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Comparison of insiders' optimal strategies depending on the type of side-information

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Abstract

In this paper, we consider a complete continuous-time financial market with discontinuous prices and different types of side-information (initial or progressive strong information, weak information). The agents strive to maximize the expectation of the logarithm of their terminal wealth. Our purpose is to explicit and to simulate the optimal strategy of the insiders in some examples of side-information. We compare those optimal strategies, depending on the type of side-information.

Keywords: Strong and weak information; Risk neutral probability measure and minimal probability measure; Optimal strategy; Simulation

1. Introduction

This paper deals with the simulation of the optimal strategy of agents who have some side-information about the market structure, and to compare it to non-insider's optimal strategy. Thus, we fix a finite horizon time $T>0$ and a filtered probability space $(\Omega, \mathrm{F}_T, \mathbb{P})$, with a multidimensional Brownian motion W and a multidimensional Poisson process N. A market model is built, with one bond and d

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discontinuous risky assets. The agents strive to maximize their expected discounted utility of terminal wealth on $[0, A]$, $A < T$.

Previous works about optimal strategies, such as Karatzas et al. [21], are written in the classical setting of financial markets where the agents share the same information flow, which is conveyed by the prices. But it seems clear that financial markets inherently have asymmetry of information. In this article, we consider three types of extra information of an insider.

The first type is called "initial strong" information: from the beginning the insider has an extra information available about the outcome of some variable L of the prices. The cornerstone of this modelization is the theory of initial enlargement of filtration by a random variable, which was developed by Jeulin [19], in the series of papers in the "Séminaire de Calcul Stochastique (1982/83)" of the University Paris VI, by Jacod [17], Jeulin and Yor [20], Föllmer and Imkeller [10] and further by Amendinger et al. [3] and Amendinger [1]. Karatzas and Pikovsky [22] studied optimization strategies with some examples of real or vectorial random variables: $L = W_1(T)$, $L = (\lambda_i W_i(T) + (1 - \lambda_i)\varepsilon_i)_{i=1,\dots,d}$ with a family of independent Gaussian variables ε. Amendinger et al. [2] quantified the value of this noisy signal about the terminal value of the Brownian motion driving the asset prices. In several papers, including Grorud and Pontier [12] or Grorud [11], the expression of the compensator, or information drift, is given in examples where the insider knows, respectively, the ratio at time T of two assets prices or knows that one asset price at time T will be greater (or smaller) than a given constant. All these examples were given in a purely diffusive market model. In a mixed diffusive-jump market model, Elliott and Jeanblanc [9] and Grorud [11] studied the case where L is the number of jumps over the interval $[0, T]$.

The second type is called "progressive strong" information: the insider's information is perturbed by an independent noise changing throughout time. This case deals with the theory of a progressive enlargement of filtration. In the pioneering papers on the enlargement of filtrations (see [20]), the progressive enlarged filtration is by definition the smallest one that makes a given random time a stopping time. Imkeller studied this theory in the context of insider models in [15]. The progressive enlargement of filtration I consider in this paper is in the sense of Corcuera et al. [8]. Using Malliavin's calculus, they obtained in [8] the formula of the compensator in some examples of information, such as a function of the terminal value of the Brownian motion, disturbed by an independent noise.

The third type is called "weak" information or anticipation: the insider anticipates the law of a random variable L that will be realized at a future date. The main difference with a strong information is that there is no change of filtration but only a change of a probability measure. This notion of weak information is defined by Baudoin [5,6].

In this paper, we consider the agent's problem of maximizing the expectation of the logarithm of his terminal wealth. In some examples of side-information, we compute the explicit formula of the insiders' strategy (optimal wealth and portfolio) in our mixed diffusive-jump market model. We simulate them and compare them between each other and to the non-insider's optimal strategy.

This article is organized as follows.

In Section 2, we define the market and introduce the general framework and notations that are valid throughout the paper. In Section 3, we recall some technical results about these three types of side-information: we recall briefly the construction of a risk neutral probability measure for a strong-informed agent, and a "minimal probability measure" associated with L for a weak-informed agent. These probability measures summarize the side-information of each insider. Then we give the solution of their optimization problem. In Section 4, we explicit the insiders' strategies in some examples of side-information. First, we explain how to compute the density process of an initial strong insider's risk neutral probability measure with respect to the effective probability measure $\mathbb{P}$. Then, we compute the strategy (optimal wealth and portfolio) for two examples of initial strong information that can occur in a case of a merger between two companies: the insider knows that an asset price will be or not at time T in a given margin, or he knows the ratio at time T of two assets prices. We explicit the difference of strategy between a non-insider and an insider in both cases: a purely diffusive market model or a mixed diffusive-jump market model. In Section 5, we simulate the above strategies in a diffusive-jump market model with four risky assets and we explicit qualitatively the optimal investments. Finally, we compare the optimal wealth in the three types of side-information, each information being a variant of the value at time T of the first component of the Brownian motion. We show that for the same noise dynamics, the wealths are plausibly ordered in the sense of increasing relevance of side-information. Figures are given in Appendix A.

2. The market

Let W be a real m-dimensional Brownian motion on its canonical probability space $(\Omega^W, \mathrm{F}_T^W := (\mathscr{F}_t^W)_{t\in[0,T]}, \mathbb{P}^W)$. Let N be a n-dimensional Poisson process on its canonical probability space $(\Omega^N, \mathrm{F}_T^N := (\mathscr{F}_t^N)_{t\in[0,T]}, \mathbb{P}^N)$, with a positive, F_T^N-predictable intensity κ satisfying $E_{\mathbb{P}^N}[\int_0^T \kappa(t)\,\mathrm{d}t] < +\infty$. The process M defined by $M(t) := N(t) - \int_0^t \kappa(s)\,\mathrm{d}s$ is a $(\mathrm{F}_T^N, \mathbb{P}^N)$-martingale, called the compensated martingale of the Poisson process N. Let $(\Omega, \mathrm{F}_T, \mathbb{P}) := (\Omega^W \times \Omega^N, \mathrm{F}_T^W \otimes \mathrm{F}_T^N, \mathbb{P}^W \otimes \mathbb{P}^N)$ be the product space. W and N are independent. Let $\mathscr{A}$ be a σ-algebra of Ω. Let $d = m + n$.

The agents on the financial market are able to invest in $d+1$ assets, the prices of which are driven by the following stochastic differential equations:

$$P_0(t) = \exp\left(\int_0^t r(s)\,\mathrm{d}s\right), \tag{2.1}$$

$$\mathrm{d}P_i(t) = P_i(t^-)\left[b_i(t)\,\mathrm{d}t + \sum_{j=1}^{d} \sigma_{ij}(t)\,\mathrm{d}(W^*, N^*)_j^*(t)\right], \quad i = 1, \ldots, d. \tag{2.2}$$

X^* denotes the transposed process of process X. σ is a given strongly non-degenerate deterministic $d \times d$-matrix-valued process. The processes r and b are assumed deterministic and bounded on $[0, T]$. We assume that $\sigma_{ij} > -1$ for all $m+1 \leqslant j \leqslant d$ and $1 \leqslant i \leqslant d$. Thus F_T and the filtration generated by the prices are identical.

If the agent has a strong information (initial or progressive), he receives an individual flow of side-information, represented by the filtration $\mathrm{H}_T := (\mathscr{H}_t)_{t\in[0,T]}$. Thus, we introduce the individual agent's filtration $\mathrm{G}_T := (\mathscr{G}_t)_{t\in[0,T]}$ of available information, with

$$\mathscr{G}_t := \mathscr{F}_t \vee \mathscr{H}_t, \quad 0 \leqslant t \leqslant T.$$

In other words, this agent possesses all information about the market up to the present time t, plus his own side-information that he does not reveal to the other agents (except indirectly, by his market strategies). Besides, if the agent has a weak information, he only has the filtration $\mathrm{G}_T := \mathrm{F}_T$ of the market available.

Definition 2.1. A G_T-admissible strategy is a portfolio π such that for all $i = 1, \ldots, d$, $\frac{\pi_i}{P_i}$ is G_T-predictable and satisfies the integrability requirement $\int_0^T \|\sigma^*(t)\pi(t)\|^2\,\mathrm{d}t < \infty$ $\mathbb{P}$ almost surely and so that the corresponding wealth process X is bounded from below and satisfies $X(T) \geqslant 0$ $\mathbb{P}$ almost surely.

$\pi_i(t)$ represents the amount invested by the agent at time t in the ith stock $(i = 1, \ldots, d)$. The agent has an initial wealth $X(0) \in L^1(\mathscr{G}_0)$. As usually, we assume that the strategy is self-financing, so the agent's discounted wealth is given by

$$\beta(t)X(t) = X(0) + \int_0^t \beta(s)\pi^*(s)(b - rI_d)(s)\,\mathrm{d}s + \int_0^t \beta(s)\pi^*(s^-)\sigma(s)\,\mathrm{d}(W^*, N^*)^*(s). \tag{2.3}$$

$\beta(t) := (P_0(t))^{-1}$ is the deflator process and $I_d = (1, \ldots, 1)^* \in \mathbb{R}^d$. But on an enlarged filtration $(\mathrm{G}_T, \mathbb{P})$, the process (W^*, N^*) could no more be a semi-martingale. We will add in Sections 3.1 and 3.2 sufficient conditions to obtain a meaningful wealth equation for a strong insider. The agent chooses his strategy so as to optimize his terminal wealth.

Let us introduce some notations

- $I_n = (1, \ldots, 1)^* \in \mathbb{R}^n$.
- If v_1 et v_2 are two vectors of same dimension d, we note $v_1.v_2$ the vector with components $(v_1.v_2)_i = v_{1,i}.v_{2,i}$, $i = 1, \ldots, d$.
- $\mathscr{E}$ denote the Doléans exponential.

We define $\Theta(t) := m$ first lines of $(\sigma(t))^{-1}(b(t) - r(t)I_d)$ and the n-dimensional process q such that $q(t).\kappa(t) := n$ last lines of $-(\sigma(t))^{-1}(b(t) - r(t)I_d)$. We assume that

Assumption 2.2. q is a process with positive components

Otherwise arbitrage opportunities can occur (see [18]).

$$\widehat{W}(t) := W(t) + \int_0^t \Theta(s)\,\mathrm{d}s, \quad \widehat{M}(t) := N(t) - \int_0^t q(s)\kappa(s)\,\mathrm{d}s, \quad t \in [0, T]. \tag{2.4}$$

We denote $\widehat{S}:=(\widehat{W}^*, \widehat{M}^*)^*$. Eq. (2.3) implies

$$\beta(t)X(t) = X(0) + \int_0^t \beta(s)\pi^*(s^-)\sigma(s)\,\mathrm{d}\widehat{S}(s). \tag{2.5}$$

$$\widehat{\mathbb{P}_0}:=Y_0\mathbb{P} \quad \text{where } Y_0 = \mathscr{E}\left(\int_0^\cdot (-\Theta^*(s)\,\mathrm{d}W(s) + (q(s)-I_n)^*\,\mathrm{d}M(s))\right) \tag{2.6}$$

is the "risk neutral probability measure" for a non-insider.

3. Modelization of three types of side-information

We consider the agent's problem of maximizing his terminal wealth, in each of the three settings of side-information. For sake of completeness, we recall here some technical results of Hillairet [14].

3.1. Initial strong information

We suppose in this subsection that the agent knows a functional L ω-wise from the beginning. Let us make more precise the nature of this initial strong information:

Assumption 3.1. $\forall t \in [0,T]$, $\mathscr{H}_t = \sigma(L)$ where L is an $\mathscr{A}$-measurable random variable with values in a Polish space $(E,\mathscr{E})$ (meaning that the agent receives his additional information immediately and only at time $t=0$) and moreover, L satisfies the assumption: $\mathbb{P}(L \in \cdot|\mathscr{F}_t)(\omega) \sim \mathbb{P}(L \in \cdot)$ for all $t \in [0,T[$ for $\mathbb{P}$ almost all $\omega \in \Omega$.

Remark. Assumption 3.1 is equivalent to: there exists a probability measure equivalent to $\mathbb{P}$ and under which $\forall t \in [0,T[$, $\mathscr{F}_t$ and $\sigma(L)$ are independent. We consider the only one that is identical to $\mathbb{P}$ on $\mathscr{F}_T$ and we denote it $\mathbb{Q}^L$. We introduce the density process

$$Z(t):=E_{\mathbb{Q}^L}\left[\frac{\mathrm{d}\mathbb{P}}{\mathrm{d}\mathbb{Q}^L}\middle|\mathscr{G}_t\right], \tag{3.1}$$

which satisfies $\mathrm{d}Z(t) = Z(t^-)[\rho_1^*(t)\,\mathrm{d}W(t) + (\rho_2(t)-I_n)^*\,\mathrm{d}M(t)]$, where ρ_1 and ρ_2 are G_T-predictable processes. $\widetilde{W}(\cdot):=W(\cdot) - \int_0^\cdot \rho_1(t)\,\mathrm{d}t$ is a $(\mathrm{G}_T,\mathbb{P})$-Brownian motion and $\widetilde{M}(\cdot):=N(\cdot) - \int_0^\cdot \kappa.\rho_2(t)\,\mathrm{d}t$ is the compensated process of a $(\mathrm{G}_T,\mathbb{P})$-Poisson process with intensity $(\kappa.\rho_2)$. Therefore the wealth (2.3) is meaningful under Assumption 3.1.

Definition 3.2.

$$Y:=\mathscr{E}\left(\int_0^\cdot \left(-(\Theta+\rho_1(s))^*\,\mathrm{d}\widetilde{W}(s) + \left(\frac{q}{\rho_2(s)} - I_n\right)^*\mathrm{d}\widetilde{M}(s)\right)\right).$$

Y is a positive $(\mathrm{G}_T, \mathbb{P})$-local martingale. A straightforward calculus yields:

$$\mathrm{d}(Y^{-1})(t) = Y^{-1}(t^-) l^*(t)\,\mathrm{d}\widehat{S}(t) \quad \text{with } l^* := \left((\Theta + \rho_1)^*, \left(\frac{\rho_2}{q} - I_n \right)^* \right).$$

Theorem 3.3. *We assume* 3.1 *and that Y defined in* (3.2) *is a* $(\mathrm{G}_T, \mathbb{P})$*-martingale. Then $\widehat{\mathbb{P}} := Y\mathbb{P}$ is a "risk neutral probability measure" for the insider. $\widehat{W}$ and $\widehat{M}$ (definition* (2.4)) *are, respectively, a* $(\mathrm{G}_T, \widehat{\mathbb{P}})$*-Brownian motion and the compensated process of a* $(\mathrm{G}_T, \widehat{\mathbb{P}})$*-Poisson process with intensity* $(q.\kappa)$*. Furthermore, the market is complete for the insider: if $\widehat{A}$ is a $(\mathscr{G}_T, \widehat{\mathbb{P}})$-local martingale, there exists $\psi \in L^1_{\mathrm{loc}}(\widehat{S}, \mathrm{G}_T, \widehat{\mathbb{P}})$ such that* $\forall (t,\omega) \in [0,T] \times \Omega$, $\widehat{A}(t) = \widehat{A}(0) + \int_0^t \psi^*(s)\,\mathrm{d}\widehat{S}(s)$.

3.2. Progressive strong information

In this subsection, we consider an agent whose additional information changes through time. His knowledge is perturbed by an independent noise, and is getting to him clearer as time evolves.

Assumption 3.4. $\forall t \in [0,T]$, $\mathscr{H}_t = \sigma(L(s), s \leqslant t)$ where $L(s) = \mathscr{L}(J, \mathscr{B}(s))$ with

- $\mathscr{L} : \mathbb{R}^2 \to \mathbb{R}$ is a given measurable function.
- $\mathscr{B} = \{\mathscr{B}(t), 0 \leqslant t \leqslant T\}$ is independent of $\mathscr{F}_T$.
- $\mathscr{J}$ is a $\mathscr{F}_T$-measurable random variable such that: $\mathbb{P}(\mathscr{J} \in \cdot | \mathscr{F}_t)(\omega) \ll \mathbb{P}(\mathscr{J} \in \cdot)$ for all $t \in [0,T[$ for $\mathbb{P}$ almost all $\omega \in \Omega$.

G_T denote the usual enlarged filtration $(\mathscr{G}_t = \cap_{u>t}(\mathscr{F}_u \vee \sigma(L(s), s \leqslant u)))_{t\in[0,T[}$. $\mathscr{J}$ contains the additional information available to the insider, and $\mathscr{B}$ represents an additional noise that perturbs this side-information. Therefore, one expects in general that $\mathscr{B}(T) = 0$ and that the variance of the noise decreases to zero as revelation time T approaches. We denote by $P_t(\omega, \mathrm{d}x)$ a regular version of the conditional law of $\mathscr{J}$ given $\mathscr{F}_t$ and by $P_{\mathscr{J}}$ the law of $\mathscr{J}$. According to Jacod [17] and Proposition 12 of Grorud [11], there exists a measurable version of the conditional density $p(t,x)(\omega) = \frac{\mathrm{d}P_t}{\mathrm{d}P_{\mathscr{J}}}(\omega, x)$ which is a $(\mathrm{F}_T, \mathbb{P})$-martingale and can be written, $\forall x \in \mathbb{R}$, as

$$p(t,x) = p(0,x) + \int_0^t \alpha(s,x)\,\mathrm{d}W(s) + \int_0^t \beta(s,x)\,\mathrm{d}M(s),$$

where for all x, $s \to \alpha(s,x)$ and $s \to \beta(s,x)$ are F_T-predictable processes. Moreover, for all $s < T$, $p(s, \mathscr{J}) > 0$ $\mathbb{P}$ almost surely.

Theorem 3.5. *We assume that* $\forall t \in [0,T[$, $E_{\mathbb{P}}\left(\| \frac{\alpha(t,\mathscr{J})}{p(t,\mathscr{J})} \| + \| I_n + \frac{\beta(t,\mathscr{J})}{p(t^-,\mathscr{J})} \| \right) < +\infty$ *and that* $\left(I_n + \frac{\beta(t,\mathscr{J})}{p(t^-,\mathscr{J})} \right)$ *has positive components. Setting* $\rho_1(t) = E_{\mathbb{P}}\left(\frac{\alpha(t,\mathscr{J})}{p(t,\mathscr{J})} \mid \mathscr{G}_t \right)$ *and* $\rho_2(t) = E_{\mathbb{P}}\left(\left(I_n + \frac{\beta(t,\mathscr{J})}{p(t^-,\mathscr{J})} \right) \mid \mathscr{G}_t \right)$, *we assume that* $\int_0^T (\|\rho_1(t)\| + \|(\kappa.\rho_2)(t)\|)\,\mathrm{d}t < +\infty$ $\mathbb{P}$ *almost surely. Then* $\widetilde{W}(\cdot) := W(\cdot) - \int_0^{\cdot} \rho_1(s)\,\mathrm{d}s$ *is a* $(\mathrm{G}_T, \mathbb{P})$*-Brownian motion and* $\widetilde{M}(\cdot) := N(\cdot) - \int_0^{\cdot} (\kappa.\rho_2)(s)\,\mathrm{d}s$ *is the compensated process of a* $(\mathrm{G}_T, \mathbb{P})$*-Poisson process with intensity* $(\kappa.\rho_2)$.

Therefore the wealth equation (2.3) is meaningful and we are now able to construct a risk neutral probability measure for this progressive strong informed agent.

Definition 3.6.

$$l^* := \left((\Theta+\rho_1)^*, \left(\frac{\rho_2}{q} - I_n\right)^* \right),$$

$$Y := \mathscr{E}\left(\int_0^{\cdot} \left(-(\Theta+\rho_1)^*(s)\,\mathrm{d}\widetilde{W}(s) + \left(\frac{q(s)}{\rho_2(s)} - I_n\right)^* \mathrm{d}\widetilde{M}(s) \right) \right).$$

Y is a $(\mathrm{G}_T, \mathbb{P})$-local martingale and $d(Y^{-1})(t) = Y^{-1}(t^-) l^*(t)\,\mathrm{d}\widehat{S}(t)$.

Theorem 3.7. *We assume* 3.4 *and the assumptions of Theorem* 3.5. *We assume that Y defined in* (3.6) *is a* $(\mathrm{G}_T, \mathbb{P})$*-martingale. Then* $\widehat{\mathbb{P}} := Y(T)\mathbb{P}$ *is a risk neutral probability measure for the agent.* $\widehat{W}$ *is a* $(\mathrm{G}_T, \widehat{\mathbb{P}})$*-Brownian motion and* $\widehat{M}$ *is the compensated process of a* $(\mathrm{G}_T, \widehat{\mathbb{P}})$*-Poisson process with intensity* $(q.\kappa)$.

3.3. Weak information

Here we consider that the true model of the stock prices is partially observed. More precisely, the effective probability measure $\mathbb{P}$ of the market is unknown, but the agents know the risk neutral probability measure $\widehat{\mathbb{P}_0}$ of a non-insider (cf. Eq. (2.6)). The discounted prices are $(\mathrm{F}_T, \widehat{\mathbb{P}_0})$-local martingales. The insider knows there will be a release of information about the outcome of some variables L of the prices, but in contrary to a strong information, he does not observe it, therefore he anticipates its law. This agent is weakly informed on this $\mathscr{F}_T$-measurable random variable L, meaning that he only has the filtration F_T available (thus his strategy is F_T-admissible), but he anticipates the law of L under $\mathbb{P}$. Let $L : \Omega \longrightarrow \mathbb{R}^j$ be a $\mathscr{F}_T$-measurable random variable. As for a initial strong information, we assume that

Assumption 3.8. $\widehat{\mathbb{P}_0}(L \in \cdot | \mathscr{F}_t)(\omega) \sim \widehat{\mathbb{P}_0}(L \in \cdot)$ for all $t \in [0, T[$ for $\mathbb{P}$ almost all $\omega \in \Omega$.

For example, $L = \widehat{W}_1(T)$, $L = \mathbb{1}_{[a,b]}(P_i(T))$ or $L = \ln(P_{i_1}(T)) - \ln(P_{i_2}(T))$ satisfy Assumption 3.8. With L we associate a probability measure v on $\mathbb{R}^j$ which admits an almost surely positive bounded density ξ with respect to the law of L under $\widehat{\mathbb{P}_0}$. The insider knows that the law of L under the effective probability measure $\mathbb{P}$ is v. v is called a weak information on the functional L.

Proposition 3.9 (*Baudoin [6, Proposition 3]*). *On* $\mathscr{F}_T$, *there exists a unique probability measure* $\mathbb{P}^v$ *such that*

(i) *For any* $\mathscr{F}_T$*-measurable bounded random variable* X, $E_{\mathbb{P}^v}(X|L) = E_{\widehat{\mathbb{P}_0}}(X|L)$.
(ii) *The law of* L *under* $\mathbb{P}^v$ *is* v.
$\mathbb{P}^v$ *is called the minimal probability associated with the conditioning* (T, L, v).

We define ζ the density process of $\mathbb{P}^v$ with respect to $\widehat{\mathbb{P}_0}$

Definition 3.10.

$$\zeta(t) := \left(\frac{\mathrm{d}\mathbb{P}^{\nu}}{\mathrm{d}\widehat{\mathbb{P}_0}}\right)_{|\mathscr{F}_t},$$

$$Y := \frac{1}{\zeta}.$$

Since $\zeta = \frac{1}{Y}$ is a positive $(\mathrm{F}_T, \widehat{\mathbb{P}_0})$-local martingale, there exists a F_T-predictable process l such that $\mathrm{d}\frac{1}{Y}(t) = \frac{1}{Y}(t^-)l^*(t)\,\mathrm{d}\widehat{S}(t)$.

3.4. The optimization problem

Let $A < T$. A progressive or initial strong insider wants to maximize the mapping

$$(\pi, X) \rightarrow V(\pi, X) := E_{\mathbb{P}}[\ln(X(A)) \mid \mathscr{G}_0]$$

over all G_T-admissible strategies, whereas a weak insider wants to maximize

$$(\pi, X) \rightarrow V(\pi, X) := E_{\mathbb{P}^{\nu}}[\ln(X(A))]$$

over all F_T-admissible strategies.

We assume that the initial wealth is positive: $X(0) > 0$ $\mathbb{P}$ almost surely. Then, for those three types of insiders, the optimal wealth and portfolio are given by

$$\forall t \in [0, A], \quad \begin{cases} \beta(t)\widehat{X}(t) = X(0)\dfrac{1}{Y}(t), \\ \widehat{\pi}(t) = (\sigma^*(t))^{-1}\dfrac{Y(t)}{Y(t^-)}\,\widehat{X}(t)l(t) \end{cases} \tag{3.2}$$

with the corresponding processes $\frac{1}{Y}$ and l. We remark here that the optimal wealth is proportional to the process $\frac{1}{Y}$. Moreover, this process "summarizes" the information available for the agent. Therefore, the key of the following computations will be the computation of the process Y.

Remark. If the insider maximizes the expectation of the logarithm of both his consumption and his terminal wealth, the optimal strategy is given by a very similar formula

$$\forall t \in [0, A], \quad \begin{cases} \beta(t)\widehat{c}(t) = \dfrac{X(0)}{A+1}\dfrac{1}{Y}(t), \\ \beta(t)\widehat{X}(t) = \dfrac{X(0)(A+1-t)}{A+1}\dfrac{1}{Y}(t), \\ \widehat{\pi}(t) = (\sigma^*(t))^{-1}\dfrac{Y(t)}{Y(t^-)}\,\widehat{X}(t)l(t). \end{cases}$$

4. Comparison insider/non-insider's optimal strategies in some examples of side-information

First we give some technical results to compute the process $\frac{1}{Y}$ of an initial strong insider. Then, for two examples of initial strong side-information that can occur in a case of a merger between two companies, we compute the optimal strategy (wealth and part of the wealth invested in each asset) and we compare it to the optimal strategy of a non-insider.

4.1. Technical results for an initial strong insider

We recall here a result of Jeulin [19] (cf. [12, Lemma 3.1]).

Lemma 4.1. *We assume there exists $A \in]0,T[$ and a $\mathcal{F}_A \otimes \mathcal{E}$-measurable function $p(A,\cdot)$ such that, for all bounded $\mathcal{E}$-measurable function f,*

$$E_{\mathbb{P}}[f(L)|\mathcal{F}_A] = \int_E f(x)p(A,\omega,x)\mathbb{P}_L(\mathrm{d}x) \tag{4.1}$$

with $p(A,\omega,x)>0$ $\mathbb{P}\otimes\mathbb{P}_L$ a.s., where $\mathbb{P}_L$ is the law of L. Then the probability measure $\mathbb{Q}^L := \frac{1}{p(A,L)}\mathbb{P}$ is equivalent to $\mathbb{P}$, $\mathbb{Q}^L = \mathbb{P}$ on $\mathcal{F}_A$ and $\mathcal{F}_A$ and $\sigma(L)$ are independent under $\mathbb{Q}^L$.

Remark 4.2. If the assumptions of Lemma 4.1 are satisfied for a function $p(A,\cdot)$, then $\forall t<A$, $p(t,\cdot)$ defined by $p(t,\cdot) := E_{\mathbb{Q}^L}[p(A,\cdot)|\mathcal{G}_t]$ satisfies, for all bounded $\mathcal{E}$-measurable function f,

$$E_{\mathbb{P}}[f(L)|\mathcal{F}_t] = \int_E f(x)p(t,\omega,x)\mathbb{P}_L(\mathrm{d}x) \tag{4.2}$$

meaning that, $\forall t\leqslant A$, the measure $p(t,x)\mathbb{P}[L\in \mathrm{d}x]$ *on* $(E,\mathcal{E})$ is a version of the conditional distributions $\mathbb{P}[L\in \mathrm{d}x|\mathcal{F}_t]$.

Proof. First, we prove that if $Y \in L^\infty(\Omega,\mathcal{F}_A,\mathbb{Q}^L)$, then for all $t\leqslant A$, $E_{\mathbb{Q}^L}[Y|\mathcal{G}_t] = E_{\mathbb{P}}[Y|\mathcal{F}_t]$. Indeed, let $B\in\mathcal{F}_t$ and $g\in L^1(E,\mathbb{P}_L)$. Because of the independence of $\sigma(L)$ and $\mathcal{F}_A$ under $\mathbb{Q}^L$ and the fact that $\mathbb{Q}^L=\mathbb{P}$ on $\mathcal{F}_A$, we have

$$\begin{aligned} E_{\mathbb{Q}^L}[Yg(L)\mathbb{1}_B] &= E_{\mathbb{Q}^L}[Y\mathbb{1}_B]E_{\mathbb{Q}^L}[g(L)] \\ &= E_{\mathbb{Q}^L}[E_{\mathbb{P}}[Y\mathbb{1}_B|\mathcal{F}_t]]E_{\mathbb{Q}^L}[g(L)] \\ &= E_{\mathbb{Q}^L}[E_{\mathbb{P}}[Y|\mathcal{F}_t]\mathbb{1}_B g(L)]. \end{aligned}$$

Then we obtain (4.2) by taking $E_{\mathbb{Q}^L}[\cdot|\mathcal{G}_t]$ of the two members of relation (4.1) and by using this result with $Y = E_{\mathbb{P}}[f(L)|\mathcal{F}_A]$. □

Therefore, the process Z introduced in (3.1), satisfies $\forall t\in[0,A]$ $Z(t)=p(t,L)$. We recall that Y_0 (cf. (2.6)) denotes the process Y of a non-insider (for who $\rho_1^0 = 0_{\mathbb{R}^m}$ and $\rho_2^0 = I_n$).

Lemma 4.3. *The link between* Y, Y_0 *and* Z *is given by* $Y = \frac{Y_0}{Z}$.

Proof. We recall that $Z(t) = E_{\mathbb{Q}^L}[\frac{\mathrm{d}\mathbb{P}}{\mathrm{d}\mathbb{Q}^L}|\mathcal{G}_t]$ and that

$$\mathrm{d}Z(t) = Z(t^-)[\rho_1^*(t)\,\mathrm{d}W(t) + (\rho_2(t) - I_n)^*\,\mathrm{d}M(t)].$$

Therefore

$$\frac{1}{Z} = \mathscr{E}\left(\int_0^{\cdot}\left(-\rho_1^*(s)\,\mathrm{d}\widetilde{W}(s) + \left(\frac{1}{\rho_2(s)} - I_n\right)^*\mathrm{d}\widetilde{M}(s)\right)\right).$$

Furthermore

$$Y_0 = \mathscr{E}\left(\int_0^{\cdot}(-\Theta^*(s)\,\mathrm{d}W(s) + (q(s) - I_n)^*\,\mathrm{d}M(s))\right).$$

Since $\mathscr{E}(X)\mathscr{E}(Y) = \mathscr{E}(X + Y - [X, Y])$ and with the Definition 3.2 of Y, we obtain

$$\frac{Y_0}{Z} = \mathscr{E}\left(\int_0^{\cdot}\left(-(\Theta + \rho_1)^*(s)\,\mathrm{d}\widetilde{W}(s) + \left(\frac{q}{\rho_2}(s) - I_n\right)^*\mathrm{d}\widetilde{M}(s)\right)\right) = Y. \qquad \square$$

The two following Examples 4.2 and 4.3 are initial strong side-information. We imagine a case of a merger between two companies. The acquirer can buy the target in cash or in shares at the time T. If he does it in cash, the acquirer bids for a given price P_{bid} for each share of the target, at the maturity T. Therefore an insider will know that the spot of the target at time T will be in a range around P_{bid}: this situation corresponds to the information 4.2. If the acquisition is done with shares, the acquirer will exchange N_t shares of the target for N_a shares of the acquirer. Therefore an insider will know that at the acquisition time T, the ratio of the assets prices of the target and the acquirer will be equal to $\frac{N_t}{N_a}$: this time the situation corresponds to the information 4.3.

We denote σ_{iW} (respectively, σ_{iN}) the m first (respectively, the n last) components of σ_i the ith line of $\sigma : \sigma_i = (\sigma_{iW}, \sigma_{iN})$. For sake of simplicity, we assume κ constant.

4.2. There exists i, $1 \leqslant i \leqslant d$*, such that* $L = \mathbb{1}_{[a,b]}(P_i(T))$*,* $0 < a < b$

The insider knows whether the terminal price of the ith asset will be or not between a and b. We introduce $\Sigma_t := \int_t^T \|\sigma_{iW}(s)\|^2\,\mathrm{d}s$,

$$\begin{aligned} k_x(t) := \ln x - \ln(P_i(0)) &- \int_0^T \left(b_i(s) - \frac{1}{2}\|\sigma_{iW}(s)\|^2\right)\mathrm{d}s \\ &- \left(\int_0^t \sigma_{iW}(s)\,\mathrm{d}W(s) + \ln(1 + \sigma_{iN}(s))\,\mathrm{d}N(s)\right), \end{aligned} \tag{4.3}$$

$$(F_{t,T,k_j})^n := \left\{(t_{jl_j})_{\substack{1\leqslant j\leqslant n\\ 1\leqslant l_j\leqslant k_j}} \in \mathbb{R}^{k_1+\cdots+k_n} | \forall 1 \leqslant j \leqslant n,\ t \leqslant t_{jl_1} \leqslant \cdots \leqslant t_{jl_{k_j}} \leqslant T\right\} \tag{4.4}$$

and for $(t_{jl_j})_{\substack{1\leqslant j\leqslant n\\ 1\leqslant l_j\leqslant k_j}} \in (F_{t,T,k_j})^n$,

$$f_{(t_{jl_j})}(x) := \exp\left(\frac{-\left(x-\sum_{j=1}^{n}\sum_{l_j=1}^{k_j}\ln(1+\sigma_{ij}(t_{jl_j}))\right)^2}{2\Sigma_t}\right). \tag{4.5}$$

Let us introduce the scalar $a(t)$ for $t\in[0,A]$

$$a(t) := \frac{\prod_{j=1}^{n}\sum_{k_j\geqslant 0}\frac{e^{-\kappa_j(T-t)}}{\sqrt{2\pi\Sigma_t}}\kappa_j^{k_j}\int_{(F_{t,T,k_j})^n}\left(f_{(t_{jl_j})}(k_a(t))-f_{(t_{jl_j})}(k_b(t))\right)\prod_{l_j=1}^{k_j}\mathrm{d}t_{jl_j}\mathbb{1}_{(L=1)}}{\int_{\mathbb{R}}\prod_{j=1}^{n}\sum_{k_j\geqslant 0}\frac{e^{-\kappa_j(T-t)-\frac{x^2}{2\Sigma_t}}}{\sqrt{2\pi\Sigma_t}}\kappa_j^{k_j}\int_{(F_{t,T,k_j})^n}\mathbb{1}_{(k_a(t)\leqslant x+\sum_{j=1}^{n}\sum_{l_j=1}^{k_j}\ln(1+\sigma_{ij}(t_{jl_j}))\leqslant k_b(t))}\mathrm{d}x\prod_{l_j=1}^{k_j}\mathrm{d}t_{jl_j}}$$
$$+\frac{\prod_{j=1}^{n}\sum_{k_j\geqslant 0}\frac{e^{-\kappa_j(T-t)}}{\sqrt{2\pi\Sigma_t}}\kappa_j^{k_j}\int_{(F_{t,T,k_j})^n}\left(f_{(t_{jl_j})}(k_b(t))-f_{(t_{jl_j})}(k_a(t))\right)\prod_{l_j=1}^{k_j}\mathrm{d}t_{jl_j}\mathbb{1}_{(L=0)}}{\int_{\mathbb{R}}\prod_{j=1}^{n}\sum_{k_j\geqslant 0}\frac{e^{-\kappa_j(T-t)-\frac{x^2}{2\Sigma_t}}}{\sqrt{2\pi\Sigma_t}}\kappa_j^{k_j}\int_{(F_{t,T,k_j})^n}\mathbb{1}_{[k_a(t),k_b(t)]^C}(x+\sum_{j=1}^{n}\sum_{l_j=1}^{k_j}\ln(1+\sigma_{ij}(t_{jl_j})))\,\mathrm{d}x\prod_{l_j=1}^{k_j}\mathrm{d}t_{jl_j}}.$$

Proposition 4.4. *If the insider knows $L=\mathbb{1}_{[a,b]}(P_i(T))$ for some i ($0<a<b$ and $1\leqslant i\leqslant d$), then $Z(t)=p(t,L)=p(t,1)\mathbb{1}_{(L=1)}+p(t,0)\mathbb{1}_{(L=0)}$ with*

$$p(t,1) = \frac{\int_{\mathbb{R}}\prod_{j=1}^{n}\sum_{k_j\geqslant 0}\frac{e^{-\kappa_j(T-t)-\frac{x^2}{2\Sigma_t}}}{\sqrt{2\pi\Sigma_t}}\kappa_j^{k_j}\int_{(F_{t,T,k_j})^n}\mathbb{1}_{\left(k_a(t)\leqslant x+\sum_{j=1}^{n}\sum_{l_j=1}^{k_j}\ln(1+\sigma_{ij}(t_{jl_j}))\leqslant k_b(t)\right)}\mathrm{d}x\prod_{l_j=1}^{k_j}\mathrm{d}t_{jl_j}}{\int_{\mathbb{R}}\prod_{j=1}^{n}\sum_{k_j\geqslant 0}\frac{e^{-\kappa_jT-\frac{x^2}{2\Sigma_0}}}{\sqrt{2\pi\Sigma_0}}\kappa_j^{k_j}\int_{(F_{0,T,k_j})^n}\mathbb{1}_{\left(k_a(0)\leqslant x+\sum_{j=1}^{n}\sum_{l_j=1}^{k_j}\ln(1+\sigma_{ij}(t_{jl_j}))\leqslant k_b(0)\right)}\mathrm{d}x\prod_{l_j=1}^{k_j}\mathrm{d}t_{jl_j}},$$

$$p(t,0) = \frac{\int_{\mathbb{R}}\prod_{j=1}^{n}\sum_{k_j\geqslant 0}\frac{e^{-\kappa_j(T-t)-\frac{x^2}{2\Sigma_t}}}{\sqrt{2\pi\Sigma_t}}\kappa_j^{k_j}\int_{(F_{t,T,k_j})^n}\mathbb{1}_{[k_a(t),k_b(t)]^C}(x+\sum_{j=1}^{n}\sum_{l_j=1}^{k_j}\ln(1+\sigma_{ij}(t_{jl_j})))\,\mathrm{d}x\prod_{l_j=1}^{k_j}\mathrm{d}t_{jl_j}}{\int_{\mathbb{R}}\prod_{j=1}^{n}\sum_{k_j\geqslant 0}\frac{e^{-\kappa_jT-\frac{x^2}{2\Sigma_0}}}{\sqrt{2\pi\Sigma_0}}\kappa_j^{k_j}\int_{(F_{0,T,k_j})^n}\mathbb{1}_{[k_a(0),k_b(0)]^C}(x+\sum_{j=1}^{n}\sum_{l_j=1}^{k_j}\ln(1+\sigma_{ij}(t_{jl_j})))\mathrm{d}x\prod_{l_j=1}^{k_j}\mathrm{d}t_{jl_j}},$$

$$\rho_1(t)=a(t)\sigma_{iW}(t) \quad \textit{and} \quad \rho_2(t)-\mathbb{1}_n=a(t)\ln(1+\sigma_{iN}(t)).$$

Proof. cf. Appendix A.4.

Remark. For a purely diffusive market model, the formulas are more simple:

$$p(t,1)=\frac{F(t,k_b(t))-F(t,k_a(t))}{F(0,k_b(0))-F(0,k_a(0))}, \quad p(t,0)=\frac{1-F(t,k_b(t))+F(t,k_a(t))}{1-F(0,k_b(0))+F(0,k_a(0))},$$

$$a(t) = \frac{\frac{\partial F}{\partial x}(t, k_b(t)) - \frac{\partial F}{\partial x}(t, k_a(t))}{1 - F(t, k_b(t)) + F(t, k_a(t))} \mathbb{1}_{(L=0)} - \frac{\frac{\partial F}{\partial x}(t, k_b(t)) - \frac{\partial F}{\partial x}(t, k_a(t))}{F(t, k_b(t)) - F(t, k_a(t))} \mathbb{1}_{(L=1)},$$

where $F(t, \cdot)$ is the cumulative distribution function of a Gaussian $\mathcal{N}(0, \Sigma_t)$ with mean 0 and variance $\Sigma_t = \int_t^T \|\sigma_i(s)\|^2 \, ds$.

We will compare the insider's strategy to the optimal strategy of a non-insider.

Proposition 4.5. *If the insider knows $L = \mathbb{1}_{[a,b]}(P_i(T))$ for some i $(0 < a < b$ and $1 \leqslant i \leqslant d)$, then if t is not a time of jump*

$$\frac{\widehat{\pi}}{\widehat{X}}(t) - \frac{\widehat{\pi_0}}{\widehat{X_0}}(t) = (\sigma^*(t))^{-1} \left(\sigma_{iW}(t), \frac{\ln(1 + \sigma_{iN}(t))}{q} \right)^* a(t).$$

In a purely diffusive market model, the part of the wealth invested in the assets j $(1 \leqslant j \leqslant d$ and $j \neq i)$ are the same for this insider and a non-insider.

Proof. Let t be not a time of jump. Then yields $\frac{\widehat{\pi}}{\widehat{X}}(t) = (\sigma^*(t))^{-1} l(t)$. Therefore the difference between the part of the wealth invested in the four assets by an insider and a non-insider is

$$\frac{\widehat{\pi}}{\widehat{X}}(t) - \frac{\widehat{\pi_0}}{\widehat{X_0}}(t) = (\sigma^*(t))^{-1}(l(t) - l_0(t)) = (\sigma^*(t))^{-1} \left((\rho_1)^*, \left(\frac{\rho_2 - \mathbb{1}_2}{q} \right)^* \right)^* (t)$$
$$= (\sigma^*(t))^{-1} \left(\sigma_{iW}(t), \frac{\ln(1 + \sigma_{iN}(t))}{q} \right)^* a(t),$$

where $a(t)$ is a scalar. In a purely diffusive market model,

$$\frac{\widehat{\pi}}{\widehat{X}}(t) - \frac{\widehat{\pi_0}}{\widehat{X_0}}(t) = (\sigma^*(t))^{-1} \sigma_i^*(t) a(t) = (0, \ldots, 0, \overbrace{a(t)}^{\text{component } i}, 0, \ldots, 0)^*$$

because $(\sigma^*(t))^{-1} \sigma_1^*(t)$ is the ith row of $(\sigma^*(t))^{-1} \sigma^*(t) = ((\sigma(t)\sigma(t))^{-1})^*$ which is the identity $d \times d$-matrix. Therefore, in a purely diffusive market model, the part of the wealth invested in the assets j $(1 \leqslant j \leqslant d$ and $j \neq i)$ are the same for an insider and a non-insider. But this is not true in a mixed diffusive-jump market model (cf. Figs. 3 and 4, for a simulation in a diffusive-jump market model with four risky assets) because $(\sigma^*(t))^{-1}(\sigma_{iW}(t), \frac{\ln(1+\sigma_{iN}(t))}{q})^* \neq (0, \ldots, 0, \overbrace{1}^{\text{component } i}, 0, \ldots, 0)^*$. From a mathematical point of view, this difference comes from the expression of the logarithm of the prices, because the ln function appears in the jump part. From an heuristic point of view, it seems that the insider uses his side-information to learn about the jump process and reflect it in all the assets. $\square$

4.3. $L = \ln(P_{i_1}(T)) - \ln(P_{i_2}(T))$

The insider knows the ratio of assets i_1 and i_2 prices at time T. We introduce

$$m_t := \int_0^T \left((b_{i_1} - b_{i_2}) - \frac{1}{2}(\|\sigma_{i_1 W}\|^2 - \|\sigma_{i_2 W}\|^2) \right)(s)\,\mathrm{d}s + \int_0^t (\sigma_{i_1 W} - \sigma_{i_2 W})(s)\,\mathrm{d}W(s) + \int_0^t \ln\left(\frac{1+\sigma_{i_1 N}}{1+\sigma_{i_2 N}}\right)(s)\,\mathrm{d}N(s).$$

Then we have

$$L = m_t + \int_t^T (\sigma_{i_1 W} - \sigma_{i_2 W})(s)\,\mathrm{d}W(s) + \int_t^T \ln\left(\frac{1+\sigma_{i_1 N}}{1+\sigma_{i_2 N}}\right)(s)\,\mathrm{d}N(s).$$

$(F_{t,T,k_j})^n$ and $f_{(t_{jl_j})}$ are defined in (4.4) and (4.5).

In this subsection, $\Sigma_t := \int_t^T \|\sigma_{i_1 W} - \sigma_{i_2 W}\|^2(s)\,\mathrm{d}s$ and

$$a(t) := \frac{\prod_{j=1}^n \sum_{k_j \geq 0} \frac{1}{\Sigma_t} \kappa_j^{k_j} \int_{(F_{t,T,k_j})^n} f_{(t_{jl_j})}(L - m_t)\left(L - m_t - \sum_{j=1}^n \sum_{l_j=1}^{k_j} \ln\left(\frac{1+\sigma_{i_1 j}}{1+\sigma_{i_2 j}}\right)(t_{jl_j})\right) \prod_{l_j=1}^{k_j} \mathrm{d}t_{jl_j}}{\prod_{j=1}^n \sum_{k_j \geq 0} \kappa_j^{k_j} \int_{(F_{t,T,k_j})^n} f_{(t_{jl_j})}(L - m_t) \prod_{l_j=1}^{k_j} \mathrm{d}t_{jl_j}}.$$

Using similar methods as for 4.2, we obtain

Proposition 4.6. *If the insider knows $L = \ln(P_{i_1}(T)) - \ln(P_{i_2}(T))$, then $Z(t) = p(t, L)$ with*

$$p(t,x) = \frac{\prod_{j=1}^n \sum_{k_j \geq 0} \frac{\mathrm{e}^{-\kappa_j (T-t)}}{\sqrt{2\pi\Sigma_t}} \kappa_j^{k_j} \int_{(F_{t,T,k_j})^n} \mathrm{e}^{\left(\frac{-\left(x - m_t - \sum_{j=1}^n \sum_{l_j=1}^{k_j} \ln\left(\frac{1+\sigma_{i_1 j}}{1+\sigma_{i_2 j}}\right)(t_{jl_j})\right)^2}{2\Sigma_t}\right)} \prod_{l_j=1}^{k_j} \mathrm{d}t_{jl_j}}{\prod_{j=1}^n \sum_{k_j \geq 0} \frac{\mathrm{e}^{-\kappa_j T}}{\sqrt{2\pi\Sigma_0}} \kappa_j^{k_j} \int_{(F_{0,T,k_j})^n} \mathrm{e}^{\left(\frac{-\left(x - m_0 - \sum_{j=1}^n \sum_{l_j=1}^{k_j} \ln\left(\frac{1+\sigma_{i_1 j}}{1+\sigma_{i_2 j}}\right)(t_{jl_j})\right)^2}{2\Sigma_0}\right)} \prod_{l_j=1}^{k_j} \mathrm{d}t_{jl_j}}.$$

$$\rho_1(t) = a(t)(\sigma_{i_1 W} - \sigma_{i_2 W})(t) \quad \textit{and} \quad \rho_2(t) - \mathbb{1}_n = a(t) \ln\left(\frac{1+\sigma_{i_1 N}(t)}{1+\sigma_{i_2 N}(t)}\right).$$

Remark. For a purely diffusive market model, the formulas are more simple

$$Z(t) = p(t,L) = \frac{\sqrt{2\pi\Sigma_0}}{\sqrt{2\pi\Sigma_t}} \exp\left(\frac{-(\int_t^T (\sigma_{i_1} - \sigma_{i_2})(s)\,\mathrm{d}W(s))^2}{2\Sigma_t} + \frac{(\int_0^T (\sigma_{i_1} - \sigma_{i_2})(s)\,\mathrm{d}W(s))^2}{2\Sigma_0} \right),$$

$$a(t) = \frac{\int_t^T (\sigma_{i_1} - \sigma_{i_2})(s)\,\mathrm{d}W(s)}{\Sigma_t} \quad \text{where } \Sigma_t := \int_t^T \|\sigma_{i_1} - \sigma_{i_2}\|^2(s)\,\mathrm{d}s. \quad \square$$

The following proposition compare this insider's strategy to the optimal strategy of a non-insider.

Proposition 4.7. *If the insider knows* $L=\ln(P_{i_1}(T))-\ln(P_{i_2}(T))$, *then if* t *is not a time of jump*

$$\frac{\widehat{\pi}}{\widehat{X}}(t)-\frac{\widehat{\pi_0}}{\widehat{X_0}}(t)=(\sigma^*(t))^{-1}\left((\sigma_{i_1W}-\sigma_{i_2W})(t),\frac{1}{q}\ln\left(\frac{1+\sigma_{i_1N}}{1+\sigma_{i_2N}}\right)(t)\right)^* a(t).$$

In a purely diffusive market model, the part of the wealth invested in the bond and the assets j $(1\leqslant j\leqslant d,\ j\neq i_1$ *and* $j\neq i_2)$ *are the same for this insider and a non-insider.*

Proof. Let t be not a time of jump. Then the difference between the part of the wealth invested in the four assets by an insider and a non-insider is

$$\frac{\widehat{\pi}}{\widehat{X}}(t)-\frac{\widehat{\pi_0}}{\widehat{X_0}}(t)=(\sigma^*(t))^{-1}\left((\sigma_{i_1W}-\sigma_{i_2W})(t),\frac{1}{q}\ln\left(\frac{1+\sigma_{i_1N}}{1+\sigma_{i_2N}}\right)(t)\right)^* a(t),$$

where $a(t)$ is a scalar. In a purely diffusive market model,

$$\begin{aligned}&(\sigma^*(t))^{-1}(\sigma_{i_1W}-\sigma_{i_2W})^*(t)\\ &\quad=(0,\dots,0,\overbrace{1}^{\text{component } i_1},0,\dots,0,\overbrace{-1}^{\text{component } i_2},0,\dots,0)^*.\end{aligned}$$

Since the fraction of the wealth invested in the bond is equal to $1-\sum_{i=1}^{d}\frac{\widehat{\pi_i}}{\widehat{X}}$, the part of the wealth invested in the bond and the assets j $(1\leqslant j\leqslant d,\ j\neq i_1$ and $j\neq i_2)$ are the same for an insider and a non-insider. But this is not true in a mixed diffusive-jump market model (cf. Figs. 6 and 9 for a simulation in a diffusive-jump market model with four risky assets). $\square$

5. Simulations

We simulate the strategy of an insider optimizing his terminal wealth on $[0,A]$ $(A<T)$. Here are the data we have used for our simulations. $A=0.95$ and $T=1$ and the initial wealth is equal to 1: $X(0)=1$. Both Brownian motion and Poisson process are 2-dimensional: $m=n=2$. The intensity of the Poisson process is $\kappa=(3,2)$. We choose constant market coefficients, but the simulations could be easily extended with time-varying market coefficients. The annual interest rate is 0.02: $r(t)=0.02$ for all t.

$$\text{The drift } b(t)=\begin{pmatrix}0.15\\0.1\\0.084\\0.1\end{pmatrix}\quad\forall t\in[0,T].$$

$$\text{The volatility } \sigma(t)=\begin{pmatrix}-0.4&-0.1&-0.15&0.17\\-0.09&-0.4&-0.03&0.035\\0.048&-0.12&0.1&-0.12\\0.075&0.26&0.31&-0.28\end{pmatrix}\quad\forall t\in[0,T].$$

Therefore $q = (1.3496, 1.9731)$ has positive components. The first two rows of σ are the components of the volatility on the diffusion part (in the practice the standard variation of a purely diffusive asset is in the range [0.1,0.4]). The last two rows of σ are the components of the volatility on the jump part. We choose σ_{3i} and σ_{4i} ($i = 1, \dots, 4$) in the same range and of opposite sign so that the price of asset i does not increase (or decrease) at each jump time. The prices of the assets at time $t = 0$ are $1 : P_i(0) = 1,\ i = 0, \dots, 4$.

We simulate the optimal strategy (wealth and part of the wealth invested in each asset) of a non-insider and an insider. In 5.1 (respectively, in 5.2), the insider has the side-information of Example 4.2 (respectively, of Example 4.3). Our aim is to determine qualitatively the optimal strategy and to show that the simulated optimal investments are in agreement with what we could have expected. Finally, we compare in 5.3 the processes $\frac{1}{Y}$ for one example of each type of side-information. Here, we want to show that for the same noise dynamics, the wealths are plausibly ordered in the sense of increasing relevance of side-information.

5.1. $L = \mathbb{1}_{[a,b]}(P_1(T)),\ 0 < a < b$

cf. Appendix A.1, p. 18 for the figures. In our simulation,

$$E_{\mathbb{P}}(P_1(T)) = P_1(0)\exp(T(b_1 - \tfrac{1}{2}\|\sigma_{1W}\|^2))(1 + \sigma_{13})^{\kappa_1 T}(1 + \sigma_{14})^{\kappa_2 T} = 0.8971.$$

We notice on Fig. 1 that $\frac{1}{Y}$ is bigger than $\frac{1}{Y_0}$, therefore the wealth of the insider is larger than those of a non-insider. If $P_1(T)$ is in $[a, b]$, the more $[a, b]$ is away from its expectation $E_{\mathbb{P}}(P_1(T))$, the better the gain is. We can explain this by the fact that in this case, the insider knows an event that occurs with a low probability, therefore his side-information is important. Thus, the more relevant a side-information is, the bigger the process $\frac{1}{Y}$ (and therefore the optimal wealth) is. We notice also that the bigger $P_1(T)$ is, the bigger $\frac{1}{Y}$ and $\frac{1}{Y_0}$ are.

Furthermore, as noticed in Proposition 4.5, the parts of the wealth invested in each asset by an insider and a non-insider are different (cf. Figs. 2–4). If t is not a time of jump, the fact that the market coefficients r, b and σ are constant implies that $\frac{\widehat{\pi_0}}{X_0}$ of a non-insider is constant, and that the parts of wealth invested by the insider in the bond and each of the four risky assets are proportional to each other.

We notice that the insider's strategy in our simulations is very rational and is as we can expect: the strategy depends on the value of L and if $E_{\mathbb{P}}(P_1(T))$ is in $[a, b]$ or not.

- *Case* 1: *$P_1(T) \notin [a, b]$ and $E_{\mathbb{P}}(P_1(T)) \notin [a, b]$*: The side-information is not very relevant. The insider's strategy is the following.
  If $a < b < E_{\mathbb{P}}(P_1(T))$, then $\mathbb{P}[P_1(T) > E_{\mathbb{P}}(P_1(T))] > \mathbb{P}[P_1(T) < E_{\mathbb{P}}(P_1(T))]$. Thus the insider (compared to a non-insider) invests more in the first asset and less in the bond, regardless of the first asset price. Conversely if $E_{\mathbb{P}}(P_1(T)) < a < b$, the insider invests less in the first asset and more in the bond.
- *Case* 2: *$P_1(T) \notin [a, b]$ and $E_{\mathbb{P}}(P_1(T)) \in [a, b]$*: The insider follows the first asset price. At time t, if $P_1(t) < E_{\mathbb{P}}(P_1(T))$ the insider thinks that $P_1(T) < a$ and invests

less in the first asset and more in the bond. Conversely if $P_1(t) > E_{\mathbb{P}}(P_1(T))$, the insider invests more in the first asset and less in the bond.

- *Case* 3: $P_1(T) \in [a, b]$ *and* $E_{\mathbb{P}}(P_1(T)) \notin [a, b]$: The side-information is very relevant, the insider knows exactly if $P_1(T)$ is bigger or not than its expectation. His gain is important. If $P_1(T) > E_{\mathbb{P}}(P_1(T))$, he invests more in the first asset. Conversely if $P_1(T) < E_{\mathbb{P}}(P_1(T))$, he invests less in the first asset.
- *Case* 4: $P_1(T) \in [a, b]$ *and* $E_{\mathbb{P}}(P_1(T)) \in [a, b]$: The insider follows the first asset price. At time t, he invests more in the first asset if $P_1(t)$ is near the bottom range a (because he knows that $P_1(T) > a$ and thus P_1 will increase) and if $P_1(t)$ is near the top range b he invests less in the first asset.

5.2. $L = \ln(P_1(T)) - \ln(P_2(T))$

cf. Appendix A.2, p. 21 for the figures. In our simulation,

$$\begin{aligned} E_{\mathbb{P}}(\ln(P_1(T)) - \ln(P_2(T))) &= \left(T(b_1 - b_2) - \frac{T}{2}(\|\sigma_{1W}\|^2) - \left(\frac{T}{2}\|\sigma_{2W}\|^2\right)\right) \\ &\quad + \kappa_1 T \ln\left(\frac{1+\sigma_{13}}{1+\sigma_{23}}\right) + \kappa_2 T \ln\left(\frac{1+\sigma_{14}}{1+\sigma_{24}}\right) \\ &= -0.1019. \end{aligned}$$

We notice on Fig. 5 that $\frac{1}{Y}$ is bigger than $\frac{1}{Y_0}$. The insider strategy on assets 1 and 2 depends on the position of $\ln(\frac{P_1(t)}{P_2(t)})$ (the ratio of assets 1 and 2 at time t) compared to the value of L (which is the same ratio at time T). If $\ln(\frac{P_1}{P_2})(t) > L$ (cf. Fig. 7 for the graph of $\ln(\frac{P_1}{P_2})$), the fraction of the wealth at time t invested in asset 1 (respectively, in asset 2) is smaller (respectively, bigger) than those of a non-insider. Indeed the insider knows that at maturity time T, the ratio $\ln(\frac{P_1}{P_2})$ will be smaller than it is now, meaning that the price of asset 1 will decrease compared to the price of asset 2. Inversely if $\ln(\frac{P_1}{P_2})(t) < L$, the insider invests more in asset 1 and less in asset 2 (cf. Fig. 8).

Furthermore, as noticed in Proposition 4.7, the parts of the wealth invested in each asset by an insider and a non-insider are different (cf. Figs. 6–9).

5.3. *Comparison between the three types of side-information*

According to Eq. (3.2), the process Y is sufficient to explicit the optimal strategy. That is why we focus our attention on this process. We would like to compare the processes Y for the three types of side-information of Section 3 and show that the more relevant a side-information is, the bigger the process $\frac{1}{Y}$ (and thus the optimal wealth) is. To do that, we simulate simultaneously (i.e. for the same realization of ω) the process $\frac{1}{Y}$ of four agents.

- First, we simulate the process $\frac{1}{Y_0}$ of a non-insider.
- The first insider is an initial strong one. Since the beginning he knows the functional $L_1 = W_1(T)$. Since the law of L_1 given $\mathscr{F}_t$ is a Gaussian law with mean

$W_1(t)$ and variance $(T-t)$,

$$p(t,x)=\frac{\sqrt{T}}{\sqrt{T-t}}\exp\left(-\frac{(x-W_1(t))^2}{2(T-t)}+\frac{x^2}{2T}\right)$$

and

$$\frac{1}{Y_1}=\frac{Z_1}{Y_0}\quad\text{with}$$
$$Z_1(t)=p(t,L_1)=\frac{\sqrt{T}}{\sqrt{T-t}}\exp\left(-\frac{(W_1(T)-W_1(t))^2}{2(T-t)}+\frac{W_1(T)^2}{2T}\right).$$

- The second insider is a progressive strong one, at time t (for all $0\leqslant t\leqslant T$) he knows the functional $L_2(t)=2W_1(T)+B_{T-t}$ where B is a Brownian motion independent of F_T. Then $\rho_2^2=0$, $\rho_3^2=1$, $\rho_4^2=1$ and

$$\rho_1^2(t)=\frac{2(2(W_1(T)-W_1(t))+B_{(T-t)})}{5(T-t)}$$

$$\frac{1}{Y_2}(t)=\frac{1}{Y_0}(t)\mathscr{E}\left(\int_0^t\rho_1^2(s)\,\mathrm{d}W_1(s)\right).$$

- The third insider is a weak insider, he anticipes that the law of $L_3=\widehat{W}_1(T)$ will be a Gaussian law with mean $m=\int_0^T\sum_{j=1}^d(\sigma)_{1j}^{-1}(s)(b_j-r)(s)\,\mathrm{d}s$ and variance T. Then (cf. [5])

$$\frac{1}{Y_3}(t)=\int\frac{1}{\sqrt{2\pi(T-t)}}\exp\left(\frac{y^2}{2T}-\frac{(y-\widehat{W}_1(t))^2}{2(T-t)}-\frac{(y-m)^2}{2T}\right)\mathrm{d}y$$

  that we simulate by mean of a Monte-Carlo method.

Results: We notice (cf. Fig. 10) that $\frac{1}{Y_0}\leqslant\frac{1}{Y_3}\leqslant\frac{1}{Y_2}\leqslant\frac{1}{Y_1}$: the more informed an agent is, the bigger his process $\frac{1}{Y}$. $\frac{1}{Y_0}$ and $\frac{1}{Y_3}$ are very close together and the distance between them stay quite constant as time evolves, whereas $\frac{1}{Y_2}$ and $\frac{1}{Y_1}$ get further away from $\frac{1}{Y_0}$ as time T approaches.

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Appendix A

A.1. $L = \mathbb{1}_{[a,b]}(P_1(T))$

In this simulation, $a = 0.7$, $b = 1.1$, $P_1(T) = 1.5422$. The first Poisson process jumps at time 0.7039 and 0.9695, the second at time 0.8280 and 0.9129. cf. Figs. 1–4.

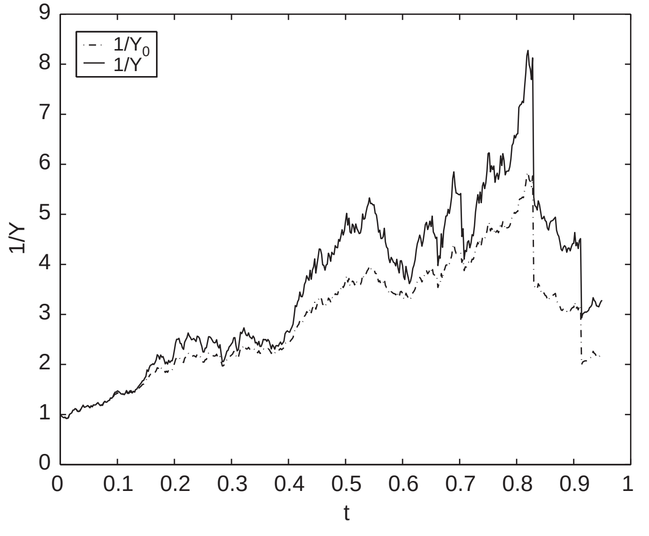

Fig. 1. Processes $\frac{1}{Y_0}$ and $\frac{1}{Y}$: $a = 0.7$, $b = 1.1$, $P_1(T) = 1.5422$.

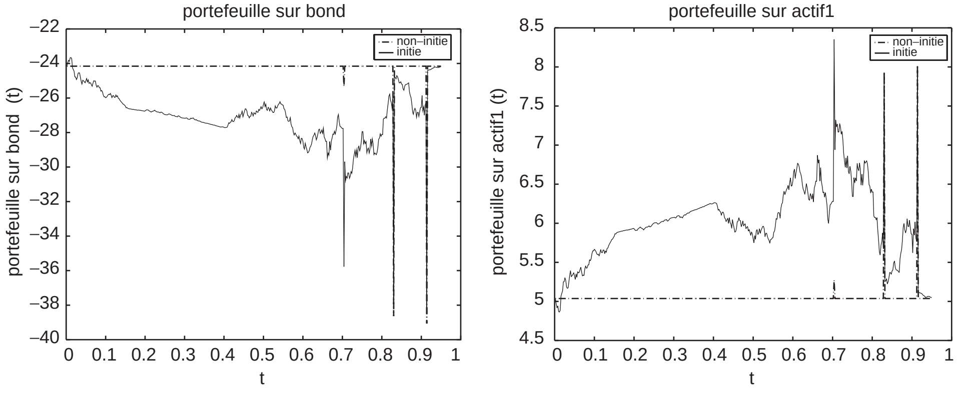

Fig. 2. Portfolio on the bond and portfolio on asset 1: $a = 0.7$, $b = 1.1$, $P_1(T) = 1.5422$.

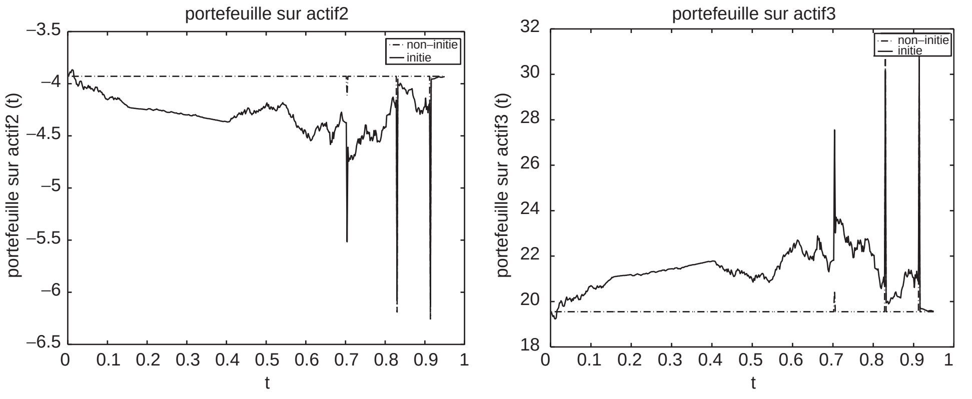

Fig. 3. Portfolio on asset 2 and portfolio on asset 3: $a = 0.7$, $b = 1.1$, $P_1(T) = 1.5422$.

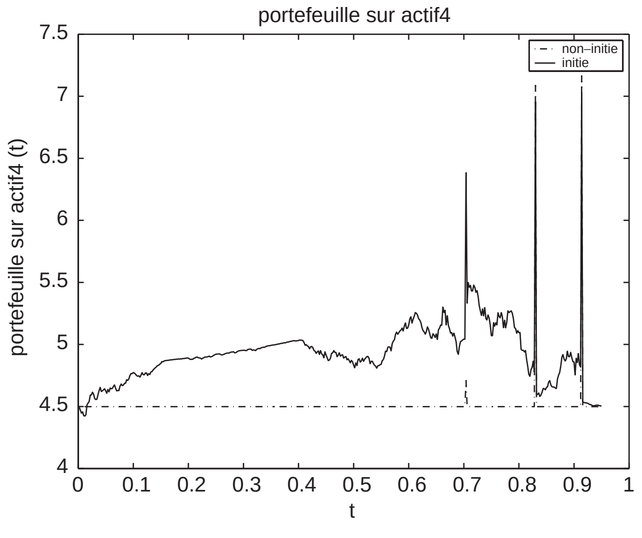

Fig. 4. Portfolio on asset 4: $a = 0.7$, $b = 1.1$, $P_1(T) = 1.5422$.

A.2. $L = \ln(P_1(T)) - \ln(P_2(T))$

In this simulation, $L = -0.316$. The first Poisson process jumps at time 0.0227 and 0.466, the second at time 0.1875 and 0.718. cf. Figs. 5–9.

A.3. Comparison between the three types of side-information

In this simulation, the first Poisson process jumps at time 0.0369, 0.0567, 0.4211, 0.6968 and 0.9477, the second at time 0.0111, 0.2338. cf. Fig. 10.

We recall that $\beta(t)\widehat{X}(t) = X(0)\frac{1}{Y}(t)$.

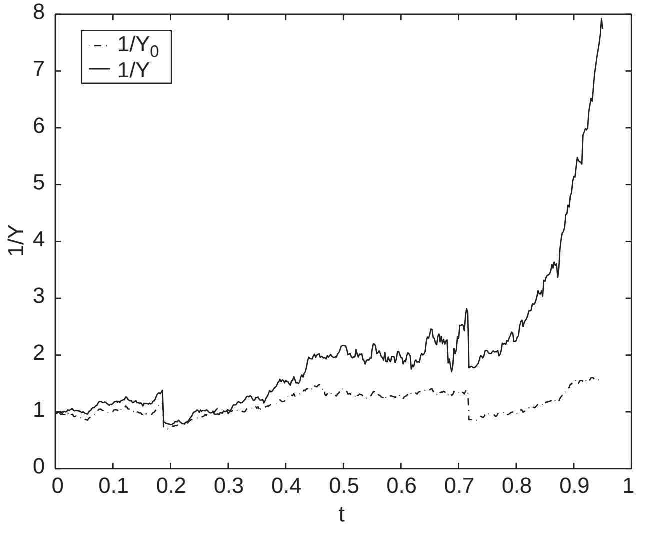

Fig. 5. Processes $\frac{1}{Y_0}$ and $\frac{1}{Y}$: $L = -0.316$.

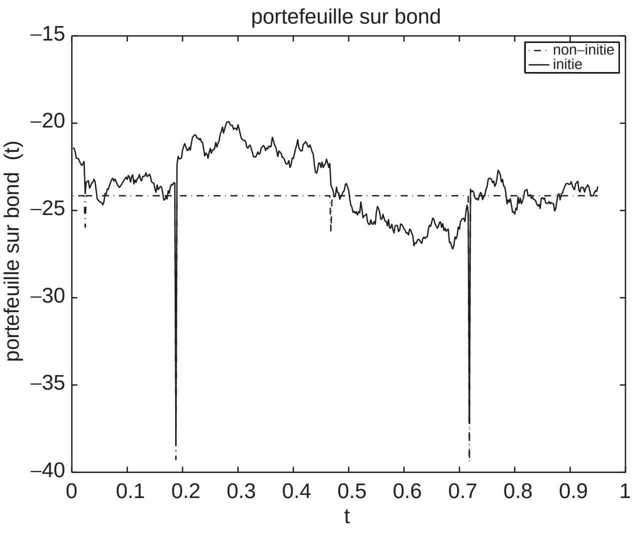

Fig. 6. Portfolio on the bond: $L = -0.316$.

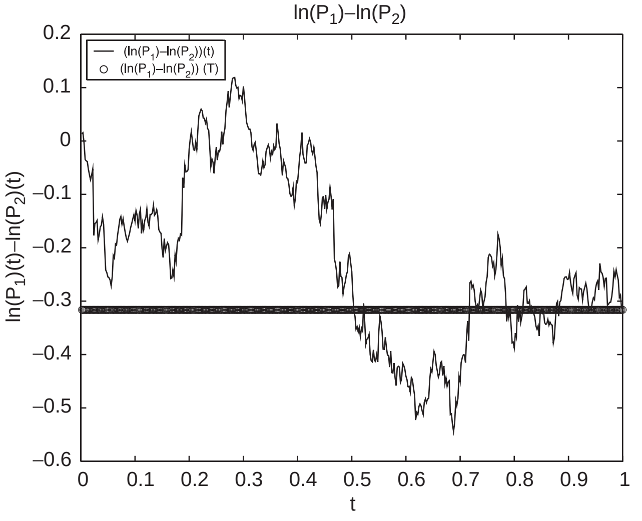

Fig. 7. $\ln(P_1(t)) - \ln(P_2(t))$: $L = -0.316$.

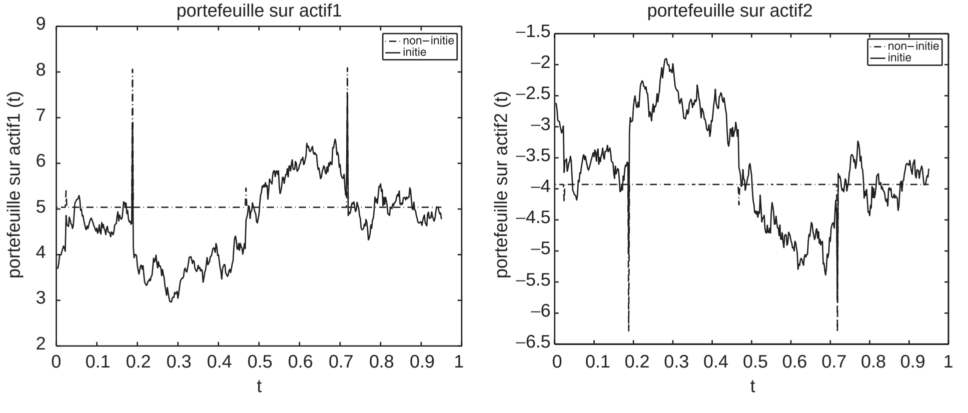

Fig. 8. Portfolio on asset 1 and portfolio on asset 2: $L = -0.316$.

A.4. Proof of Proposition 4.4

The insider knows whether the terminal price of the ith asset will be or not between a and b: $L = \mathbb{1}_{[a,b]}(P_i(T))$.

$$\ln(P_i(T)) = \ln(P_i(0)) + \int_0^T \left(b_i(s) - \frac{1}{2}\|\sigma_{iW}(s)\|^2\right) \mathrm{d}s + \int_0^T \sigma_{iW}(s)\,\mathrm{d}W(s)$$
$$+ \int_0^T \ln(1+\sigma_{iN}(s))\,\mathrm{d}N(s),$$

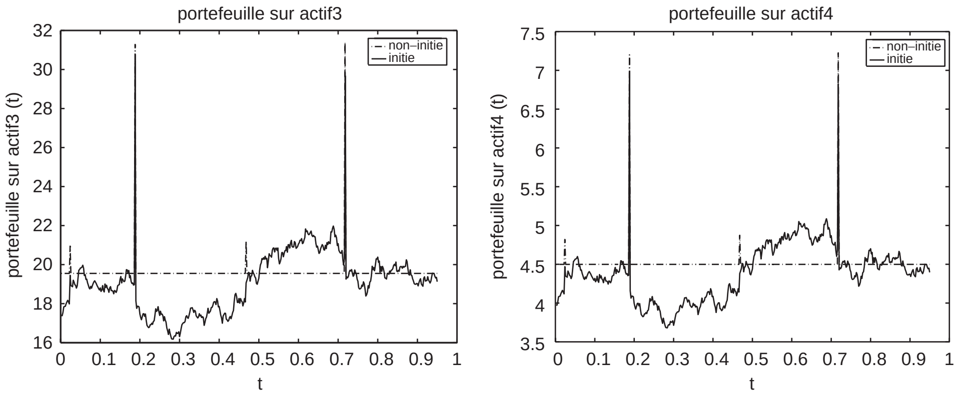

Fig. 9. Portfolio on asset 3 and portfolio on asset 4: $L = -0.316$.

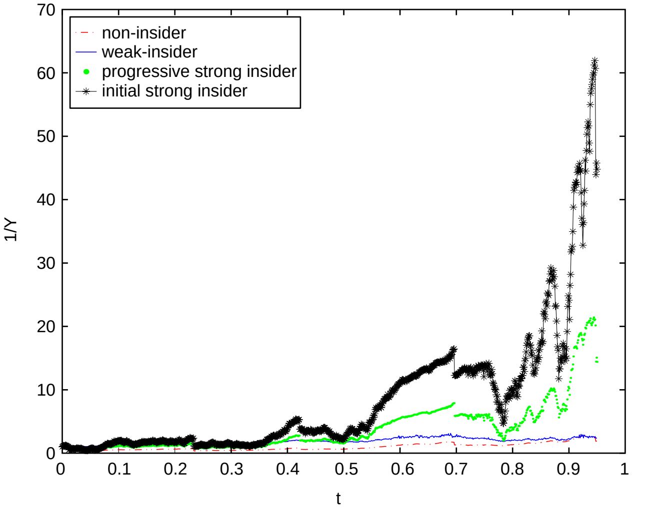

Fig. 10. Comparison of the processes $\frac{1}{Y}$ for four agents.

$$\begin{aligned}\mathbb{P}[L = 1|\mathscr{F}_t] &= \mathbb{P}[\ln a \leqslant \ln(P_i(T)) \leqslant \ln b|\mathscr{F}_t] \\ &= \mathbb{P}\left[k_a(t) \leqslant \int_t^T \sigma_{iW}(s)\,\mathrm{d}W(s) + \ln(1+\sigma_{iN}(s))\,\mathrm{d}N(s) \leqslant k_b(t)\right]\end{aligned}$$

because $\int_t^T \sigma_{iW}(s)\,\mathrm{d}W(s) + \ln(1+\sigma_{iN}(s))\,\mathrm{d}N(s)$ is independent of $\mathscr{F}_t$ and $\forall x \in \mathbb{R}$, k_x (defined in (4.3)) is $\mathscr{F}_t$-measurable. What is the law of $\int_t^T \sigma_{iW}(s)\,\mathrm{d}W(s) + \ln(1+\sigma_{iN}(s))\,\mathrm{d}N(s)$? Since N_j is a Poisson process with intensity κ_j, if τ_i^j is the ith time of jump of N_j, the random variables $(\tau_{i+1}^j - \tau_i^j)_{i \geqslant 1}$ are independent; their law is exponential with parameter κ_j. Thus, the law of $(\tau_1^j, \ldots, \tau_k^j)$ has for density $\mathbb{1}_{0\leqslant t_1\leqslant\cdots\leqslant t_k}\kappa_j^k e^{-\kappa_j t_k}\,dt_1\cdots dt_k$ with respect to the Lebesgue measure on $\mathbb{R}^k$. Therefore, let f be a bounded real measurable function and we introduce the sets

$$\begin{aligned}D_{t,T,k} = \{(t_1,\ldots,t_{p+k}) \in \mathbb{R}^{p+k} | 0\leqslant t_1 \leqslant \cdots \leqslant t_{p-1} \leqslant t \leqslant t_p \\ \leqslant \cdots \leqslant t_{p+k-1} \leqslant T \leqslant t_{p+k}\},\end{aligned}$$

$$F_{t,T,k} = \{(t_1,\ldots,t_k) \in \mathbb{R}^k | 0 \leqslant t \leqslant t_1 \leqslant \cdots \leqslant t_k \leqslant T\}.$$

$$\begin{aligned}
&E_{\mathbb{P}}\left(f\left(\int_t^T \ln(1+\sigma_{ij}(s))\,dN_j(s)\right)\right)\\
&= \sum_{k\geqslant 0}\sum_{p\geqslant 1}\kappa_j^{p+k}\int_{D_{t,T,k}} f\left(\sum_{l=0}^{k-1}\ln(1+\sigma_{ij}(t_{p+l}))\right)e^{-\kappa_j t_{p+k}}\,dt_1\cdots dt_{p+k}\\
&= \sum_{k\geqslant 0}\kappa_j^k\sum_{p\geqslant 1}\kappa_j^p\left(\int_{0\leqslant t_1\leqslant\cdots\leqslant t_{p-1}\leqslant t} dt_1\cdots dt_{p-1}\right)\left(\int_{T\leqslant t_{p+k}} e^{-\kappa_j t_{p+k}}\,dt_{p+k}\right)\\
&\quad\times\left(\int_{t\leqslant t_p\leqslant\cdots\leqslant t_{p+k-1}\leqslant T} f\left(\sum_{l=0}^{k-1}\ln(1+\sigma_{ij}(t_{p+l}))\right)dt_p\cdots dt_{p+k-1}\right)\\
&= \sum_{k\geqslant 0} e^{-\kappa_j(T-t)}\kappa_j^k\int_{F_{t,T,k}} f\left(\sum_{l=1}^{k}\ln(1+\sigma_{ij}(t_l))\right)dt_1\cdots dt_k.
\end{aligned}$$

We use the convention that $\int_{F_{t,T,k}} f(x+\sum_{l=1}^k \ln(1+\sigma_{ij}(t_l)))\,dt_1\cdots dt_k = f(x)$ if $k=0$. The last equality follows from a change of index in the last bracket and from the fact that

$$\begin{aligned}
&\sum_{p\geqslant 1}\kappa_j^p\left(\int_{0\leqslant t_1\leqslant\cdots\leqslant t_{p-1}\leqslant t} dt_1\cdots dt_{p-1}\right)\left(\int_{T\leqslant t_{p+k}} e^{-\kappa_j t_{p+k}}\,dt_{p+k}\right)\\
&\quad= \sum_{p\geqslant 1}\frac{(\kappa_j t)^{p-1}}{(p-1)!}\,e^{-\kappa_j T} = e^{-\kappa_j(T-t)}.
\end{aligned}$$

Since the Poisson processes $N_1,\ldots,N_n$ are mutually independent,

$$\begin{aligned}
&E_{\mathbb{P}}\left(f\left(\int_t^T \ln(1+\sigma_{iN}(s))\,dN(s)\right)\right)\\
&= \prod_{j=1}^n\sum_{k_j\geqslant 0} e^{-\kappa_j(T-t)}\kappa_j^{k_j}\int_{(F_{t,T,k_j})^n} f\left(\sum_{j=1}^n\sum_{l_j=1}^{k_j}\ln(1+\sigma_{ij}(t_{jl_j}))\right)\prod_{l_j=1}^{k_j} dt_{jl_j}
\end{aligned} \tag{A.1}$$

with $(F_{t,T,k_j})^n$ defined in (4.4). $\int_t^T \sigma_{iW}(s)\,\mathrm{d}W(s)$ is independent of $(N_1,\ldots,N_n)$, its law is a Gaussian law with mean 0 and variance $\Sigma_t = \int_t^T \|\sigma_{iW}(s)\|^2\,\mathrm{d}s$, thus

$$
\begin{aligned}
&E_{\mathbb{P}}\left(f\left(\int_t^T \sigma_{iW}(s)\,\mathrm{d}W(s) + \ln(1+\sigma_{iN}(s))\,\mathrm{d}N(s)\right)\right)\\
&\quad = \int_{\mathbb{R}} \prod_{j=1}^{n} \sum_{k_j \geqslant 0} \mathrm{e}^{-\kappa_j(T-t)} \kappa_j^{k_j} \int_{(F_{t,T,k_j})^n} f\left(x + \sum_{j=1}^{n} \sum_{l_j=1}^{k_j} \ln(1+\sigma_{ij}(t_{jl_j}))\right)\\
&\qquad \times \frac{\exp\left(\dfrac{-x^2}{2\Sigma_t}\right)}{\sqrt{2\pi\Sigma_t}}\,\mathrm{d}x \prod_{l_j=1}^{k_j} \mathrm{d}t_{jl_j}.
\end{aligned}
$$

Therefore

$$
\begin{aligned}
\mathbb{P}[\mathbb{1}_{[a,b]}(P_i(T)) = 1 | \mathscr{F}_t] = {} & \int_{\mathbb{R}} \prod_{j=1}^{n} \sum_{k_j \geqslant 0} \mathrm{e}^{-\kappa_j(T-t)} \kappa_j^{k_j}\\
& \times \int_{(F_{t,T,k_j})^n} \mathbb{1}_{(k_a(t) \leqslant x + \sum_{j=1}^{n} \sum_{l_j=1}^{k_j} \ln(1+\sigma_{ij}(t_{jl_j})) \leqslant k_b(t))}\\
& \times \frac{\exp\left(\dfrac{-x^2}{2\Sigma_t}\right)}{\sqrt{2\pi\Sigma_t}}\,\mathrm{d}x \prod_{l_j=1}^{k_j} \mathrm{d}t_{jl_j}.
\end{aligned}
$$

This yields the formulas of $p(t,1)$ and $p(t,0)$ given in Proposition 4.4.

The coefficient of $\mathrm{d}p(t,1)$ on $\mathrm{d}W(t)$ is

$$
\frac{\prod_{j=1}^{n} \sum_{k_j \geqslant 0} \dfrac{\mathrm{e}^{-\kappa_j(T-t)}}{\sqrt{2\pi\Sigma_t}} \kappa_j^{k_j} \int_{(F_{t,T,k_j})^n} \left(f_{(t_{jl_j})}(k_a(t)) - f_{(t_{jl_j})}(k_b(t))\right) \prod_{l_j=1}^{k_j} \mathrm{d}t_{jl_j} \sigma_{iW}(t)}{\int_{\mathbb{R}} \prod_{j=1}^{n} \sum_{k_j \geqslant 0} \dfrac{\mathrm{e}^{-\kappa_j T - \frac{x^2}{2\Sigma_0}}}{\sqrt{2\pi\Sigma_0}} \kappa_j^{k_j} \int_{(F_{0,T,k_j})^n} \mathbb{1}_{\left(k_a(0) \leqslant x + \sum_{j=1}^{n} \sum_{l_j=1}^{k_j} \ln(1+\sigma_{ij}(t_{jl_j})) \leqslant k_b(0)\right)} \mathrm{d}x \prod_{l_j=1}^{k_j} \mathrm{d}t_{jl_j}}
$$

with $f_{(t_{jl_j})}$ defined in (4.5). The coefficient of $\mathrm{d}p(t,0)$ on $\mathrm{d}W(t)$ is

$$
\frac{\prod_{j=1}^{n} \sum_{k_j \geqslant 0} \dfrac{\mathrm{e}^{-\kappa_j(T-t)}}{\sqrt{2\pi\Sigma_t}} \kappa_j^{k_j} \int_{(F_{t,T,k_j})^n} \left(f_{(t_{jl_j})}(k_b(t)) - f_{(t_{jl_j})}(k_a(t))\right) \prod_{l_j=1}^{k_j} \mathrm{d}t_{jl_j} \sigma_{iW}(t)}{\int_{\mathbb{R}} \prod_{j=1}^{n} \sum_{k_j \geqslant 0} \dfrac{\mathrm{e}^{-\kappa_j T - \frac{x^2}{2\Sigma_0}}}{\sqrt{2\pi\Sigma_0}} \kappa_j^{k_j} \int_{(F_{0,T,k_j})^n} \mathbb{1}_{[k_a(0),k_b(0)]^C}\left(x + \sum_{j=1}^{n} \sum_{l_j=1}^{k_j} \ln(1+\sigma_{ij}(t_{jl_j}))\right) \mathrm{d}x \prod_{l_j=1}^{k_j} \mathrm{d}t_{jl_j}}.
$$

Thus we obtain the expression of $\rho^k(t)$ given in Proposition 4.4, with $a(t)$ defined in p. 11. $\square$

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