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Robust PID controller design for unstable processes with parametric uncertainty

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Abstract

The paper presents a method for robust PI controller design for system affected by parametric uncertainty. The proposed method is based on plotting the stability boundary locus in the plane of controller parameters that is called (k_p, k_i) - plane. The design approach is verified using simulations of control of the continuous stirred tank reactor (CSTR) with hydrolysis of propylen oxide to propylene glycol. The reactor which has three uncertain parameters is controlled into its unstable steady-state.

Keywords: Robust PI controller; parametric uncertainty ; chemical reactor; unstable steady-state

1. Introduction

The CSTRs belong to the most important plants in chemical and food technology. Their operation is corrupted with various uncertainties. Some of them arise from varying or not exactly known parameters. In others cases, operating point of reactor vary or reactor dynamics is affected by various changes of parameters of inlet stream. All these uncertainties can cause poor performance or even instability of closed-loop control systems. Application of robust control is one of the ways to overcome these problems, [2], [4]. In case when the unstable steady-state coincides with the point that yields the maximum reaction

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rate at a prescribed temperature, it is necessary to control CSTRs into the prescribed open-loop unstable steady-state [1], [2], [6].

For design of robust PI controllers, which stabilize nonlinear system of CSTR with uncertain parameters [5] in unstable steady-state, is simple and fast method presented [7]. The method is based on plotting the stability boundary locus in the (k_p, k_i) – plane. The parameters of robust stabilizing PI controller are determined from the stability region [10]. Robust PI controllers stabilize a controlled system with parametric uncertainties, if the stability region is found for sufficient number of Kharitonov plants [3].

2. Robust PI controller design

Consider a single-input single-output uncertain control system in the form

$$G(s)=\frac{N(s)}{D(s)} \tag{1}$$

where $N(s)$ is uncertain polynomial of system numerator and $D(s)$ is uncertain polynomial of denominator. $C(s)$ is a PI controller in the form

$$C(s)=k_p+\frac{k_i}{s}=\frac{k_p s+k_i}{s} \tag{2}$$

The problem is to find the parameters of the PI controller (2). Decomposing the numerator and the denominator polynomials in (1) [8] into their even and odd parts, and substituting $s=j\omega$, where ω is the frequency, gives

$$G(j\omega)=\frac{N_e\left(-\omega^2\right)+j\omega N_o\left(-\omega^2\right)}{D_e\left(-\omega^2\right)+j\omega D_o\left(-\omega^2\right)} \tag{3}$$

Using (3) the closed loop characteristic equation $\Delta(j\omega)$ can be written as

$$\begin{aligned}\Delta(j\omega)=&\left[k_i N_e\left(-\omega^2\right)-k_p\omega^2 N_o\left(-\omega^2\right)-\omega^2 D_o\left(-\omega^2\right)\right]+\\&+j\left[k_p\omega N_e\left(-\omega^2\right)+k_i\omega N_o\left(-\omega^2\right)+\omega D_e\left(-\omega^2\right)\right]=0\end{aligned} \tag{4}$$

Then, equating the real and the imaginary parts of $\Delta(j\omega)$ to zero, one obtains [7], [9]

$$k_p\left(-\omega^2 N_o\left(-\omega^2\right)\right)+k_i\left(N_e\left(-\omega^2\right)\right)=\omega^2 D_o\left(-\omega^2\right) \tag{5}$$

and

$$k_p\left(N_e\left(-\omega^2\right)\right)+k_i\left(N_o\left(-\omega^2\right)\right)=-D_e\left(-\omega^2\right) \tag{6}$$

Solving the equations (5), (6) simultaneously for $\omega\geq 0$, the set of parameters k_p and k_i is obtained. Then, it is possible to plot the dependence of k_i on k_p, and the stability boundary locus $l(k_p,k_i,\omega)$ in the (k_p,k_i)-plane is obtained. The stability boundary divides the parameter plane into stable and unstable regions.

The described robust PI controller design is fast and effective. The controller problem consists in finding a proper interval of frequency ω. Hence is for this method necessary to find real values of ω that satisfy condition

$$\mathrm{Im}\left[G(j\omega)\right]=0 \tag{7}$$

3. Description of the controlled processes

As a controlled process the continuous stirred tank reactor for hydrolysis of propylene oxide to propylene glycol was chosen, see e.g. [5]. The reaction is described according to the scheme C_3H_6O + $H_2O \rightarrow C_3H_8O_2$, where C_3H_6O and H_2O represent source substances propylene and water, respectively and $C_3H_8O_2$ is product propylene glycol. Chemical reaction run in a reaction vessel and reaction heat generated by exothermic reaction is removed from the reactor by coolant in a reactor jacket. The excess of water provides higher selectivity to propylene glycol and eliminates consecutive reactions of propylene oxide as a key component. The mathematical model of CSTR consist of Arrhenius equation which describe the exponential dependency of reactant concentrations on the temperature of the reaction mixture, the mass balance for any species in the system, the simplified enthalpy balance of the reacting mixture and the simplified enthalpy balance of the cooling medium, see e.g. [2].

The values of constant parameters and steady-state inputs of the reactor are summarized in Tab. 1.

Table 1. Constant parameters and steady-state inputs of the chemical reactor

Variable	Description	Value	Unit
V_r	volume of the reaction mixture	2.407	m^3
V_c	volume of coolant	2.000	m^3
ρ_r	density of the reaction mixture	974.19	kg m^{-3}
ρ_c	density of the coolant	998.00	kg m^{-3}
c_{pr}	specific heat capacity of the reaction mixture	3.7187	kJ kg^{-1} K^{-1}
c_{pc}	specific heat capacity of the coolant	4.182	kJ kg^{-1} K^{-1}
A	heat exchange surface area	8.695	kJ min^{-1} K^{-1}
$g=(E/R)$	activation energy divided by gas constant	10183	K
q_r	volumetric flow rates of reaction mixture	0.072	m^3 min^{-1}
q_c	volumetric flow rates of coolant	0.6307	m^3 min^{-1}
T_{rf}	inlet temperature of the reaction mixture	299.05	K
T_{cf}	inlet temperature of the coolant	288.15	K
$c_{f,A}$	inlet concentration of the C_3H_6O	0.0824	kmol m^{-3}
$c_{f,B}$	inlet concentration of the $C_3H_8O_2$	0	kmol m^{-3}

The model of the CSTR contains two inputs: volumetric flow rates of reaction mixture q_r and coolant q_c and three outputs: concentration of the C_3H_6O, the reaction mixture temperature in the reaction vessel T_r and the coolant temperature in the jacket T_c. Model uncertainties of the reactor follow from the fact that are three physical parameters in this reactor: the reaction enthalpy $\Delta_r H$, the pre-exponential factor k_∞ and the overall heat transfer coefficient α. The boundaries values of which vary within certain intervals are in Tab. 2.

Table 2. Uncertain parameters in the CSTR

Variable	Description	Minimal Value	Maximal value	Unit
$\Delta_r H$	reaction enthalpy	-5.508×10^6	-5.412×10^6	kJ mol^{-1}
k_∞	pre-exponential factor	2.5867×10^{11}	3.0667×10^{11}	min^{-1}
α	heat transfer coeficient	13.0	14.6	kJ min^{-1} K^{-1}

The steady-state behavior of the model CSTR was studied. It can be stated the CSTR has three steady-states, two of them are stable and one is unstable. From the viewpoint of safety operation or in the case of unstable steady-state coinciding with the point that yields the maximum reaction rate at a prescribed temperature, it is necessary to control CSTRs near the prescribed open-loop unstable steady-state [2]. Therefore the main operating point is described by unstable steady-state values of state variables. The heat analysis the nominal model is shown in Fig. 2, where Q_{GEN} is the heat generated by chemical reactions and Q_{OUT} is the heat removed by the jacket and the product stream. The operating point of the nominal system is described using $[c_A^s, T_r^s, T_c^s]$=[0.0371 kmol m^{-3}, 343.0992 K, 290.5456 K].

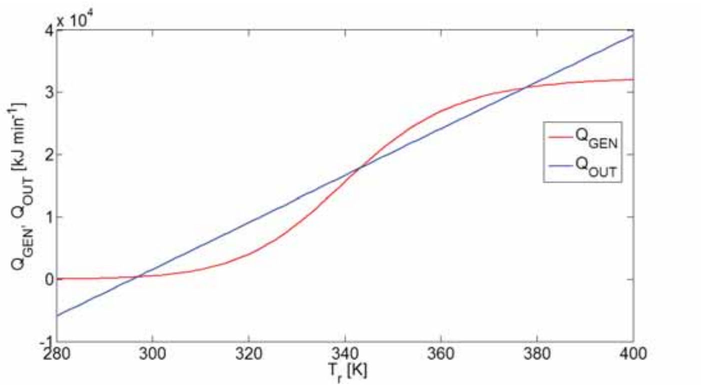

Fig. 1. Steady-states of nominal model of CSTR obtained as intersections of Q_{GEN} and Q_{OUT}

4. Control of the CSTR

Robust controller has been designed to achieve control of the reaction mixture temperature in the reaction vessel. Control input is volumetric flow rate of the reaction mixture q_r and control output is temperature of the reaction mixture T_r. For controller design, the mathematical model of the continuous stirred tank reactor with three uncertain parameters (Tab. 2) is obtained in the form of a transfer function

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0} \tag{8}$$

where the values of the uncertain parameters are shown in Tab. 3. Using sixteen Kharitonov plants [3] are generated for the CSTR controlled system (8) to design robust PI controller.

Table 3. Uncertain parameters

Parameter	Minimal value	Maximal value	Parameter	Minimal value	Maximal value
b_2	-20.0485	-16.8807	a_2	0.26	0.2672
b_1	-6.6816	-5.7959	a_1	-0.0259	-0.0228
b_0	-0.0802	-0.0180	a_0	-0.8117×10^{-3}	-0.7658×10^{-3}

Using the (4), (5) to obtain the PI controller tuning rules in the form (9), (10)

$$k_i = \frac{b_2 a_3 \omega^6 + (a_2 b_1 - a_1 b_2 - a_3 b_0)\omega^4 + (-a_0 b_1 + a_1 b_0)\omega^2}{b_2^2 \omega^4 + \left(-2b_0 b_2 + b_1^2\right)\omega^2 + b_0^2} \qquad (9)$$

and

$$k_p = \frac{a_3 \omega^4 + (-a_1 - b_2 k_i)\omega^2 + b_0 k_i}{b_1 \omega^2} \qquad (10)$$

After a suitable choice of $\omega \in (0;0.3289)$ (7), the stability boundary locus as the dependence of $k_i = f(\omega)$ on $k_p = f(\omega)$ is plotted. In Fig. 2 are shown the stability regions of sixteen Kharitonov plants, where intersection of these regions represents the stable region

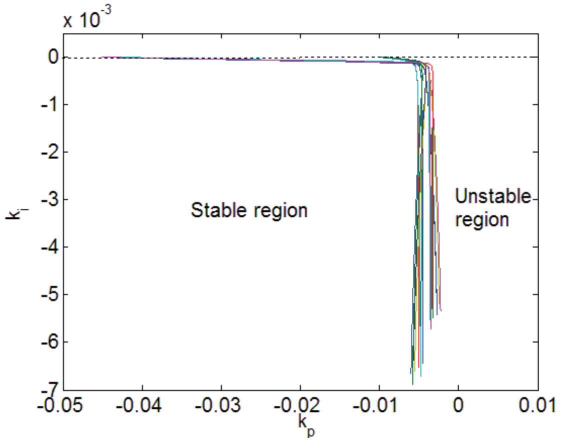

Fig. 2. Stable region for sixteen Kharitonov plants

Each Kharitonov system divided the (k_p, k_i) – plane into two parts, stable and unstableregion which are shown in the Fig.3. Robust stability region is obtained as a intersection of all particular stability regions generated by 16 Kharitonov plants. The required robust PI controller is tuned using the parameters from the stable region. The robust controller parameters k_p and k_i which stabilizes all sixteen Kharitonov systems are chosen from the stable region and design PI controller is described for example by (11)

$$C(s) = \frac{-0.03s - 0.004}{s} \qquad (11)$$

5. Simulation results

The designed PI controller (11) for control of nonlinear uncertain model of unstable CSTR by simulations in MATLAB-Simulink environment has been verified.

Fig. 4 presents simulation results obtained with the designed controller (11) and nonlinear models of the CSTR - nominal and two systems obtained with minimal and maximal values of uncertain parameters. The control output is the reaction mixture temperature T_r. The setpoint changes at time 0 min from initial value 345.0992 K to unstable working point 345.0992 K, then at time 150 min to new value of temperature 347.0992 K, then at time 450 min to 344.0992 K and at time 600 min again to unstable working point 345.0992 K. At time 300 min the disturbance has been generated which is represented by change of the inlet temperature T_{rf} from 299.05 K to 300.05 K. In Figures 4 are shown control inputs represented by volumetric flow rates of reaction mixture q_r for nominal system and two systems obtained with minimal and maximal values of uncertain parameters.

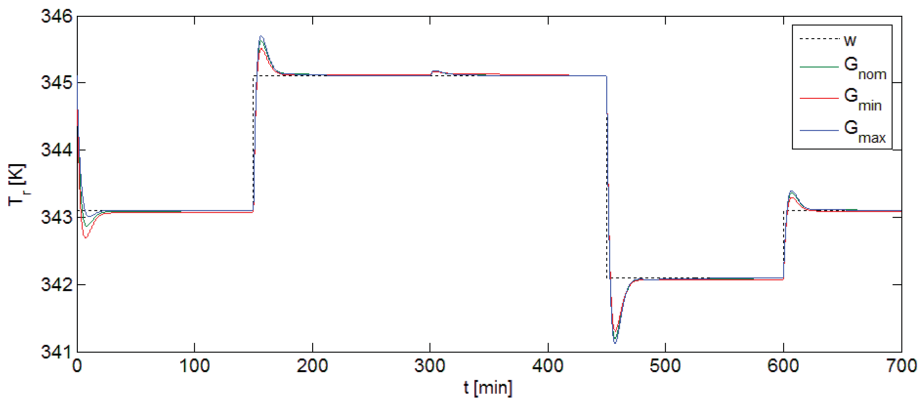

Fig. 3. Control responses for controller C_1

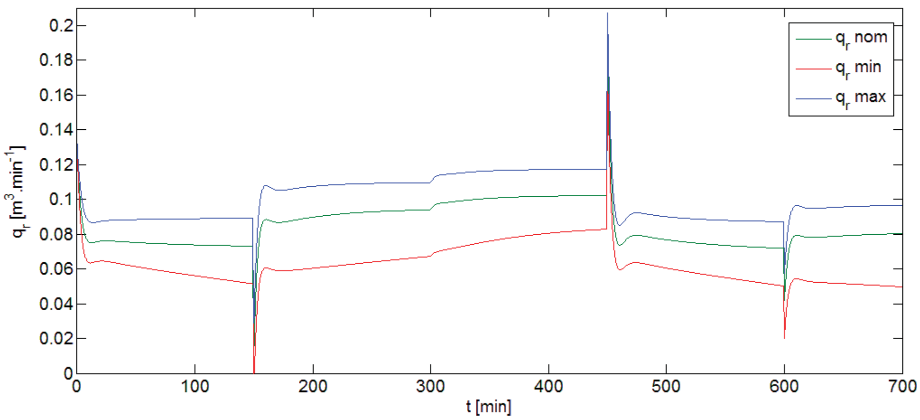

Fig. 4. Control inputs generated by controller C_1

Robust PI controller (11) which has been designed using the robust stability region of all stabilizing parameters k_p and k_i was able stabilize nonlinear model of CSTR with uncertainties into unstable working point and his surroundings. The controller also has been able to remove the disturbance which was generated by change of the inlet temperature T_{rf}.

6. Conclusion

In this paper, robust PI controller for control uncertain system in unstable steady-state has been designed. For calculation of robust PI controller was used method which is based on the plotting the stability boundary locus in the (k_p, k_i)-plane. Design controllers were used for control of nonlinear model of CSTR with three uncertain parameters. The aim was control of the reactor in unstable state for example from the viewpoint of safety operation. Therefore, the reactor was in the unstable steady-state stabilized which coincides with the point that yields the maximum reaction rate at a prescribed temperature.

Presented simulation results shows that designed robust PI controllers is able to stabilize the uncertain CSTR in unstable working point and his surroundings.

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