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Frequency Domain Study of Vibrations Above and Under Stability Lobes in Machining Systems

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Abstract

Using modified Nyquist contours, the dominant poles of the closed loop delay-differential equation for machining systems such as milling are identified. Contours with constant damping ratio of the dominant poles are constructed using this method. These contours are similar in shape to the stability lobes, but move upwards and to the right as the instability parameter increases. Additionally, it is possible to study the movement of the dominant poles to the right-hand side of the complex plane as the system becomes unstable by increasing the depth of cut at a constant spindle speed. The movement of the dominant pole is shown to be towards the right (unstable) and upward (higher vibration frequency) of the complex plane. In some cases, there would be a jump of vibration frequency due to the change of the lobe number. It is also shown that the damping ratio of the structure strongly affects both the vibration frequency and the damping ratio of the dominant poles in the closed loop system. Finally, in two milling experiments with two different spindle speeds and continuously increasing depth of cuts, vibration frequencies are measured and compared to the theoretical predictions. The measurements agree with the theoretical predictions, particularly in the unstable cutting conditions.

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Keywords: Stability Chart, Chatter, Nyquist Stability Criterion, Vibration Frequency, Dominant Poles

1. Introduction

In a metal cutting operation, a closed loop is formed between the machining process and the dynamic vibrations of the structure that positions the tool against the workpiece. This closed loop might have unstable characteristics and self-exited vibrations might grow depending on the dynamic compliance of the structure and the sensitivity of the cutting forces to the vibrations. The time delay between consecutive machining passes, which depends on spindle speed, controls the phase difference between the vibration waves on inside and outside of machined chips and plays a significant role in the instability dynamics. On the other hand, the depth of cut controls the sensitivity of the cutting forces to the vibrations; therefore, the depth of cut at threshold of stability changes as a function of spindle speed, in a diagram known as the stability lobe diagram.

The past research has established the methods for calculation of the stability lobe diagrams and the vibration frequencies at the threshold of stability. After identification of regeneration of vibration force due to the phase difference between the vibration waves inside and outside of the chips by Doi and Kato [1], width or depth of cut at the threshold of stability are calculated [2, 3], and the saw-tooth plot for chatter frequencies has been presented using tools such as Nyquist stability criterion [4]. In addition to the orthogonal turning problems, Nyquist stability criterion or its equivalents have been used in prediction of chatter in 3D turning problems [5] and milling problems [6, 7].

Chatter predictions and construction of stability lobe diagrams are used for selection of optimal cutting conditions (i.e. optimal spindle speed and depth of cut); however, impact tests are often needed for identification of structural parameters such as modal stiffness, natural frequency and damping ratio of the modes that may contribute to chatter. These parameters are then used in the machining dynamics model to calculate the stability lobes and to select optimal cutting parameters as depicted in Fig. 1(a).

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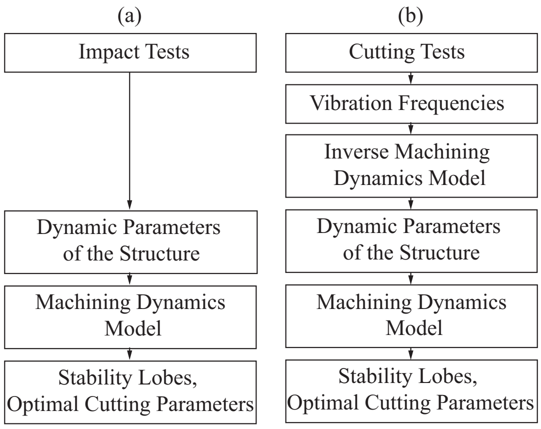

Fig. 1. (a) The conventional approach; (b) the inverse approach for construction of stability lobes

An alternative algorithm is to record sound, displacement, force or acceleration during the cutting tests, extract the vibration frequencies, and use an inverse machining dynamics model to extract the dynamic parameters of the structure. These parameters are then used in the machining dynamics model to find optimal cutting parameters [8, 9], as depicted in Fig. 1(b). While this approach needs more calculations, it has the advantage of eliminating the need for impact test equipment in one hand, and extraction of the dynamic parameters of the structure during cutting, thus avoiding the changes in the dynamic parameters that may arise due to the spindle's rotation or preloads and nonlinearities. Some of such changes are discussed in references [10, 11].

In order to extract the structural parameters from the vibration signals, the relationships between the cutting conditions, structural parameters and vibration frequencies should be embedded in the inverse machining dynamics model. Consequently, having a clear picture about the vibration frequencies above and under stability lobes would improve the extraction of the dynamic parameters of the structure in these algorithms. Additionally, a clear understanding of the vibration frequencies above and under the stability lobes would help in distinguishing regenerative chatter from forced vibrations that arise in interrupted machining with flexible structures by answering the following questions:

- What is the frequency of unstable vibrations (ω_d) at a depth of cut (a) above threshold depth of cut ($a > a_{cr}$)? Is it equal to instability frequency at the threshold of instability (ω_{cr})?
- What is the frequency of damped vibration of a stable machining system ($a < a_{cr}$) in response to disturbance forces? Is it equal to the vibration frequency at the threshold (ω_{cr}), or is it equal to the natural frequency of the flexible mode (ω_n)?

Nomenclature

ω_n	natural frequency of the structure
ω_t	tooth passing frequency
s	dominant pole
ω_d	vibration frequency of the dominant pole
ω_{cr}	vibration frequency at the threshold of stability, changes as a function of spindle speed
ζ	damping ratio of the dominant pole
τ	time constant of the dominant pole
ζ_{xx}	damping ratio of the structure
a	depth of cut
a_{min}	smallest depth of cut at the threshold of stability
a_{cr}	depth of cut at threshold of stability, which changes as a function of spindle speed
j	imaginary unit
N_z	number of flutes on a milling tool
FDR	Fixed Damping Ratio
FTC	Fixed Time Constant

The reminder of the paper is organized as follows. In the second section, the analytical/numerical method for calculation of the complex poles is explained. The method is used to study the effects of depth of cut, spindle speed and damping ratio of the structure on vibration frequencies in the third section. The fourth section presents the results of an experiment is where the changes in vibration frequencies due to the increase of depth of cut at two different spindle speeds are compared to the analytical predictions. Finally, section 5 presents conclusions and suggestions for the future work.

2. Modified Nyquist Contours

The standard Nyquist contour, Fig. 2(a), maps the imaginary axis by the closed loop characteristic equation of closed loop machining system to the complex plane and if the mapped contour encircles the center of the complex plane in the clockwise direction, it indicates the presence of unstable poles (poles in the right side of the imaginary axis) in the closed loop system. A pole is on the imaginary axis if the mapped contour passes through the center of the complex plane. While the standard Nyquist contour reveals the presence of an unstable pole, it cannot provide information about its location on the complex plane. On the other hand, no analytical method exists to determine the poles of the delay differential equation of a machining system, even though there are numerical methods based on Cauchy's argument principle that find the complex roots by sectioning of the complex plane. In past modified Nyquist contours have been used in controller design in applications such as spacecraft control systems [12, 13]. Two different contours could be constructed, Fixed Damping Ratio

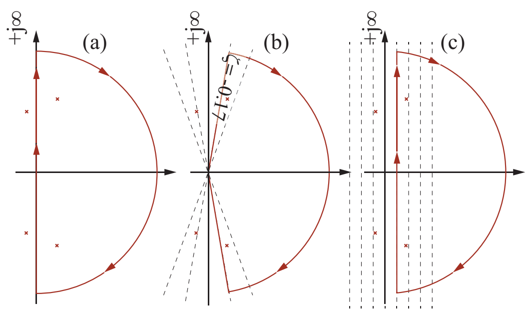

Fig. 2. (a) Nyquist contour, (b) Fixed Damping Ratio (FDR) contour; (c) Fixed Time Constant contour

(FDR), shown in Fig. 2(b) and Fixed Time Constant (FTC), shown in Fig. 2(c). In the remaining sections, only the FDR contours are employed, since with a known damping ratio and vibration frequency; it is possible to calculate the time constant.

3. Results

3.1. FDR contours

The vibration frequencies (the imaginary part of the dominant complex pole) at different spindle speeds for FDR contours with different damping ratio of the closed loop poles are presented in Fig. 3 (a), along with the depth of cut-spindle speed contours in Fig. 3 (b). The contours are for slot milling with a 3 flute end mill tool on a workpiece with flexibility in the feed direction. The characteristic equation for the stability prediction is calculated using the zero order solution of Budak and Altintas [7], while the Nyquist stability criterion is used in the manner described in [14], the only difference is that modified Nyquist contours are used instead of standard Nyquist contour. The values for the parameters such as stiffness, cutting coefficient (specific cutting pressures in the cutting speed and radial directions) and the natural frequency are taken from Table 1. Normalized values for frequencies and depth of cut are also presented. In the top horizontal axis for both Fig. 3 (a) and Fig. 3(b), the tooth-passing frequency is divided to the natural frequency (ω_n) to obtain normalized tooth passing frequency. In Fig. 3 (a) the vibration frequency is divided to the natural frequency to obtain normalized vibration frequency and in Fig. 3(b), the depth of cut (a) is divided to a_{min} to result in the normalized (unit-less) depth of cut. The $\zeta = 0$ contour in Fig. 3(b) is the conventional stability chart.

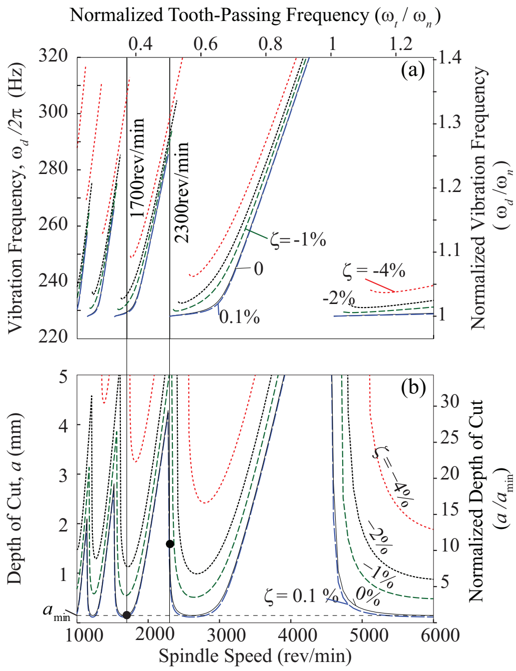

Fig. 3. (a) Vibration frequency and (b) depth of cut versus spindle speed for different FDR contours.

Table 1. Cutting and structural parameters of the milling system

Parameter	Symbol and value
Cutting coefficient in the cutting speed direction	$K_t = 1730MPa$
Cutting coefficient in the radial direction	$K_r = 712MPa$
Number of evenly spaced flutes	$N_z = 3$
Tool diameters	$D = 12mm$
Natural frequency in the feed direction	$\omega_n = 2\pi.(228\ Hz)$
Damping ratio (of the structure)	$\zeta_{xx} = 0.486\%$
Stiffness in feed direction	$K_{xx} = 8.03MN/m$

The FDR contours move to the right (higher spindle speeds) as the instability parameter (negative damping ratio) increases. The frequency contours also move in the same manner. In addition, as the spindle speed increases, the distance between the stable contours (e.g. $\zeta = 0.1\%$) and unstable contours (e.g. $= -4\%$) decreases. This means that at high speeds the boundary between the stable and unstable becomes sharper, in other words, a small change in the depth of cut is enough for rapid change of the characteristics of the machining system. On the other hand, at lower speeds, a larger change in the depth of cut is needed to observe the similar change of damping ratio of the dominant poles.

As it is seen in Fig. 3 (a), while the vibration frequency at the threshold of stability (ω_{cr}) always starts values close the natural frequency, the frequency of vibrations for systems with higher instability (e.g. $\zeta = -4\%$), particularly in lower spindle speeds are higher. As an example, ω_d is about $1.2\,\omega_n$ for spindle speeds around 1000rev/min. Comparing Fig. 3(a) and Fig. 3(b), it is possible to see how at a certain spindle speed, vibration frequency could change by increasing the depth of cut. In spindle speeds such as 1700rev/min, in the valleys of the stability lobe diagram, the vibration frequency first changes in the ($\omega_n - 1.1\omega_n$) range before jumping to a range above $1.3\omega_n$ by intersecting a cluster to the left side of the original cluster. On the other hand, for spindle speeds such as 2300rev/min, which is just to the right side of a peak in Fig. 3(b), after a couple of intersects at ($\omega_n - 1.05\omega_n$) range, the constant spindle speed line intersects the left side cluster at higher frequency range ($1.25\omega_n - 1.35\omega_n$). These changes in vibration frequencies are the results of the movements of the dominant poles that are discussed in the next subsection.

3.2. Movement of the dominant poles

Fig. 4. shows the movement of the dominant pole with positive imaginary part from the stable region to unstable region at two different spindle speeds of 1700 rev/min and 2300 rev/min. Each point on this figure belongs to a certain depth of cut with corresponding damping ratio of the closed loop pole. There is a notable jump for the 2300 rev/min case which causes a large change in the frequency of vibration,$\omega_d = imag(s)$. This jump was also possible to see in Fig. 3 (a), on the fixed spindle speed line of 2300 rev/min.

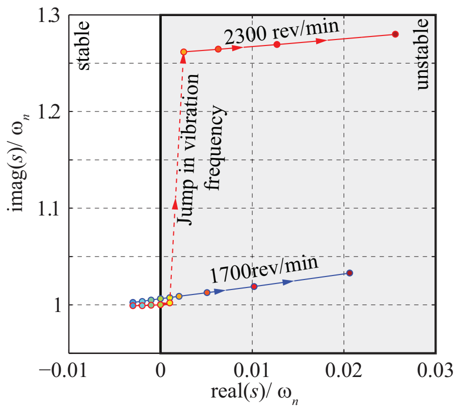

Fig. 4. Movement of the dominant pole due to the increase in the depth of cut at two different spindle speeds. The jump in the vibration frequency happens at the 2300rev/min spindle speed, which is just to the right side of a local peak in the stability chart.

The effect of large depths of cut on the vibration frequency is presented in Fig. 5, which shows that the frequency jump is possible for both spindle speeds, provided that the depth of cut is increased to the large enough values. While the jump to the high frequency vibration happens for both cases, the jump happens at a smaller depth of cut for the 2300rev/min case. In addition, the change of vibration frequency is faster in low depth of cuts for 1700 rev/min compared to 2300 rev/min. While the 1700 rev/min case reaches earlier to instability, the 2300rev/min case reaches to a large instability parameter of $\zeta = -2\%$ at a smaller depth of cut, while a twice as large depth of cut is needed to create the same instability for 1700rev/min case. There is a big depth of cut gap for the system with $\zeta = -0.5$ and $\zeta = -1.0$ for the spindle speed of 1700 rev/min, this means that a very large change of depth of cut at this speed would result in a minor change in the instability level of the machining system.

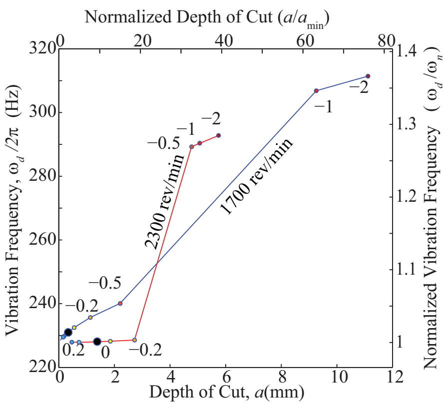

Fig. 5. Vibration frequency versus depth of cut at two different spindle speeds. The numbers next to the points indicate damping ratios of the closed loop poles, and the bigger dots indicate the critical depth of cut (threshold of stability).

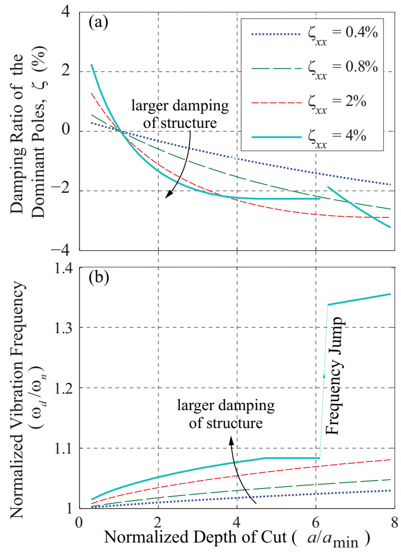

Fig. 6. Effect of structural damping ratio on frequencies and stability of the system at the spindle speed of 1700rev/min.

Another interesting observation from Fig. 5 is that the transition of vibration frequencies from the stable regime (positive damping ratios) to the unstable cases (negative damping ratios) is smooth. This suggests that the vibration frequencies are not sensitive to the stability condition of the poles. This point is important since any measurable discontinuity in the vibration frequencies could have been a suitable means of detecting chatter.

3.3. Sensitivity to the damping ratio of the structure

It could be shown that the normalized vibration frequency (ω_d / ω_n) and the damping ratio of the dominant poles (ζ) are independent of the natural frequency of the structure (ω_n). On the other hand, as Fig. 6 depicts, the damping ratio of the structure (ζ_{xx}) strongly affects the vibration frequencies and damping ratio of the dominant poles of the closed loop machining system (ζ). In these calculations, all structural and cutting parameters except for ζ_{xx} are selected from Table 1 to model a system with spindle speed of 1700 rev/min. With a small damping ratio of the structure ($\zeta_{xx} = 0.4\%$), a normalized depth of cut of 8 (a / a_{min} = 8) causes 3% increase in the vibration frequency, and makes the damping ratio of the closed loop system -1.8%. When the damping ratio of structure is large ($\zeta_{xx} = 4\%$), the vibration frequency (ω_d) increases 8% before jumping to 1.36ω_n and the damping ratio of the dominant pole of the closed loop system (ζ) becomes -3.2% for the same normalized depth of cut. In other words, a large damping ratio of the structure also makes the damping ratio and the vibration frequency of the closed loop system more sensitive to the depth of cut. Another interpretation of this effect would be that the boundary between stable and unstable cutting conditions would be sharper for systems with higher damping of structure, and a small change of normalized depth of cut would be needed to create large instability in the closed

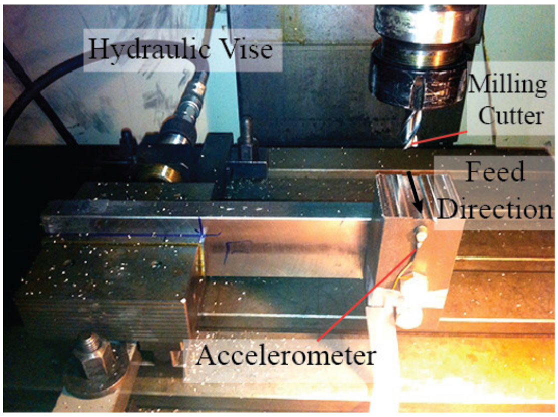

Fig. 7. The experiment setup.

loop system. In addition, as Fig. 6(b) shows, when the damping ratio of the structure is higher the jump to the higher frequency happens earlier.

4. Milling Experiments

The experiment setup consists of a 50×50×100 mm AISI1045 hot rolled steel block, bolted to a cantilever beam clamped on a hydraulic vise as shown in Fig. 7. The setup has a single dominant bending mode with a natural frequency of 228 Hz. The tool is a 3 flute cylindrical end-mill of 12 mm diameter, the cutting operation is slotting and the feed direction is oriented with the beam's flexible direction. The parameters related to the tool, cutting coefficients and dynamic properties of the workpiece are listed in Table 1. The dynamic stiffness values are extracted by an instrumented impact hammer test and the cutting coefficients (K_r and K_t) are identified by slot milling of the same material at a cutting speed of 240 m/min at five different feed rates and using an averaging method as described in [15]. The theoretical minimum stable depth of cut at all cutting speeds (a_{min}) is ~0.15 mm as shown in Fig. 3.

A sloped surface is prepared for each cutting test, where the depth cut is increased from 0 to 1 mm on a 40 mm span of cutting. Two tests with spindle speeds of 1700 rev/min and 2300 rev/min have been performed. As indicated in the stability chart diagram (Fig. 3 b), the critical depth of cuts for 1700 rev/min and 2300 rev/min are 0.15 mm and 1.5 mm respectively. A small accelerometer, shown in Fig. 7, measures the acceleration of the beam during cutting.

The measured acceleration, identified frequency and amplitude of vibration and the depth of cut are plotted as a function of the position of the center of the milling tool in Fig. 8. and Fig. 9. for 1700 rev/min and 2300 rev/min respectively. Considering that the tool has a 6mm radius, care has been taken to separate the phases corresponding to the entry, rubbing of the flat region with 0 depth of cut, the cutting of the 0-1mm ramp and the exit. Amplitude and frequency of the vibration are identified by fitting a stable or unstable harmonic vibration of single frequency to the short samples of data, containing about six vibration cycles using the active-set nonlinear constrained multivariable optimization algorithm. In 1700 rev/min (Fig. 8.), the amplitude of vibration shows a sudden jump at a depth of cut about 0.6 mm at 1700 rev/min and the frequency of vibration shows a distinct increase to 235 Hz, which indicates the start of chatter at this depth of cut. The difference between this depth of cut and the predicted critical depth of cut for chatter might be attributed to the process damping [6] which is not investigated here.

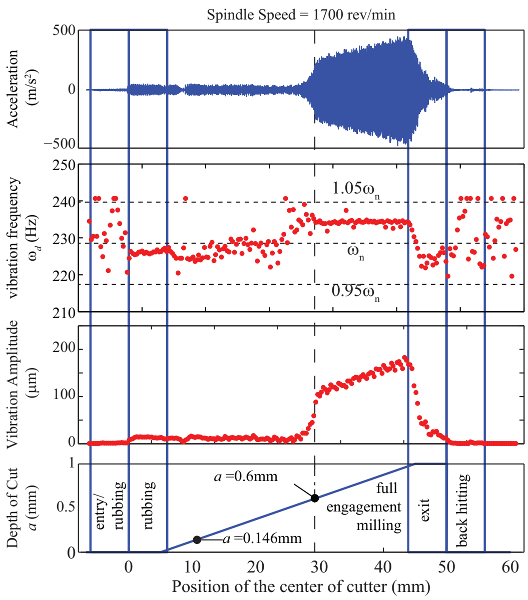

Fig. 8. Acceleration, vibration frequency, amplitude and depth of cut during the test at 1700 rev/min.

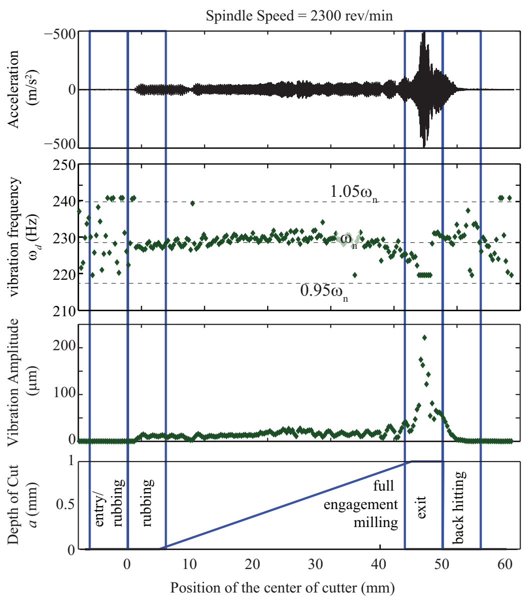

Fig. 9. Acceleration, vibration frequency, amplitude and depth of cut during the test at 2300 rev/min.

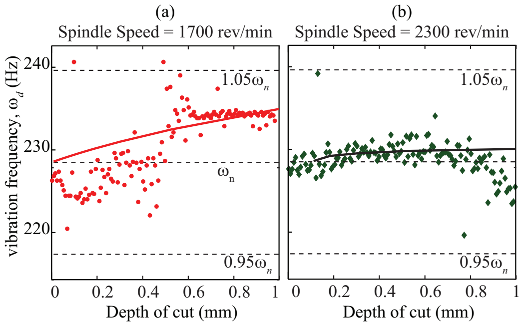

Fig. 10. Comparison of measured vibration frequency (dots) to the predicted vibration frequency (solid lines) at (a) 1700 rev/min and (b) 2300 rev/min spindle speeds.

The amplitude of vibration at spindle speed of 2300 rev/min (Fig. 9.) remains smaller than 50 μm during full engagement cutting and only shows a rapid increase during the exit phase, where burr-formation mechanism [16] could be blamed.

Theoretical vibration frequency for depth of cuts of 0 – 12 mm is presented earlier in Fig. 5. Vibration frequencies in the 1700 rev/min and 2300 rev/min cutting tests are compared to the theoretical predictions for depth of cut range of 0-1.0 mm in Fig. 10. The vibration frequency is higher at larger depth of cuts, which is correctly predicted by the method presented in this paper. The spread of frequencies also decrease as the depth of cut is increased at 1700 rev/min and show a better agreement with the theoretical prediction, but in 2300 rev/min case, for large depth of cuts (a >0.8 mm), the vibration frequency decrease to values smaller than the natural frequency, which is not explained by the method. Also in 1700 rev/min case, for the small depth of cuts (a <0.6 mm), the vibration frequency is smaller than the one predicted by the theory.

5. Conclusions

The presented method predicts the increase or jump of the vibration frequencies when the depth of cut is increased at constant spindle speeds. This is due to the upward and rightward movement of the dominant poles of the machining system in the complex plane as the depth of cut increases. In the studied case, this meant increase of the vibration frequency from values close to the natural frequency (ω_n) to frequencies about 140% of the natural frequency ($1.4\omega_n$), depending on the spindle speed, damping ratio of the structure and the level of instability. In some regions of stability chart, an increase in the depth of cut causes a sudden jump of the vibration frequency due to the change of lobe numbers.

Furthermore, it has been observed that the boundary between stable and unstable conditions become sharper at higher spindle speeds, i.e. smaller changes in the depth of cut would lead to more significant changes in the time constant and the damping ratio of the dominant pole. The theoretical predictions of vibration frequencies are in agreement with the experimental measurements, particularly at unstable cutting conditions.

The method used in this paper treats the machining system as a linear system; therefore, its findings are applicable in early stages of chatter. Different methods such as time domain simulation may be necessary when cases such as tool jumping out of cut or entry and exit effects are investigated.

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References

[1] Doi, S. and S. Kato, Chatter vibration of lathe tools, Transactions of ASME, vol. 78, pp. 1127-1134, 1956.
[2] Tlusty, J. and M. Polacek, The stability of machine tools against self excited vibrations in machining, in Proceedings of the ASME International Research in Production Engineering, 1963, pp. 465-474.
[3] Tobias, S. and W. Fishwick, Theory of regenerative machine tool chatter, The Engineer, vol. 205, pp. 199-203, 1958.
[4] Merritt, H. E., Theory of Self-Excited Machine-Tool Chatter: Contribution to Machine-Tool Chatter Research—1, Journal of Engineering for Industry, vol. 87(4), pp. 447-454, 1965.
[5] Eynian, M. and Y. Altintas, Chatter Stability of General Turning Operations With Process Damping, Journal of Manufacturing Science and Engineering, vol. 131(4), p. 041005, 2009.
[6] Eynian, M. and Y. Altintas, Analytical Chatter Stability of Milling With Rotating Cutter Dynamics at Process Damping Speeds, Journal of Manufacturing Science and Engineering, vol. 132(2), p. 021012, 2010.
[7] Altintaş, Y. and E. Budak, Analytical Prediction of Stability Lobes in Milling, CIRP Annals - Manufacturing Technology, vol. 44(1), pp. 357-362, 1995.
[8] Suzuki, N., Y. Kurata, T. Kato, R. Hino, and E. Shamoto, Identification of transfer function by inverse analysis of self-excited chatter vibration in milling operations, Precision Engineering, vol. 36(4), pp. 568-575, 2012.
[9] Pejryd, L. and M. Eynian, Minimization of chatter in machining by the use of mobile platform technologies, in Proceedings of the 5th International Swedish Production Symposium, SPS12 2012, pp. 179-189.
[10] Rantatalo, M., J.-O. Aidanpää, B. Göransson, and P. Norman, Milling machine spindle analysis using FEM and non-contact spindle excitation and response measurement, International Journal of Machine Tools and Manufacture, vol. 47(7–8), pp. 1034-1045, 2007.
[11] Cao, Y. and Y. Altintas, A General Method for the Modeling of Spindle-Bearing Systems, Journal of Mechanical Design, vol. 126(6), p. 1089, 2004.
[12] Greensite, A. L., Analysis and design of space vehicle flight control systems, Volume III, Linear systems, San Diego, California1967.
[13] Wang, Q.-G., Z. Zhang, K. J. Astrom, and L. S. Chek, Guaranteed dominant pole placement with PID controllers, Journal of Process Control, vol. 19(2), pp. 349-352, 2009.
[14] Eynian, M., Chatter stability of turning and milling with process damping, PhD Thesis, Department of Mechanical Engineering, University of British Columbia, Vancouver, 2010.
[15] Altintas, Y., Manufacturing automation: metal cutting mechanics, machine tool vibrations, and CNC design: Cambridge University Press, 2010.
[16] Park, I. and D. Dornfeld, A study of burr formation processes using the finite element method: part II-The influences of exit angle, rake angle, and backup material on burr formation processes, Journal of engineering materials and technology, vol. 122(2), pp. 229-237, 2000.