nucleon deep inelastic scattering, and found that the value is three standard deviations from the standard model prediction. This result is obtained under the assumption that the strange quark–antiquark sea of nucleons are symmetric, and that the up and down quark distributions are symmetric with the simultaneous interchange of \( u \leftrightarrow d \) and \( p \leftrightarrow n \). We discuss the contribution of asymmetric strange–antistrange sea to the Paschos–Wolfenstein relation in the extraction of weak mixing angle \( \sin^2 \theta_w \). We also point out that the contribution of asymmetric strange–antistrange sea should remove roughly 30–80% of the discrepancy between the NuTeV result and other determinations of \( \sin^2 \theta_w \) when using the light-cone meson–baryon model to calculate the contribution of the strange–antistrange sea.

1. Introduction

It is widely believed that the standard model is a low energy remnant of some more fundamental theory. In the standard model, the weak mixing angle \( \sin^2 \theta_w \) is one of the important quantities. The precise determination of \( \sin^2 \theta_w \) plays a key role in testing the standard model of electroweak interaction. Its present value was consistent with all the known electroweak observables [1], until the NuTeV Collaboration reported a value of \( \sin^2 \theta_w \) measured in neutrino–nucleon deep inelastic scattering (DIS) with both neutrino and antineutrino beams. The value [2]

\[
\sin^2 \theta_w = 0.2277 \pm 0.0013 \text{ (stat)} \pm 0.0009 \text{ (syst)},
\]

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which is three standard deviations larger than the value,
\[
\sin^2 \theta_w = 0.2227 \pm 0.0004,
\]
measured in other electroweak processes [1]. Various sources of systematic errors have been clearly identified
and examined. For extracting \( \sin^2 \theta_w \) the NuTeV Collaboration measured the ratio of neutrino neutral-current
and charge-current cross sections on iron [2]. This procedure is closely related to the Paschos–Wolfenstein relation [3]:
\[
R^-=\frac{\sigma^\nu_{NC}-\sigma^{\bar{\nu}N}_{NC}}{\sigma^\nu_{CC}-\sigma^{\bar{\nu}N}_{CC}}=\frac{1}{2}-\sin^2 \theta_w.
\]
Because the NuTeV Collaboration did not strictly measure the Paschos–Wolfenstein relation, Eq. (1), there are a
number of corrections that need to be considered, such as charge symmetry violation [4], which should reduce
roughly one-third of the discrepancy between the NuTeV result and all accepted average value of \( \sin^2 \theta_w \), nuclear
effect, which arises from the higher twist effect of nuclear shadowing [5], neutron excess [6], although such
modification are not measured, differences in shadowing from photons, \( W^\pm \) and \( Z^0 \) [7], asymmetry in the \( s \) and \( \bar{s} \)
distributions [8], nuclear correction, discussed in Ref. [9] by noting nuclear modification of \( F_2 \), also recently QCD
correction [10], and so on. In addition, the discussion of possible uncertainties and physics behind the anomaly can
be found in Ref. [11].

Eq. (1) is based on the assumption of symmetric quark and antiquark distributions in the nucleon sea. In fact,
the study of the quark sea in the nucleon is important to understand the nucleon structure and the strong
interaction. Usually, we assume that the quark and antiquark sea are symmetric, but we should note that it may
have asymmetry to some extent [12]. It is rather difficult to study the asymmetry of the up and down sea in
experiment, because we hardly can distinguish the up and down sea quarks from the corresponding valence quarks
in the nucleon bound state. However, for the strange quark sea, it is relatively accessible and there have been
analyses of experimental data [13–16], which suggest the asymmetry of \( s \) and \( \bar{s} \) distributions in the nucleon sea.
Also, there are some theoretical discussions on this issue [12,17–21]. Brodsky and Ma [12] proposed a light-cone
meson–baryon fluctuation model to describe the \( s-\bar{s} \) distributions and found that \( s<\bar{s} \) in small \( x \) region and
\( s>\bar{s} \) in large \( x \) region. A significantly different conclusion was obtained by Holtmann, Szczurek and Speth [19]
from Ref. [12] by using the meson cloud model with fluctuation function [17,19]. Cao and Signal [21] obtained
a phenomenological analysis of \( s-\bar{s} \) asymmetry in the nucleon sea when using two different models: light-cone
model [12] and meson cloud model [17,21]. In this Letter, we consider the role of the \( s-\bar{s} \) asymmetry in the
nucleon sea by using the light-cone meson–baryon fluctuation model to calculate the contribution, and find that
it should account for roughly 30–80% of the discrepancy between the NuTeV result and other accepted value of
\( \sin^2 \theta_w \). Our result is different from the previous conclusion [8] that the effect of asymmetric strange–antistrange
sea is fairly small and does not affect the NuTeV extraction of \( \sin^2 \theta_w \).

2. Modified Paschos–Wolfenstein relation

The Paschos–Wolfenstein relation was derived for \( s(x)=\bar{s}(x) \) in the nucleon sea. In this section, we shall
derive a revised expression for \( s(x) \neq \bar{s}(x) \). The cross sections for neutrino– and antineutrino–nucleon neutral
current interaction have the form [22]
\[
\frac{d^2 \sigma^{\nu(x)}_{NC}}{dx \, dy} = \pi s \left( \frac{\alpha}{2 \sin^2 \theta_w \cos^2 \theta_w M_Z^2} \right)^2 \left( \frac{M_Z^2}{M_Z^2 + Q^2} \right)^2 \times \left[ \frac{x y F_2^\nu(x, \, Q^2) + \left( 1 - \frac{y m_n^2}{s} \right) F_2^\bar{\nu}(x, \, Q^2) \pm \left( \frac{y^2}{2 \, x} \right) x F_2^\nu(x, \, Q^2) \right].
\]
and the cross sections for neutrino– and antineutrino–nucleon charge current reaction have the form [22]

\[
\frac{d^2\sigma_{CC}^{(i)}}{dx\,dy} = \frac{\alpha}{2\sin^2\theta_W} \left( \frac{M_W^2}{M_W^2 + Q^2} \right)^2 \times 
\left[ xy F_{1}^{W\pm}(x, Q^2) + \left( 1 - y - \frac{x M_N^2}{s} \right) F_{2}^{W\pm}(x, Q^2) \pm \left( y - \frac{y^2}{2} \right) x F_{3}^{W\pm}(x, Q^2) \right],
\]

(3)

where \( Q^2 = -q^2 \) is the square of the four momentum transfer for the reaction, \( M_W (M_Z) \) is the mass of the charge (neutral) current interacting weak vector boson, \( \theta_W \) is the Weinberg angle, and \( x = Q^2/2p \cdot q, y = p \cdot q / p \cdot k, \) and \( s = (k + p)^2 \) are the DIS variables for four momentum \( k(p) \) of the initial state neutrino or antineutrino (nucleon). The structure functions \( F_i^{W\pm}(x, Q^2) \) on proton \( (p) \), which only depend on \( x, Q^2 \rightarrow \infty \), were given by [22]

\[
\lim_{Q^2 \rightarrow \infty} F_{1}^{W\pm}(x, Q^2) = dP(x) + \bar{u}P(x) + sP(x) + cP(x),
\]

\[
\lim_{Q^2 \rightarrow \infty} F_{2}^{W\pm}(x, Q^2) = uP(x) + \bar{d}P(x) + \bar{c}P(x) + cP(x),
\]

\[
\frac{1}{2} \lim_{Q^2 \rightarrow \infty} F_{3}^{W\pm}(x, Q^2) = dP(x) - \bar{u}P(x) + sP(x) - \bar{c}P(x),
\]

\[
\frac{1}{2} \lim_{Q^2 \rightarrow \infty} F_{3}^{W\pm}(x, Q^2) = uP(x) - \bar{d}P(x) - \bar{s}P(x) + cP(x),
\]

\[
F_{2}^{W\pm}(x, Q^2) = 2x F_{1}^{W\pm}(x, Q^2).
\]

(4)

The structure functions of neutral current reaction take the form [22]

\[
\lim_{Q^2 \rightarrow \infty} F_{1}^{Z\pm}(x, Q^2) = 1/2\left[ (uV + uA)(uP(x) + \bar{u}P(x) + cP(x) + \bar{c}P(x)) \right.
\]

\[
+ \left. (dV + dA)(dP(x) + \bar{d}P(x) + sP(x) + \bar{s}P(x)) \right],
\]

\[
\lim_{Q^2 \rightarrow \infty} F_{2}^{Z\pm}(x, Q^2) = 2[ uV uA (uP(x) - \bar{u}P(x) + cP(x) - \bar{c}P(x))]
\]

\[
+ dV uA (dP(x) - \bar{d}P(x) + sP(x) - \bar{s}P(x))],
\]

\[
F_{2}^{Z\pm}(x, Q^2) = 2x F_{1}^{Z\pm}(x, Q^2).
\]

(5)

and the corresponding structure functions \( F_i^{W\pm(Z)}(x, Q^2) \) for neutrons are given by replacing superscripts \( p \rightarrow n \) in Eqs. (4), (5), with the assumption of charge symmetry for parton distributions

\[
dP(x) = uP(x), \quad u^n(x) = dP(x),
\]

\[
sP(x) = sP(x), \quad c^n(x) = cP(x) = c(x).
\]

(6)

In Eqs. (5), \( uV, dV, u_V \) and \( d_A \) are vector and axial-vector couplings:

\[
uV = \frac{1}{2} - \frac{4}{3} \sin^2\theta_w, \quad u_A = \frac{1}{2},
\]

\[
dV = \frac{1}{2} + \frac{2}{3} \sin^2\theta_w, \quad d_A = -\frac{1}{2}.
\]

Using these equations, we obtain the modified Paschos–Wolfenstein relation:

\[
R_N = \frac{\sigma_{NC}^{\nu N} - \sigma_{NC}^{\bar{\nu} N}}{\sigma_{CC}^{\nu N} - \sigma_{CC}^{\bar{\nu} N}} = R^- + \delta R^-.
\]

(7)
Here, $\delta R_{s}^{-}$ is the correction to the Paschos–Wolfenstein relation $R^{-}$ from the asymmetry of $s - \bar{s}$ distribution in the nucleon sea,

$$\delta R_{s}^{-} = -\left(-1 + \frac{7}{3} \sin^2 \theta_W \right) \frac{S^{-}}{Q_V + 3S^{-}},$$

(8)

where $Q_V \equiv \int_0^1 x [\nu_V(x) + d_V(x)] dx$ and $S^{-} \equiv \int_0^1 x [s(x) - \bar{s}(x)] dx$. During this procedure of getting $R_{s}^{-}$, we assume isospin symmetry and $c(x) = \bar{c}(x)$. In this way, we obtain the correction $R_{s}^{-}$ and below we shall calculate $S^{-}$ and $Q_V$ by using the light-cone two-body wave function model [12] and the light-cone spectator model [23].

3. Strange–antistrange asymmetry

We shall adapt the light-cone two-body wave function model [12] to calculate $S^{-}$. In this light-cone formalism [24], the hadronic wave function can be expressed by a series of light-cone wave functions multiplied by the Fock states, for example, the proton wave function can be written as

$$|p\rangle = |uud\rangle \Phi_{uud/p} + |uudg\rangle \Phi_{uudg/p} + \sum_{q\bar{q}} |uudq\bar{q}\rangle \Phi_{uudq\bar{q}/p} + \cdots.$$  

(9)

Brodsky and Ma made an approximation [12], which suggests that the intrinsic sea part of the proton function can be expressed as a sum of meson–baryon Fock states. For example:

$$P(uuds\bar{s}) = K + (u\bar{s}) + \Lambda(uds)$$

for the intrinsic strange sea, the higher Fock states are less important, the $ud$ in $\Lambda$ serves as a spectator in the quark-spectator model [23], for which we choose

$$\Phi_D(x, k_\perp) = A_D \exp \left(-\frac{M^2}{8\alpha_D^2}\right),$$

(10)

$$\Phi_D(x, k_\perp) = A_D \left(1 + \frac{M^2}{\alpha_D^2}\right)^{-P},$$

(11)

where $\Phi_D(x, k_\perp)$, is a two-body wave function which is a function of invariant masses for meson–baryon state:

$$M^2 = \frac{m_1^2 + k_\perp^2}{x} + \frac{m_2^2 + k_\perp^2}{1-x},$$  

(12)

where $k_\perp$ is the initial quark transversal momentum, $m_1$ and $m_2$ are the masses for quark $q$ and spectator $D$, $\alpha_D$ sets the characteristic internal momentum scale, and $P$ is the power constant which is chosen as $P = 3.5$ here. The momentum distribution of the intrinsic $s$ and $\bar{s}$ in the $K^+\Lambda$ state can be modelled from the two-level convolution formula:

$$s(x) = \int_x^1 \frac{dy}{y} f_{A/K^+\Lambda}(y) q_{s/A}(y/x),$$

$$\bar{s}(x) = \int_x^1 \frac{dy}{y} f_{K^+/K^+\Lambda}(y) q_{s/K^+}(y/x).$$

(13)

where $f_{A/K^+\Lambda}(y)$, $f_{K^+/K^+\Lambda}(y)$ are the probabilities of finding $\Lambda$, $K^+$ in the $K^+\Lambda$ state with the light-cone momentum fraction $y$.

$$f_{A/K^+\Lambda}(y) = \int_{-\infty}^{+\infty} dk_\perp \left| A_D \exp \left[-\frac{1}{8\alpha_D^2} \left(\frac{m_A^2 + k_\perp^2}{y} + \frac{m_{K^+}^2 + k_\perp^2}{1-y}\right)\right]\right|^2,$$
The result of our calculation is 0

\[ f_{K^+/K^+}(y) = \int_{-\infty}^{+\infty} d\mathbf{k}_\perp |A_D| \exp \left[ -\frac{1}{8\alpha_D^2} \left( \frac{m_{K^+}^2 + \mathbf{k}_\perp^2}{y} + \frac{m_{K^+}^2 + \mathbf{k}_\perp^2}{1-y} \right) \right]^2, \]

(14)

and \( q_s/\Lambda(x/y), q_{\bar{s}}/K^+(x/y) \), are the probabilities of finding \( s, \bar{s} \) quarks in \( \Lambda, K^+ \) state with the light-cone momentum fraction \( x/y \).

\[ q_s/\Lambda(x/y) = \int_{-\infty}^{+\infty} d\mathbf{k}_\perp |A_D| \exp \left[ -\frac{1}{8\alpha_D^2} \left( \frac{m_s^2 + \mathbf{k}_\perp^2}{x/y} + \frac{m_D^2 + \mathbf{k}_\perp^2}{1-x/y} \right) \right]^2; \]

\[ q_{\bar{s}}/K^+(x/y) = \int_{-\infty}^{+\infty} d\mathbf{k}_\perp |A_D| \exp \left[ -\frac{1}{8\alpha_D^2} \left( \frac{m_{\bar{s}}^2 + \mathbf{k}_\perp^2}{x/y} + \frac{m_D^2 + \mathbf{k}_\perp^2}{1-x/y} \right) \right]^2. \]

(15)

Two wave function models, the Gaussian type and the power-law type, are adopted [12] to evaluate the asymmetry of strange–antistrange sea, and almost identical distributions of \( s - \bar{s} \) are obtained in the nucleon sea. Here, we also consider the two kinds of wave functions, Eqs. (10) and (11).

The up and down valence quark distributions in the proton are calculated by using the quark–diquark model. The unpolarized valence quark distribution in the proton is [23]

\[ u_V(x) = \frac{1}{2} a_S(x) + \frac{1}{6} a_V(x), \quad d_V(x) = \frac{1}{3} a_V(x), \]

(16)

where \( a_D(x) (D = S \text{ or } V, \text{ with } S \text{ standing for scalar diquark Fock state and } V \text{ standing for vector diquark state}) \) denotes that the amplitude for the quark \( q \) is scattered while the spectator is in diquark state \( D \) [25], and can be written as:

\[ a_D(x) \propto \int |\Phi_D(x, \mathbf{k}_\perp)|^2. \]

(17)

The values of parameters \( \alpha_D, m_q, \) and \( m_D \) can be adjusted by fitting the hadronic properties. In this Letter, we simply choose \( m_q = 330 \text{ MeV} \). For light-flavor quarks, \( \alpha_D = 330 \text{ MeV}, m_S = 600 \text{ MeV}, m_V = 900 \text{ MeV} \) and \( m_s = m_{\bar{s}} = 480 \text{ MeV} \). Because the fluctuation functions were normalized to 1 in Ref. [12], we can obtain the different distributions for \( s \) and \( \bar{s} \) in the nucleon. In the same way, we can get the distributions of the up and down valence quarks, for which the integrated amplitude \( \int_0^1 dx a_D(x) \) must be normalized to 3 in a spectator model [23,26]. Assuming isospin symmetry, we can get the valence distributions in the nucleon which implies \( N = (p+n)/2 \)

\[ u_V^N(x) = \frac{1}{2} \left[ \frac{1}{2} a_S(x) + \frac{1}{2} a_V(x) \right], \quad d_V^N(x) = \frac{1}{2} \left[ \frac{1}{2} a_S(x) + \frac{1}{2} a_V(x) \right]. \]

(18)

Thus, using this model, we obtain the distributions of \( s \) and \( \bar{s} \) in the nucleon sea. The numerical result is given in Fig. 1. One can find that \( s < \bar{s} \) as \( x < 0.235, s > \bar{s} \) as \( x > 0.235 \), this result is opposite to the prediction from the meson cloud model [8]. Similarly, one can obtain the shape of \( x(s-\bar{s}) \) in Fig. 2. From Eq. (7), one can find that a shift of \( \delta R^- \) should lead to a shift in the \( R^- \), which affect the extraction of \( \sin^2\theta_w \), Eq. (8).

The result of our calculation is 0.0042 < \( S^- < 0.0016 \) (0.0035 < \( S^- < 0.0087 \)) for the Gaussian wave function (for the power-law wave function), which corresponds to \( P_{K^+} = 4\%, 10\% \). Hence, 0.0017 < \( \delta R^- < 0.0041 \) (0.0014 < \( \delta R^- < 0.0034 \)), for the Gaussian wave function (the power-law wave function). The shift in \( \sin^2\theta_w \) can reduce the discrepancy from 0.005 to 0.0033 (0.0036) (\( P_{K^+} = 4\% \)) or 0.0009 (0.0016) (\( P_{K^+} = 10\% \)).
Fig. 1. Distributions for $s(x)$ and $\bar{s}(x)$. $P(s)$ ($G(s)$) is the $s$ distribution with the power-law wave function (the Gaussian wave function) and $P(\bar{s})$ ($G(\bar{s})$) is the $\bar{s}$ distribution with the power-law wave function (the Gaussian wave function).

Fig. 2. Distributions for $x\delta_s(x)$, with $\delta_s(x) = s(x) - \bar{s}(x)$. The solid curve is for the power-law wave function and the dash curve is for the Gaussian wave function.

4. Summary

Intrinsic sea quarks play a crucial role in understanding the structure of the nucleon and strong interaction, such as the effect of the strange and antistrange quark distributions to the nucleon structure. In this Letter, we have re-examined the asymmetry of $s - \bar{s}$ distribution in the nucleon with the light-cone meson–baryon model. Considering this asymmetry, we derived a modified Paschos–Wolfenstein relation. Though there have been evidences for the asymmetry of $s - \bar{s}$ distribution in the nucleon sea suggested by analyses [13–15], this asymmetry need to be directly confirmed experimentally. We have strong theoretical arguments about the sign and magnitude of the correction to the Paschos–Wolfenstein relation. In particular, this correction should make a significant contribution to the NuTeV extraction of the weak mixing angle $\sin^2 \theta_W$ by a deviation 30–80%, which corresponding to the assumption that the probability is 4–10% for the $K^+\Lambda$ state. Therefore it is important to investigate the effect of asymmetric strange–antistrange sea more carefully in future experiments.

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