Image coding based on multiband wavelet and adaptive quad-tree partition

Bi Ning\textsuperscript{a,*,1}, Dai Qinyun\textsuperscript{a,b}, Huang Daren\textsuperscript{a}, Fang Ji\textsuperscript{b}

\textsuperscript{a}School of Information Science and Technology, Sun Yat-Sen University, Guangzhou, 510275, PR China
\textsuperscript{b}Faculty of Information Engineering, Guangdong University of Technology, Guangzhou, 510643, PR China

Received 15 August 2004; received in revised form 30 March 2005

Abstract

In this paper, a new family of multiband wavelets with a parameter $\lambda$ is introduced for image coding. In our method of image coding, subbands in the wavelet decomposition are adaptively divided into insignificant subbands and significant subbands while the latter are further partitioned by a significance benchmark and by the quad-tree partition algorithm. Our experimental results show less computational cost and better capability for our method than those based on two-band wavelets.

© 2005 Elsevier B.V. All rights reserved.

Keywords: Multiband wavelet; Adaptive QTP; Image coding

1. Introduction

The wavelet transform has been widely used in image processing and analysis, especially in image coding. For the wavelet coefficients obtained from two-band wavelet decomposition or from wavelet packet decomposition, there are two different ways of coding: intra-subband coding and inter-subbands coding. The key issue of such a coding is how to deal with the relationship among wavelet coefficients in intra-subbands and inter-subbands. For the inter-subbands coding, two typical ways are embedded zero-tree wavelet (EZW) \cite{7} and set partitioning in hierarchical trees (SPIHT) \cite{6}. For the intra-subband...
coding, quad-tree partitioning (QTP) and embedded block coding with optimized truncation (EBCOT) [9] are widely used, while the latter is adopted in JPEG2000.

In recent years, multiband wavelets [1,8] have been effectively used in many areas, such as texture segmentation for real-life image analysis [3], context-based classification in document image [4], and palmprint recognition [2]. In this paper, we introduce a new family of multiband wavelets and apply them to image coding.

2. Structure of algorithm

The structure of our adaptive algorithm is as follows: (i) the size of band $M$ (for instance $M = 4, 8$ or $16$) is chosen according to the size of the image; (ii) the parameter $\lambda$ and the significant bands among high-frequency subbands are determined by the significance rule; (iii) a significant subband is further partitioned into blocks according to QTP; and (iv) an important block is introduced and the coefficients in important blocks are quantized by bit-plane, and arithmetic coding is used to form an embedded bitstream, while unimportant blocks are coded with one-bit data only.

3. Decomposition and reconstruction of multiband wavelets

Based on multiresolution analysis, Mallat [5] introduced a pyramid algorithm for a two-band wavelet. Similarly, we have a pyramid algorithm for multiband wavelets [8]. Let $M$ be an integer greater than two, and $H_j$, $1 \leq j \leq M$ be Laurent polynomials satisfying: $H_1(1) = 1$, $H_j(1) = 0$ for all $2 \leq j \leq M$, and $\sum_{l=0}^{M-1} H_i(zW^l)H_j(z^{-1}W^{-l}) = \delta_{ij}$ for all $1 \leq i, j \leq M$ and $z \neq 0$, where $W = \exp(-2\pi i/M)$ is the usual $M$th unit root. The templates of filter $H_{ij}(x, y)$, $1 \leq i, j \leq M$, are defined by $H_{ij}(x, y) = H_i(x)H_j(y)$, $1 \leq i, j \leq M$.

By multiband wavelet decomposition, an image is divided into $M \times M$ subbands via $M \times M$ channels (see Fig. 1 for $M = 4$). In this paper, we choose $M = 4$ and the Laurent polynomials for a filter template.
as follows:

\[
\begin{align*}
H_1(z) &= \lambda_1(1 + z^7) + \lambda_4(z + z^6) + \lambda_3(z^2 + z^5) + \lambda_2(z^3 + z^4), \\
H_2(z) &= -\lambda_4(1 - z^7) - \lambda_1(z - z^6) + \lambda_2(z^2 - z^5) + \lambda_3(z^3 - z^4), \\
H_3(z) &= -\lambda_4(1 + z^7) + \lambda_1(z + z^6) + \lambda_2(z^2 + z^5) - \lambda_3(z^3 + z^4), \\
H_4(z) &= \lambda_1(1 - z^7) - \lambda_4(z - z^6) + \lambda_3(z^2 - z^5) - \lambda_2(z^3 - z^4),
\end{align*}
\]

where \( \lambda_1 = \frac{1}{4}(1 + \sqrt{2} \cos \lambda) \), \( \lambda_2 = \frac{1}{4}(1 - \sqrt{2} \cos \lambda) \), \( \lambda_3 = \frac{1}{4}(1 + \sqrt{2} \sin \lambda) \), \( \lambda_4 = \frac{1}{4}(1 - \sqrt{2} \sin \lambda) \), and the parameter \( \lambda \) varies from 0 to \( 2\pi \). We remark that the above multiband wavelet filters become usual four-band Haar wavelet filters when \( \lambda = 0.75\pi \), and multiband wavelet filters in [3] when \( \lambda = 0.8547\pi \).

In most image coding, we are required to store images with minimal storage and minimal (visual) distortion. For an image, the part with less visual distortion usually has a high frequency and contains certain directional information. So a decomposition of an image with each part having different frequency and directional information could be convenient for image coding. A multiband wavelet decomposition is such a decomposition. In particular, after taking a multiband wavelet decomposition with dilation \( M \), an image is decomposed into one subband with a lower frequency and \( M^2 - 1 \) subbands with a high frequency, and each subband reflects different frequency and directional features of the image. We introduce a family of a four-band wavelet decomposition with parameter \( \lambda \), and observe that the frequency width of each subband in the wavelet decomposition varies according to different parameters \( \lambda \). Thus, for a given image, optimization of parameter \( \lambda \) in the four-band wavelet decomposition leads to a better image coding.

4. Encoding steps

**Step 1**: Significance rule.

For a given image, we observe that the intensity contrast of texture in high-frequency subbands is not similar to each other when wavelet filters with different parameters \( \lambda \) are applied (see Fig. 2). For a better
PSNR rate, we introduce the following significance rule:

$$\sum_{i=0}^{N_X \times N_Y - 1} (A_i - \text{Min}) > k \times \text{Aver},$$  \hspace{1cm} (1)

where $A_i$ is the $i$th wavelet coefficient, $N_X$ and $N_Y$ are the width and height of the corresponding subband, Min is the minimal coefficient in that subband, $k$ is a constant (here, we use $k = 4$), and

$$\text{Aver} = \frac{\sum_{i=0}^{N_X \times N_Y - 1} A_i}{N_X \times N_Y}.$$  \hspace{1cm} (2)

A given subband is said to be significant if rule (1) holds true, and insignificant otherwise.

**Step 2: Important blocks.**

To increase the compression rate, we introduce a new concept, an important block, which is defined by the following importance rule (3)

$$\left( \frac{\sum (A_i - \bar{X})^2}{N_W \times N_H} \right)^{1/2} > g,$$  \hspace{1cm} (3)

where $A_i$ are wavelet coefficients in that block, $N_W$ and $N_H$ are the width and height of that block, $g$ is a threshold, and $\bar{X}$ is the mean of the block.

**Step 3: Adaptive subband selection and QTP algorithm.**

Performing a multiband wavelet decomposition of a given image, we obtain $M \times M$ subbands. If a subband is insignificant, we omit that subband and code it in 1 bit. If a subband is significant, besides the lowest frequency subband, we further divide it into blocks with QTP, and determine whether the resulting subblocks are important blocks or not. If a subblock is not an important subblock, we regard the coefficients of that subblock as zeros and then code them accordingly. If that subblock is an important subblock, we store that subblock and code it further.

The above adaptive algorithm by selecting significant subbands and important blocks works well in our experiment; see Figs. 3 and 4 for the encoded Barbara image.

Fig. 3. Encoding with an adaptive subband. From left to right: (a) Original Barbara image, (b)Encoded image.
Fig. 4. Encoding with the QTP algorithm and adaptive block.

Fig. 5. EZW coding with two-band wavelet transform and our adaptive encoding algorithm. From left to right for each row: (a) Original texture image taken from Brodatz Textures; (b) Encoding under five-level wavelet decomposition (0.25 bpp); (c) Encoding with our adaptive encoding algorithm.

5. Experimental results and conclusion

In our experiment, we use Barbara, Goldhill and some texture images, and compare our adaptive encoding algorithm with EZW coding under the PSNR (see Fig. 5 for the encoded texture images using EZW coding and our adaptive encoding algorithm, and Table 1 for the PSNR result).

Our experiment shows that for those images having rich textures, such as Barbara, Goldhill or texture images, our adaptive encoding algorithm based on multiband wavelet performs better than the widely used EZW method. Also, our encoding scheme has the following advantages: (i) no error stacked by a multiband wavelet transform, because we deal with data on an original image directly instead of multi-level decomposition, and hence the host image can be preserved well; (ii) less information loss; (iii) simple structure of the algorithm; and (iv) ROI (region of interest) coding and embedded coding applicable.
Table 1
PSNR comparison of EZW and adaptive encoding

<table>
<thead>
<tr>
<th></th>
<th>Encoding under adaptive block</th>
<th>Encoding under adaptive subband</th>
<th>Encoding under fixed subband</th>
<th>Encoding under 2-level EZW (0.25 bpp)</th>
<th>Encoding under 5-level EZW (0.25 bpp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barbara</td>
<td>27.735700</td>
<td>25.789779</td>
<td>24.596710</td>
<td>24.631775</td>
<td>26.634919</td>
</tr>
<tr>
<td>Goldhill</td>
<td>34.044691</td>
<td>31.243574</td>
<td>29.827851</td>
<td>28.797251</td>
<td>30.224967</td>
</tr>
</tbody>
</table>

References