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A new efficient normal parameter reduction algorithm of soft sets

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ABSTRACT

Kong et al. [Kong, Z., Gao, L., Wang, L., and Li, S., The normal parameter reduction of soft sets and its algorithm, Computers and Mathematics with Applications 56 (12) (2008) 3029–3037] introduced the definition of normal parameter reduction in soft sets and presented a heuristic algorithm of normal parameter reduction. However, the algorithm is hard to understand and involves a great amount of computation. In this paper, firstly, we give some new related definitions and proved theorems of normal parameter reduction. Then we propose a new efficient normal parameter reduction algorithm of soft sets based on the oriented-parameter sum, which can be carried out without parameter important degree and decision partition. The comparison result on a dataset shows that the proposed algorithm involves relatively less computation and is easier to implement and understand as compared with the algorithm of normal parameter reduction proposed by Kong et al.

1. Introduction

A lot of practical and complicated problems in many fields involve uncertain, fuzzy, not clearly defined data. Uncertainties may be dealt with by making use of a wide variety of theories as diverse as probability theory, fuzzy sets [1], rough sets [2], intuitionistic fuzzy sets [3], vague sets [4] and interval mathematics [5], each of which has its inherent difficulties as pointed out in [6]. To overcome these difficulties, Molodtsov initiated soft set theory [7] as a new general mathematical tool for dealing with uncertainties which is free from the inadequacy of the parameterization tools. Therefore, it is very convenient and easy to apply soft set theory into practice. It has great potential for applications in several directions, some of which had already been demonstrated by Molodtsov [7], such as the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory, and measurement theory.

Presently, theoretic study on soft set is progressing rapidly. Maji et al. [8] firstly introduced some definitions of the related operations on soft sets. Ali et al. [9] took into account some errors of former studies and put forward some new operations on soft sets. Aktas et al. [10] initiated soft groups. Jun and Park [11] gave the definition of soft ideals and idealistic soft BCK/BCI-algebras. Feng et al. [12] depicted the definitions of soft semirings, soft subsemirings, soft ideals and idealistic soft semirings. Çağman et al. [13] defined soft matrices and their operations and described products of soft matrices and their properties. Qin et al. [14] introduced the concept of soft equality and some related properties were derived. Jiang [15] extended soft sets with description logics. Xiao et al. [16] proposed the notion of exclusive disjunctive soft sets and studied some of its operations. Xu et al. [17] gave the vague soft sets. It could be shown that soft set theory is closely associated with rough sets in [18–21]. Maji et al. [22] extended soft sets to fuzzy soft sets and Majumdar and Samanta [23] further generalized the

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concept of fuzzy soft sets. Liu and Yan [24] studied the algebraic structure of fuzzy soft sets and described the notion of fuzzy soft group. Maji [25], Maji et al. [26,27] extended soft sets to intuitionistic fuzzy soft sets. Soft set models in combination with the interval-valued fuzzy set have been proposed as the concept of the interval-valued fuzzy soft set [28] by Yang et al. Jiang et al. [29] presented the interval-valued intuitionistic fuzzy soft set theory by combining the interval-valued intuitionistic fuzzy sets and soft sets. As for practical applications of soft set theory, great progress has been made. Maji and Roy [30] employed soft sets to solve the decision-making problem. Roy and Maji [31] presented a novel method of object recognition from an imprecise multi-observer data to deal with decision making based on fuzzy soft sets, which was revised by Kong et al. [32]. Feng et al. [33] showed an adjustable approach to fuzzy soft set based decision making by means of level soft sets. Jiang et al. [34] presented an adjustable approach to intuitionistic fuzzy soft sets based decision making by using level soft sets of intuitionistic fuzzy soft sets. Feng et al. [35] gave deeper insights into decision making involving interval-valued fuzzy soft sets and put forward flexible schemes for decision making based on interval-valued fuzzy soft sets by using reduct fuzzy soft sets and level soft sets. Qin et al. [36] further presented an adjustable approach to interval-valued intuitionistic fuzzy soft sets based decision making. Zou and Xiao [37] depicted data analysis approaches of soft sets under incomplete information. Qin et al. [38] proposed a data filling approach for an incomplete soft set. Xiao et al. [39] described a combined forecasting approach based on fuzzy soft sets. Herawan and Mat Deris [40] proposed a soft set approach for association rules mining.

It is worthwhile to mention that some effort has been made towards issues concerning reduction of soft sets. Chen et al. [41] pointed out that the conclusion of soft set reduction offered in [30] was incorrect, and then present a new notion of parameterization reduction in soft sets in comparison with the definition to the related concept of attributes reduction in rough set theory. The concept of normal parameter reduction was introduced in [42], which overcomes the problem of suboptimal choice and added parameter set of soft sets. An algorithm for normal parameter reduction was also presented in [42]. However, the algorithm is hard to understand and involves a great amount of computation. In order to make the reduction of soft sets easy to implement and reduce computation, in this paper, we propose a simpler and more easily understandable algorithm which is referred to as a new efficient normal parameter reduction algorithm of soft sets.

The rest of this paper is organized as follows. Section 2 reviews the basic notions of soft set theory. Section 3 analyzes the normal parameter reduction of soft set put forward in [42]. Section 4 gives some new related definitions and proved theorems and then proposes a novel normal parameter reduction algorithm of soft sets. Section 5 firstly compares the proposed algorithm with the algorithm of [42] in terms of computational complexity, and then elaborates the comparison result for capturing the normal parameter reduction through a Boolean data set. Finally Section 6 presents the conclusion from our study.

2. Soft set theory

In this section, we review some definitions and properties with regard to soft sets. Let U be a non-empty initial universe of objects, E be a set of parameters in relation to objects in U, $P(U)$ be the power set of U, and $A \subset E$. The definition of soft set is given as follows.

Definition 2.1 (*See [6]*)**.** A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

That is, a soft set over U is a parameterized family of subsets of the universe U. As an illustration, let us consider the following example, which is quoted directly from [6].

Example 2.1. A soft set (F, E) describes the "attractiveness of houses" that Mr. X is going to purchase. Suppose that

$$U = \{h_1, h_2, h_3, h_4, h_5, h_6\} \quad \text{and} \quad E = \{e_1, e_2, e_3, e_4, e_5\},$$

where there are six houses in the universe U and E is a set of parameters, e_i $(i = 1, 2, 3, 4, 5)$ standing for the parameters "expensive", "beautiful", "wooden", "cheap", and "in green surroundings" respectively.

Suppose that we have

$$F(e_1) = \{h_2, h_4\}, \qquad F(e_2) = \{h_1, h_3\}, \qquad F(e_3) = \phi, \qquad F(e_4) = \{h_1, h_3, h_5\}, \quad \text{and} \quad F(e_5) = \{h_1\},$$

where $F(e_i)$ means a subset of U which elements match the parameter e_i. Then we can view the soft set (F, E) as a collection of approximations as below:

$$(F, E) = \left\{ \begin{array}{c} \text{expensive houses} = \{h_2, h_4\}, \\ \text{beautiful houses} = \{h_1, h_3\} \\ \text{wooden houses} = \phi \\ \text{cheap houses} = \{h_1, h_3, h_5\} \\ \text{in green surroundings houses} = \{h_1\} \end{array} \right\}.$$

Each approximation has two parts, a predicate p and an approximate value set v. For example, for the approximation "expensive houses $= \{h_2, h_4\}$", we have the predicate name of expensive houses and the approximate value set or value set is $\{h_2, h_4\}$. Thus, a soft set (F, E) can be viewed as a collection of approximations below:

$$(F, E) = \{p_1 = v_1, p_2 = v_2, p_3 = v_3, \ldots, p_n = v_n\}.$$

The soft set is a mapping from the parameter to the crisp subset of the universe. From such a case, we may see the structure of a soft set can classify the objects into two classes (yes/1 or no/0). Thus we can make a one-to-one correspondence between a Boolean-valued information system and a soft set, as stated in Proposition 2.1.

Definition 2.2. An information system is a 4-tuple (quadruple) $S = (U, A, V, f)$, where $U = \{u_1, u_2, \ldots, u_{|U|}\}$ is a non-empty finite set of objects, $A = \{a_1, a_2, \ldots, a_{|A|}\}$ is a non-empty finite set of attributes, $V = \bigcup_{a \in A} V_a$, V_a is the domain (value set) of attribute a, $f : U \times A \to V$ is an information function such that $f(u, a) \in V_a$, for every $(u, a) \in U \times A$, called the information (knowledge) function.

An information system is also called a knowledge representation system or an attribute-valued system and can be intuitively expressed in terms of an information table. In an information system $S = (U, A, V, f)$, if $V_a = \{0, 1\}$, for every $a \in A$, then S is called a *Boolean-valued information system*.

Proposition 2.1. *If (F, E) is a soft set over the universe U, then (F, E) is a Boolean-valued information system $S = (U, A, V_{\{0,1\}}, f)$.*

Proof. Let (F, E) be a soft set over the universe U, we define a mapping

$$F = \{f_1, f_2, \ldots, f_n\},$$

where

$$f_1 : U \to V_1 \quad \text{and} \quad f_1(x) = \begin{cases} 1, & x \in F(e_1) \\ 0, & x \notin F(e_1) \end{cases}$$

$$f_2 : U \to V_2 \quad \text{and} \quad f_2(x) = \begin{cases} 1, & x \in F(e_2) \\ 0, & x \notin F(e_2) \end{cases}$$

$$\vdots$$

$$f_n : U \to V_n \quad \text{and} \quad f_n(x) = \begin{cases} 1, & x \in F(e_n) \\ 0, & x \notin F(e_n) \end{cases}.$$

Thus, if $A = E$, $V = \bigcup_{e_i \in A} V_{e_i}$, where $V_{e_i} = \{0, 1\}$, then a soft set (F, E) can be considered as a Boolean-valued information system $S = (U, A, V_{\{0,1\}}, f)$. □

Obviously, for the reverse process, an information system of Boolean-value can be represented as a soft set.

According to Proposition 2.1, the soft set (F, E) mentioned in Example 2.1 can be represented as a Boolean table as shown in Table 1.

3. Analysis of the normal parameter reduction of soft sets in [42]

In this section, we briefly discuss the normal parameter reduction of soft sets and its algorithm, which were presented by Kong et al. in [42].

3.1. The normal parameter reduction of soft sets

Suppose $U = \{h_1, h_2, \ldots, h_n\}$, $E = \{e_1, e_2, \ldots, e_m\}$, (F, E) is a soft set with tabular representation. Define $f_E(h_i) = \sum_j h_{ij}$, where h_{ij} are the entries in the table of (F, E).

Definition 3.1 (*See [42]*)**.** With every subset of parameters $B \subseteq A$, an indiscernibility relation IND (B) is defined by

$$\mathrm{IND}(B) = \{(h_i, h_j) \in U \times U : f_B(h_i) = f_B(h_j)\}.$$

For soft set (F, E), $U = \{h_1, h_2, \ldots, h_n\}$, the decision partition is referred to as

$$C_E = \left\{\{h_1, h_2, \ldots, h_i\}_{f_1}, \{h_{i+1}, \ldots, h_j\}_{f_2}, \ldots, \{h_k, \ldots, h_n\}_{f_s}\right\},$$

where for subclass $\{h_v, h_{v+1}, \ldots, h_{v+w}\}_{f_i}$, $f_E(h_v) = f_E(h_{v+1}) = \cdots = f_E(h_{v+w}) = f_i$, and $f_1 \geq f_2 \geq \cdots \geq f_s$, s is the number of subclasses. In other words, objects in U are classified and ranked according to the value of $f_E(.)$ based on the indiscernibility relation.

Definition 3.2 (*See [42]*). For soft set (F, E), $E = \{e_1, e_2, \dots, e_m\}$, if there exists a subset $A = \{e'_1, e'_2, \dots, e'_p\} \subset E$ satisfying $f_A(h_1) = f_A(h_2) = \cdots = f_A(h_n)$, then A is dispensable, otherwise, A is indispensable. $B \subset E$ is a normal parameter reduction of E, if the two conditions as follows are satisfied

(1) B is indispensable
(2) $f_{E-B}(h_1) = f_{E-B}(h_2) = \cdots = f_{E-B}(h_n)$.

3.2. Algorithm of normal parameter reduction

Definition 3.3 (*See [42]*). The decision partition mentioned above is $C_E = \{E_{f_1}, E_{f_2}, \dots, E_{f_s}\}$, similarly, decision partition deleted e_i is figured as

$$C_{E-e_i} = \left\{\overline{E - e_{if'_1}}, \overline{E - e_{if'_2}}, \dots, \overline{E - e_{if'_s}}\right\}.$$

The importance degree of e_i for the decision partition is defined by

$$r_{e_i} = \frac{1}{|U|}\left(\alpha_{1,e_i} + \alpha_{2,e_i} + \cdots + \alpha_{s,e_i}\right),$$

where $|.|$ denotes the cardinality of set and

$$\alpha_{k,e_i} = \begin{cases} \left|E_{f_k} - \overline{E - e_{if_{z'}}}\right|, & \text{if there exist } z' \text{ such that } f_k = f'_z,\ 1 \le z' \le s',\ 1 \le k \le s \\ \left|E_{f_k}\right|, & \text{otherwise.} \end{cases}$$

Based on the parameter importance degree, Kong et al. [42] presented the algorithm of normal parameter reduction as shown in Fig. 1.

It is clear that this algorithm is hard to understand and implement. Besides, parameter importance degree involves a great amount of computation. In order to make reduction of soft sets easy to implement and reduce computation, we propose a new efficient normal parameter reduction algorithm of soft sets below.

4. A new efficient normal parameter reduction algorithm

4.1. The proposed technique

Given a soft set (F, E) with a tabular presentation, $U = \{h_1, h_2, \dots, h_n\}$ is the object set, $E = \{e_1, e_2, \dots, e_m\}$ is the parameter set, and h_{ij} are the entries in the table of (F, E).

Definition 4.1. For soft set (F, E), $U = \{h_1, h_2, \dots, h_n\}$, $E = \{e_1, e_2, \dots, e_m\}$, we denote $f_E(h_i) = \sum_j h_{ij}$ as an oriented-object sum.

Definition 4.2. For soft set (F, E), $U = \{h_1, h_2, \dots, h_n\}$, $E = \{e_1, e_2, \dots, e_m\}$, we denote $S(e_j) = \sum_i h_{ij}$ as an oriented-parameter sum.

Definition 4.3. For soft set (F, E), $U = \{h_1, h_2, \dots, h_n\}$, $E = \{e_1, e_2, \dots, e_m\}$, we denote $S_A = \sum_j S(e_j)$, for $A \subseteq E$ as the overall sum of A.

Theorem 4.1. *For soft set (F, E), $U = \{h_1, h_2, \dots, h_n\}$, $E = \{e_1, e_2, \dots, e_m\}$, if there exists a subset $A = \{e'_1, e'_2, \dots, e'_P\} \subset E$, such that $E - A$ is the normal parameter reduction of E, then we have $S_A = qn$, for $q = 0, 1, 2, \dots, m$, where n is the number of the universe U.*

Proof. Suppose $A = \{e'_1, e'_2, \dots, e'_P\} \subset E$. According to Definition 3.2, if $B \subset E$ is defined as a normal parameter reduction of E, then $f_{E-B}(h_1) = f_{E-B}(h_2) = \cdots = f_{E-B}(h_n)$. In other words, if $A = E - B$ can be reduced, then $f_A(h_1) = f_A(h_2) = \cdots = f_A(h_n)$. Therefore the following equations must be satisfied.

$$\begin{aligned} &h'_{11} + h'_{12} + \cdots + h'_{1P} = q \\ &h'_{21} + h'_{22} + \cdots + h'_{2P} = q \\ &\vdots \\ &h'_{n1} + h'_{n2} + \cdots + h'_{nP} = q. \end{aligned}$$

Table 1
Tabular representation of a soft set in the above example.

U	e_1	e_2	e_3	e_4	e_5
h_1	0	1	0	1	1
h_2	1	0	0	0	0
h_3	0	1	1	1	0
h_4	1	0	1	0	0
h_5	0	0	1	1	0
h_6	0	0	0	0	0

Table 2
A soft set (F, E).

U/E	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	$f(.)$
h_1	1	0	1	1	1	0	1	0	0	1	6
h_2	0	0	1	1	1	1	0	0	0	0	4
h_3	0	0	0	0	0	1	0	1	0	1	3
h_4	1	0	1	0	0	0	0	0	1	1	4
h_5	1	0	1	0	0	0	1	1	0	1	5
h_6	0	1	1	1	0	1	0	0	0	0	4
$S(e_j)$	3	1	5	3	2	3	2	2	1	4	$S_E = 26$

We can easily get

$$\begin{aligned} S_A &= S(e'_1) + S(e'_2) + \cdots + S(e'_P) \\ &= (h'_{11} + h'_{21} + \cdots + h'_{n1}) + (h'_{12} + h'_{22} + \cdots + h'_{n2}) + \cdots + (h'_{1P} + h'_{2P} + \cdots + h'_{nP}) \\ &= (h'_{11} + h'_{12} + \cdots + h'_{1P}) + (h'_{21} + h'_{22} + \cdots + h'_{2P}) + \cdots + (h'_{n1} + h'_{n2} + \cdots + h'_{nP}) \\ &= n \cdot q. \end{aligned}$$

Namely, S_A is a multiple of n. This completes the proof. □

In order to explicitly clarify this theorem, the following example is given.

Example 4.1. Let a soft set (F, E) with the tabular representation displayed as in Table 2, which is quoted from [42]. Suppose that

$$U = \{h_1, h_2, h_3, h_4, h_5, h_6\}, \qquad E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\},$$

where $n = 6$.

In [42], it turns out that $\{e_1, e_2, e_5, e_6, e_8, e_9\}$, $\{e_6, e_7, e_9\}$, or $\{e_1, e_4, e_6, e_8, e_9\}$ can be deleted from E. The subsets $\{e_3, e_4, e_7, e_{10}\}$, $\{e_1, e_2, e_3, e_4, e_5, e_8, e_{10}\}$ and $\{e_2, e_3, e_5, e_7, e_{10}\}$ are the normal parameter reductions of soft set. It is clear that

$$S_{\{e_6,e_7,e_9\}} = 6 = 1 \times 6 \quad \text{and} \quad S_{\{e_1,e_2,e_5,e_6,e_8,e_9\}} = S_{\{e_1,e_4,e_6,e_8,e_9\}} = 12 = 2 \times 6,$$

which are the multiples of 6, respectively.

Theorem 4.2. *For soft set (F, E), $U = \{h_1, h_2, \ldots, h_n\}$, $E = \{e_1, e_2, \ldots, e_m\}$, suppose a subset $A \subseteq E$ and its complement $\overline{A}$. Check whether $S_A = qn$, for $q = 0, 1, 2, \ldots, m$, at the same time, we can determine if $S_{\overline{A}} = pn$, for $p = 0, 1, 2, \ldots, m$, where n is the number of the universe U.*

Proof. Let S_E be the overall sum of the whole table. It is obvious that $S_A + S_{\overline{A}} = S_E$. Hence by checking S_A, then $S_{\overline{A}}$ can also be determined. □

Example 4.2. In Table 2, $n = 6$, $S_E = 26$, $S_{e_6,e_7,e_9} = 1 \times 6 = 6$. Due to

$$S_{\{e_6,e_7,e_9\}} + S_{\{e_1,e_2,e_3,e_4,e_5,e_8,e_{10}\}} = 26,$$

we can come to the conclusion that $S_{\{e_1,e_2,e_3,e_4,e_5,e_8,e_{10}\}} = 26 - 6 = 20$, which is not a multiple of n.

Theorem 4.3. *For soft set (F, E), $U = \{h_1, h_2, \ldots, h_n\}$, $E = \{e_1, e_2, \ldots, e_m\}$, and $A = \left\{e'_1, e'_2, \ldots, e'_P\right\} \subset E$. If the value $r_{e'_1} + r_{e'_2} + \cdots + r_{e'_P}$ is a nonnegative integer, then we have $S_A = qn$, for $q = 0, 1, 2, \ldots, m$, where n is the number of the universe U; vice versa, if $S_A = qn$, for $q = 0, 1, 2, \ldots, m$, then we have $r_{e'_1} + r_{e'_2} + \cdots + r_{e'_P}$ is a nonnegative integer.*

Proof. Let

$$C_E = \left\{E_{f_1}, E_{f_2}, \ldots, E_{f_s}\right\} = \left\{\{h_1, h_2, \ldots, h_i\}_{f_1}, \left\{h_{i+1}, \ldots, h_j\right\}_{f_2}, \ldots, \{h_k, \ldots, h_m\}_{f_s}\right\}$$

be a decision partition of objects in U. If the parameter e_i' is deleted from set E, then the decision partition is changed and it can be denoted as

$$C_{E-e_i'} = \left\{\{h_1, h_2, \ldots, h_i\}_{f_1}, \left\{h_{i+1}, \ldots, h_j\right\}_{f_2}, \ldots, \{h_{k'}, \ldots, h_{m'}\}_{f_s}\right\}.$$

Now we can calculate the importance degree of e_i'. Based on Definition 3.3, for the parameter e_i', if $h_{ij} = 0$, it means that there is no change, the corresponding object will be deleted; otherwise if $h_{ij} = 1$, it means that there is a change and the corresponding object will be reserved. In other words, for the parameter e_i', if the number of $h_{ij} = 1$ is x,

$$r_{e_i} = \frac{x}{|U|}, \quad \text{namely, } \sum_{i=1}^{s} \alpha_{i,e_j'} = S\left(e_j'\right).$$

Therefore,

$$\begin{aligned} r_{e_1'} + r_{e_2'} + \cdots + r_{e_p'} &= \frac{1}{|U|}\left(\sum_{i=1}^{s} \alpha_{i,e_1'} + \sum_{i=1}^{s} \alpha_{i,e_2'} + \cdots + \sum_{i=1}^{s} \alpha_{i,e_p'}\right) \\ &= \frac{1}{|U|}\left(S\left(e_1'\right) + S\left(e_2'\right) + \cdots + S\left(e_P'\right)\right) \\ &= \frac{1}{|U|} S_A \\ &= \frac{S_A}{n}. \end{aligned}$$

Therefore, if the value $r_{e_1'} + r_{e_2'} + \cdots + r_{e_p'}$ is a nonnegative integer, then we have $S_A = qn$, for $q = 0, 1, 2, \ldots, m$, where n is the number of the universe U; vice versa, if $S_A = qn$, for $q = 0, 1, 2, \ldots, m$, then we have $r_{e_1'} + r_{e_2'} + \cdots + r_{e_p'}$ is a nonnegative integer. That is, $S_A = qn$, for $q = 0, 1, 2, \ldots, m$ and $r_{e_1'} + r_{e_2'} + \cdots + r_{e_p'}$ being a nonnegative integer are equivalent. □

Example 4.3. In Table 2, we can get that $r_{e_1} = \frac{3}{6}, r_{e_2} = \frac{1}{6}, r_{e_3} = \frac{5}{6}, r_{e_4} = \frac{3}{6}, r_{e_5} = \frac{2}{6}, r_{e_6} = \frac{3}{6}, r_{e_7} = \frac{2}{6}, r_{e_8} = \frac{2}{6}, r_{e_9} = \frac{1}{6}$, and $r_{e_{10}} = \frac{4}{6}$. It is clear that

$$r_{e_1} + r_{e_2} + r_{e_5} + r_{e_6} + r_{e_8} + r_{e_9} = 2, \qquad r_{e_6} + r_{e_7} + r_{e_9} = 1,$$

accordingly,

$$S_{\{e_1,e_2,e_5,e_6,e_8,e_9\}} = 12 = 2 \times 6, \quad \text{and} \quad S_{\{e_6,e_7,e_9\}} = 6 = 1 \times 6.$$

Definition 4.4. For soft set (F, E), $U = \{h_1, h_2, \ldots, h_n\}$, and $E = \{e_1, e_2, \ldots, e_m\}$. For $e_j \in E$, if $h_{1j} = h_{2j} = \cdots = h_{nj} = 1$, we denote e_j as e_j^1.

Definition 4.5. For soft set (F, E), $U = \{h_1, h_2, \ldots, h_n\}$, and $E = \{e_1, e_2, \ldots, e_m\}$. For $e_j \in E$, if $h_{1j} = h_{2j} = \cdots = h_{nj} = 0$, we denote e_j as e_j^0.

4.2. The proposed algorithm

Based on above theorems and definitions, we give our algorithm in Fig. 2.

The algorithm of [42] and the proposed algorithm represent two different approaches to normal parameter reduction of the soft set. There are some differences between them as follows.

(1) We directly put e_j^1 and e_j^0 into the reduced parameter set, which leads to the number of subsets in the candidate parameter reduction set of the proposed algorithm being much less than that of subsets in the feasible parameter reduction set of the algorithm in [42]. Hence computation is reduced.
(2) We compute the oriented-parameter sum rather than parameter importance degree. It is evident that the oriented-parameter sum is much more easily obtained and involves much less relative computation than the parameter importance degree.
(3) If S_A is a multiple of $|U|$, we refer to A as a candidate parameter reduction in the proposed algorithm; while a subset is regarded as a feasible parameter reduction if the sum of all of parameter importance degrees in this subset is a nonnegative integer in [42].

Table 3
A soft set (F, E).

U/E	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}	e_{16}	$f(.)$
h_1	1	0	0	1	1	0	0	1	1	0	0	0	0	0	1	0	6
h_2	1	1	0	0	0	0	1	0	1	0	0	0	0	0	1	0	5
h_3	1	0	0	0	1	1	0	1	0	1	1	0	0	0	1	1	8
h_4	1	0	0	1	0	1	1	0	1	0	0	1	0	0	1	0	7
h_5	0	1	1	1	0	1	0	0	1	1	1	0	0	0	1	1	9
h_6	0	0	0	0	1	0	1	0	0	1	0	1	0	0	1	0	5
h_7	0	1	1	0	1	1	1	0	1	0	0	1	0	0	1	0	8
h_8	1	1	1	1	0	1	0	0	1	1	0	1	0	1	1	0	10
$S(e_j)$	5	4	3	4	4	5	4	2	6	4	2	4	0	1	8	2	$S_E = 58$

5. The comparison result

In this section, we compare the proposed algorithm with the algorithm of [42]. Firstly, we compare them in terms of computational complexity. And then, a comparison for capturing the normal parameter reduction is elaborated through a Boolean data set as shown in Table 3. Both algorithms are implemented in the C++ program. They are executed sequentially on an Intel Core 2 Duo CPU processer. The total main memory is 1 GB and the operating system is Windows XP Professional SP3.

5.1. The comparison of computational complexity

In this section, we compare the algorithm in [42] with our algorithm in view of computational complexity from three aspects, which are as follows:

5.1.1. The parameter importance degree vs. oriented-parameter sum

We estimate the computational complexity of the algorithm by counting the number of the basic operation. Because the basic operation perhaps varies with different implementation of the algorithm, we consider element access here as the basic operation.

The algorithm in [42].

For computing parameter importance degree $r_{e_j},\ 1 \le j \le m$, the following steps must be taken,

1. Compute $f_E(h_i) = \sum_j h_{ij},\ 1 \le i \le n, 1 \le j \le m$. Every entry will be accessed once, so the number of element access is $m \cdot n$.
2. Get $C_E = \{E_{f_1}, E_{f_2}, \ldots, E_{f_s}\}$, that is, classify objects according to $f_E(h_i)$. The column of $f_E(h_i)$ will be accessed once, so the number of element access is n.
3. Obtain decision partition deleted e_j. This step includes two sub steps:
   (1) Compute $f_{E-e_j}(h_i),\ 1 \le i \le n,\ 1 \le j \le m-1$.
   (2) Partition, i.e. get C_{E-e_j}.
   These two steps are similar to Steps 1 and 2. So the numbers of element access are $n \cdot (m-1)$ and n respectively.
4. Calculate parameter importance degree r_{e_j}. This step includes two sub steps:
   (1) Compute α_{k,e_j}. Every object h_i will be accessed once, so the number of element access is n.
   (2) Compute r_{e_j}. Every α_{k,e_j} will be accessed once. The maximum value of k is n. So the approximate number of element access is n.

From the above Steps 3 and 4, we obtain the total number of element access for computing one parameter importance degree, $n \cdot m + 2n$. Hence for m parameters, the number of element access is $m(n \cdot m + 2n) = m^2 n + 2mn$. Including Steps 1 and 2, the number of element access is $m^2 n + 2mn + mn + n = m^2 n + 3mn + n$. Taking big O notation, the complexity of computing the whole parameter importance degree is $O(m^2 n)$. Suppose $m = n$, the complexity will be $O(n^3)$.

The proposed algorithm.

For computing the oriented-parameter sum in our algorithm, we only need to access every entry once, so the number of element access is $m \cdot n$. Compared with $m^2 n + 3mn + n$ in computing the parameter importance degree, the amount of computation is greatly decreased. Taking big O notation, the complexity of computing oriented-parameter sum is $O(mn)$. Suppose $m = n$, the complexity will be $O(n^2)$.

(1) Input the soft set (F, E) and the parameter set E;
(2) Compute parameter importance degree r_{e_i}, $1 \leq i \leq m$;
(3) Find maximum subset $A = \{e_1', e_2', \cdots, e_p'\} \subset E$ in which that sum of $r_{e_i'}$, for $1 \leq i \leq p$, is nonnegative integer, then put the A into a feasible parameter reduction set;
(4) Filter in the feasible parameter reduction set, if $f_A(h_1) = f_A(h_2) = \ldots = f_A(h_n)$, then $E - A$ is the normal parameter reduction, otherwise A is deleted.
(5) Get the maximum cardinality of A in feasible parameter reduction set.
(6) Compute $E - A$ as the optimal normal parameter reduction.

Fig. 1. The algorithm of normal parameter reduction in [42].

5.1.2. Looking for feasible parameter reduction set vs. candidate parameter reduction set

For ease of description, we call the combination that consists of k' parameter columns combination-k'. Generally, we will test all of the combinations from combination-1 to combination-m. In fact, it is not necessary to test all of them. When we check the combination-k', at the same time, we can check the combination-$(m - k')$. So, we only need to test from combination-2 to combination-$(\lfloor m/2 \rfloor)$.

The algorithm of [42].

In the algorithm of [42], for subset $A = \{e_1', e_2', \ldots, e_p'\} \subset E$, it is necessary to check whether the sum of $r_{e_i'}$, for $1 \leq i \leq p$ is a nonnegative integer. At the same time, if its complement $\overline{A}$ is checked, the importance degree of all of the parameters in $\overline{A}$ should be accessed and then summed up. Consequently, the number of parameter importance degree access is $\left(C_m^1 + C_m^2 + \cdots + C_m^{\lfloor m/2 \rfloor}\right) \cdot m$.

The proposed algorithm.

However in our algorithm, according to Theorem 4.2, due to $S_A + S_{\overline{A}} = S_E$ we only need to have a basic operation of subtraction to determine if $S_{\overline{A}} = pn$ rather than access every oriented-parameter sum in $\overline{A}$. So the number of oriented-parameter sum access is $C_m^2 \cdot 3 + C_m^3 \cdot 4 + \cdots + C_m^{\lfloor m/2 \rfloor} \cdot (\lfloor m/2 \rfloor + 1)$. It is evident that our algorithm reduces computation.

5.1.3. Card (feasible parameter reduction set) vs. card (candidate parameter reduction set)

Suppose that the number of e_j^1 and e_j^0 is k. If card (candidate parameter reduction set) $= x$, it can be concluded that card(feasible parameter reduction set) $= x \cdot 2^k + k$, and then the difference between card (candidate parameter reduction set) and card (feasible parameter reduction set) is equal to $x \cdot (2^k - 1) + k$. It is very obvious that the larger k is, the larger the difference between them is.

As a result, it is found that our algorithm outperforms the former algorithm.

5.2. A comparison on capturing the normal parameter reduction in a Boolean data set

Example 5.1. Let (F, E) be a soft set with the tabular representation displayed in Table 3. Suppose that

$$U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8\},$$

and

$$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}\}.$$

5.2.1. The results from the algorithm in [42]

Step 1: Figuring out the oriented-object sum and then getting a decision partition.

$$C_E = \{\{h_8\}_{10}, \{h_5\}_9, \{h_3, h_7\}_8, \{h_4\}_7, \{h_1\}_6, \{h_2, h_6\}_5\}.$$

Step 2: Figuring out the oriented-object sum deleted e_i and the decision partition deleted e_i, see Fig. 3.

Step 3: Getting the importance degree of e_i.

$$r_{e_1} = \frac{5}{8}, \quad r_{e_2} = \frac{4}{8}, \quad r_{e_3} = \frac{3}{8}, \quad r_{e_4} = \frac{4}{8}, \quad r_{e_5} = \frac{4}{8}, \quad r_{e_6} = \frac{5}{8}$$

$$r_{e_7} = \frac{4}{8}, \quad r_{e_8} = \frac{2}{8}, \quad r_{e_9} = \frac{6}{8}, \quad r_{e_{10}} = \frac{4}{8}, \quad r_{e_{11}} = \frac{2}{8}, \quad r_{e_{12}} = \frac{4}{8}$$

$$r_{e_{13}} = \frac{0}{8}, \quad r_{e_{14}} = \frac{1}{8}, \quad r_{e_{15}} = \frac{8}{8}, \quad \text{and} \quad r_{e_{16}} = \frac{2}{8}.$$

(1) Input the soft set (F, E) and the parameter set E;
(2) If there exists e_j^1 and e_j^0, they will be put into the reduced parameter set denoted by C and a new soft set (F, E') will be established without e_j^1 and e_j^0, where $U = \{h_1, h_2, \cdots, h_n\}$, $E' = \{e_{1'}, e_{2'}, \cdots, e_{t'}\}$;
(3) For the soft set (F, E'), calculate $S(e_{j'})$ of $e_{j'}$ (that is, oriented-parameter sum), for $j' = 1', 2', \cdots, t'$;
(4) Find the subset $A \subset E'$ in which S_A is a multiple of $|U|$, then put A into a candidate parameter reduction set;
(5) Check every A in the candidate parameter reduction set, if $f_A(h_1) = f_A(h_2) = \cdots = f_A(h_n)$, it will be kept; otherwise it will be omitted;
(6) Find the maximum cardinality of A in the candidate parameter reduction set, then $E - A - C$ as the optimal normal parameter reduction.

Fig. 2. The proposed algorithm.

$$C_{E-e_1} = \{\{h_5, h_8\}_9, \{h_7\}_8, \{h_3\}_7, \{h_4\}_6, \{h_1, h_6\}_5, \{h_2\}_4\}$$
$$C_{E-e_2} = \{\{h_8\}_9, \{h_3, h_5\}_8, \{h_4, h_7\}_7, \{h_1\}_6, \{h_6\}_5, \{h_2\}_4\}$$
$$C_{E-e_3} = \{\{h_8\}_9, \{h_3, h_5\}_8, \{h_4, h_7\}_7, \{h_1\}_6, \{h_2, h_6\}_5\}$$
$$C_{E-e_4} = \{\{h_8\}_9, \{h_3, h_5, h_7\}_8, \{h_4\}_6, \{h_1, h_2, h_6\}_5\}$$
$$C_{E-e_5} = \{\{h_8\}_{10}, \{h_5\}_9, \{h_3, h_4, h_7\}_7, \{h_1, h_2\}_5, \{h_6\}_4\},$$
$$C_{E-e_6} = \{\{h_8\}_9, \{h_5\}_8, \{h_3, h_7\}_7, \{h_1, h_4\}_6, \{h_2, h_6\}_5\}$$
$$C_{E-e_7} = \{\{h_8\}_{10}, \{h_5\}_9, \{h_3\}_8, \{h_7\}_7, \{h_1, h_4\}_6, \{h_2, h_6\}_4\}$$
$$C_{E-e_8} = \{\{h_8\}_{10}, \{h_5\}_9, \{h_7\}_8, \{h_3, h_4\}_7, \{h_1, h_2, h_6\}_5\}$$
$$C_{E-e_9} = \{\{h_8\}_9, \{h_3, h_5\}_8, \{h_7\}_7, \{h_4\}_6, \{h_1, h_6\}_5, \{h_2\}_4\}$$
$$C_{E-e_{10}} = \{\{h_8\}_9, \{h_5, h_7\}_8, \{h_3, h_4\}_7, \{h_1\}_6, \{, h_2\}_5, \{h_6\}_4\}$$
$$C_{E-e_{11}} = \{\{h_8\}_{10}, \{h_5, h_7\}_8, \{h_3, h_4\}_7, \{h_1\}_6, \{h_2, h_6\}_5\}$$
$$C_{E-e_{12}} = \{\{h_5, h_8\}_9, \{h_3\}_8, \{h_7\}_7, \{h_1, h_4\}_6, \{h_2\}_5, \{h_6\}_4\}$$
$$C_{E-e_{13}} = \{\{h_8\}_{10}, \{h_5\}_9, \{h_3, h_7\}_8, \{h_4\}_7, \{h_1\}_6, \{h_2, h_6\}_5\}$$
$$C_{E-e_{14}} = \{\{h_5, h_8\}_9, \{h_3, h_7\}_8, \{h_4\}_7, \{h_1\}_6, \{h_2, h_6\}_5\}$$
$$C_{E-e_{15}} = \{\{h_8\}_9, \{h_5\}_8, \{h_3, h_7\}_7, \{h_4\}_6, \{h_1\}_5, \{h_2, h_6\}_4\}$$
$$C_{E-e_{16}} = \{\{h_8\}_{10}, \{h_5, h_7\}_8, \{h_3, h_4\}_7, \{h_1\}_6, \{h_2, h_6\}_5\},$$

Fig. 3. The decision partitions after deleting e_i.

Step 4: Finding the maximum subset $A = \{e'_1, e'_2, \ldots, e'_p\} \subset E$ in which that sum of $r_{e'_i}$, for $1 \leq i \leq p$ is a nonnegative integer. As a result, we get 8190 subsets which are put into a feasible parameter reduction set, such as

$$\{e_{15}\}, \{e_{11}, e_{12}, e_{16}\}, \{e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{15}, e_{16}\} \quad \text{and so on.}$$

Step 5: Filtering in the feasible parameter reduction set. We get

$$\{e_{13}\}, \quad \{e_{15}\}, \quad \{e_7, e_8, e_9, e_{10}\}, \quad \{e_7, e_8, e_9, e_{10}, e_{15}\}, \quad \{e_7, e_8, e_9, e_{10}, e_{13}\}, \quad \{e_{13}, e_{15}\}$$

and

$$\{e_7, e_8, e_9, e_{10}, e_{13}, e_{15}\}$$

satisfying $f_A(h_1) = f_A(h_2) = \cdots = f_A(h_n)$ and then the remainders are deleted.

Step 6: Getting the maximum cardinality of A in feasible parameter reduction sets $\{e_7, e_8, e_9, e_{10}, e_{13}, e_{15}\}$. So, the set $\{e_1, e_2, e_3, e_4, e_5, e_6, e_{11}, e_{12}, e_{14}, e_{16}\}$ is the optimal normal parameter reduction.

5.2.2. The results from the proposed algorithm

Step 1: Because there exists e_{15}^1 and e_{13}^0, they are put into the reduced parameter set denoted by C and a new soft set (F, E') is established without e_j^1 and e_j^0, where

$$U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8\}, \qquad E' = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{14}, e_{16}\}.$$

Table 4
The comparison result.

Comparison	The algorithm [42]	The proposed algorithm	Improvement%
Optimal normal parameter reduction	$\{e_1, e_2, e_3, e_4, e_5, e_6, e_{11}, e_{12}, e_{14}, e_{16}\}$	$\{e_1, e_2, e_3, e_4, e_5, e_6, e_{11}, e_{12}, e_{14}, e_{16}\}$	The same
The number of entry access	2440	128	94.75%
The number of parameter importance degree access and oriented-parameter access	The number of parameter importance degree access 627 232	The number of oriented-parameter sum access 301 314	51.96%
The number of candidate parameter reduction set	8190	2047	75%
The involved operation	Addition, set operation, classification for parameter importance degree	Only addition for oriented-parameter sum	

Step 2: Calculating the oriented-parameter sum $S\left(e'_j\right)$ of e'_j shown in Table 3.

Step 3: Finding the subset $A \subset E'$ in which S_A is a multiple of $|U| = 8$. As a result we put 2047 subsets such as

$$\{e_8, e_9, e_{10}, e_{12}\}, \quad \{e_7, e_8, e_9, e_{10}\}, \quad \{e_1, e_2, e_3, e_4, e_{10}, e_{12}\}, \quad \{e_1, e_2, e_3, e_4, e_7, e_8, e_{10}, e_{11}, e_{12}\},$$

and so on into a candidate parameter reduction set.

Step 4: Filtering in the candidate parameter reduction set. We get $\{e_7, e_8, e_9, e_{10}\}$ satisfying $f_A(h_1) = f_A(h_2) = \cdots = f_A(h_n)$ and the remainders are deleted.

Step 5: Finding the maximum cardinality of A in the candidate parameter reduction set. Finally $E - A - C = \{e_1, e_2, e_3, e_4, e_5, e_6, e_{11}, e_{12}, e_{14}, e_{16}\}$ is considered as the optimal normal parameter reduction.

We can draw conclusions from the above example:

(1) In order to obtain all the decision partitions, the data in Table 3 are accessed 17 times by means of the algorithm in [42]. However, the data in Table 3 are accessed only once for the oriented-parameter sums in the proposed algorithm. Consequently our algorithm involves much less relative computation compared with the former algorithm.
(2) Due to considering e_j^1 and e_j^0, the number of subsets in the candidate parameter reduction set of the proposed algorithm is much less than that of subsets in the feasible parameter reduction set of the algorithm in [42]. Hence computation is reduced.
(3) It is necessary to calculate the oriented-object sums, classify objects according to the oriented-object sums and then compute the importance degree in [42], whereas our algorithm only needs to calculate the oriented-parameter sums. As a result our algorithm is easier to implement and understand compared with the former algorithm.

Some details on the comparison result for this example are clearly depicted in Table 4. These data in Table 4 can be obtained by computational complexity analysis and the programming experiment.

6. Conclusion

Several algorithms exist to address the issues concerning reduction of soft sets. The most recent concept of normal parameter reduction is introduced in [42], which overcomes the problem of suboptimal choice and added parameter set of soft sets. However, the algorithm is hard to understand and involves a great amount of computation. In this paper, some new theorems are presented and proved. Based on the theorems, we propose a new efficient normal parameter reduction algorithm of soft sets, which can be carried out without parameter important degree and decision partition. As a result, it can involve less relative computation and is easier to understand and implement, compared with the algorithm of normal parameter reduction [42]. The example illustrates our contribution and shows that the proposed algorithm efficiently captures the reductions.

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