Fault Prognostic Algorithm Based on Multivariate Relevance Vector Machine and Time Series Iterative Prediction

Zhang Lei a*

*Electrical Department of Shanghai Aircraft Design and Research Institute, Yunjing road, Xuhui District, Shanghai 200436, CHINA

Abstract

To cope with prediction uncertainty and multiple fault features in fault prognostic application, an algorithm based on multivariate relevance vector machine (MRVM) is presented. Under certain preconditions, fault prognostics is firstly transformed to a time series prediction problem. Based on MRVM and matrix partitioning, it extends the existing time series iterative multi-step ahead prediction to the application with multiple fault features. Key variables in iteration are computed approximately by Monte Carlo sampling which effectively solves the problem that some integrals can’t have analytical solutions and at the same time overcome the constraint of vector machine kernel selection in traditional algorithm. Prognostic outputs are of the form of probability distribution which is more suitable for prognostic application. Finally, A simulation experiment is adopted which demonstrates the effectiveness of the proposed algorithm.

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Keywords: fault prognostics; multivariate relevance vector machine; Monte Carlo sampling; matrix partitioning; fault probability; uncertainty bound

1. Introduction

Commercial Aircrafts usually consist of a large amount of functional systems or parts and the interconnection between them is very complicated. With the improvement of their function and performance, the risk of system failure becomes higher and higher which leads to the great difficulty for system maintenance. To solve this problem, Prognostics and Health Management (PHM) [1] has been

* Corresponding author. Tel.: +8621-56682722-677; fax: +8621-66502877.
E-mail address: zl666_81@126.com.
proposed. PHM emphasizes the comprehensive use of advanced sensing, computing, and artificial intelligence technology to evaluate system health state, predict system failure in advance and schedule maintenance plan accordingly based on available resources. Fault prognostics as one of the key supporting techniques of PHM has become a hot topic for PHM study.

Although great attentions have been paid to prognostic study, there are still difficulties need to be addressed such as prediction uncertainty management and problem of multiple fault features utilization. In [2] time series prediction is used, but the problem is that it doesn’t consider the multiple fault features and prediction uncertainty. Chen et al. [3] studies prognostic problem with multiple features and point out the performance of multivariate algorithm is better than that of single variable ones. But its predictive results are of the form of fixed point which isn’t fully compliant with the essence of prognostics, because prediction uncertainty is the intrinsic property associated with fault prognostics [4]. Wang and Vachtsevanos [5] consider prediction uncertainty, and the predictive results are given by certain confidence intervals. But the uncertainty arose in the predictive process itself is not mentioned, and predictive results are relatively far from their real values. Girard et al. [6] adopts time series iterative multi-step ahead prediction to do fault prognostics. The uncertainty arise in the iterative process is fully considered, and the predictive results are expressed by RUL distribution. But only single dimensional fault features are considered.

Relevance vector machine (RVM) proposed by Tipping [7] is a nonlinear sparse learning model. Combining with Bayesian inference, RVM has a good capability of generalization. Based on this, Thayananthan [8] proposes MRVM which can not only preserve the advantages of RVM but extend it to suit the multivariate application. In this paper, a new fault prognostic algorithm based on MRVM which can cope with the problems of multiple fault features utilization and prediction uncertainty management simultaneously is proposed. Fault prognostics is firstly transformed to a time series prediction problem. Based on MRVM and matrix partitioning, it extends the existing time series iterative multi-step ahead prediction to the application with multiple fault features. Key variables in iteration are computed approximately by Monte Carlo sampling which effectively solves the problem that some integrals can’t have analytical solutions and overcome the constraint of vector machine kernel selection in traditional algorithm simultaneously. Finally, a simulation experiment of three vessel water tank system is adopted, which demonstrates the feasibility of proposed algorithm.

2. Problem formulation

In real applications, historical running data can include some underlying information about system fault degradation and they usually have the form of multivariate time series. If we model the underlying fault degradation process through learning these time series, then we can predict the future trend of system fault degradation. When certain prior threshold is met, then we may predict that system is going to fail at some future time. Based on this assumption, fault prognostic problem studied here can be transformed to a multivariate time series prediction problem as follows.

Given multivariate time series of system fault features \( \{ y_k \mid k = 1, 2, \ldots, N \} \), where \( N \in \mathbb{N} \) is the length of time series. \( y_k = [y_1(k), \ldots, y_l(k)]^T \) is time series vector at time step \( k \). \( l \in \mathbb{N} \) denotes its dimensional number. According to Takens theory [9], system underlying fault degradation process can be reconstructed in a high dimensional phase space. Thus for certain number \( d \) which ensures that the embedded dimension of the reconstructed phase space \( l \times d \) is large enough, then there exist a certain function \( F : \mathbb{R}^{l \times d} \rightarrow \mathbb{R}^l \) which satisfy
\[ y_{k+1} = F(X_k) = \begin{pmatrix} y_k \\ y_{k-1} \\ \vdots \\ y_{k-d+1} \end{pmatrix} \]  

Above is the one step ahead predictive equation for time series \( y_k \). Where input \( X_k \) is the time series vector of reconstructed phase space, output \( y_{k+1} \) is the time series vector of original space. So the remaining task is to approximate one step ahead predictive equation based on MRVM and to use this model in fault prognostic application.

3. **Multivariate relevance vector machine**

MRVM model is used to approximate above one step ahead predictive equation and detailed MRVM model is described in [8]. Here we need to focus on its predictive outputs.

If the input of MRVM is a fixed point \( X^* \), the output of MRVM \( y^* \) is a random variable which follows Gaussian distribution. And its mean and variance are only decided by MRVM parameters. If the input of MRVM is an arbitrary variable with distribution \( p(X^*) \) (\( X^* \in \mathbb{R}^q \), and \( q \) is the dimensional number of variable \( X^* \)), then the output of MRVM can be expressed as a integral.

\[
p(y^*) = \int p(y^* | X^*) p(X^*) dX^*
\]  

(2)

Where \( p(y^* | X^*) \) is the output distribution of MRVM with \( X^* \) as its input. Usually this integral can not be solved analytically. A useful solution for this is the Monte Carlo sampling technique, that is to draw \( N \) independent sampling points from random variable \( X^* \), and the integral can be approximated by

\[
p(y^*) \approx \frac{1}{N} \sum_{i=1}^{N} p(y^* | X^{*i})
\]  

(3)

Where \( p(y^* | X^{*i}) \) is the output distribution of MRVM with fixed point \( X^{*i} \) as its input. Because \( p(y^* | X^{*i}) \) follows Gaussian distribution, it can be concluded from above equation that the output distribution of MRVM can be approximated by a Gaussian mixture model when the input of MRVM is an arbitrary variable.

4. **Multivariate time series iterative multi-step ahead prediction**

Once one step ahead predictive equation is approximated by MRVM, long term time series prediction can be achieved through an iterative calculation process. That is the estimated value of last time step can be used as one of the inputs for next one step ahead prediction and this operation is repeated until the desired predictive time horizon is reached. However, traditional algorithms only focus on predictive mean (single point prediction), which not only do not consider predictive uncertainty but do not take full use of RVM, because uncertainty can be well represented by random variables in RVM. In this paper, a MRVM based time series iterative multi-step ahead prediction process is proposed, which not only predictive mean but also predictive variance (represent the associated predictive uncertainty) are feed back at each
iterative step. It is firstly inspired by [6]. But to suit the multivariate application, matrix partitioning and Monte Carlo sampling technique is used to extend its application area.

The key point of proposed iterative prediction process is that all the inputs and outputs of MRVM model are considered as random variables and their probability distribution can be represented by means and variances approximately. So the main problem is how to estimate the mean and variance of each MRVM input variable for the next iteration based on the previous predictive output. According to matrix partition technique, time series state vector $y_k$ (dimension $l \times 1$), mean (dimension $l \times 1$), variance (dimension $l \times l$), and covariance between two random state vectors $\text{cov}(y_{k+1}, y_k)$ (dimension $l \times l$) can all be regarded as basic elements of certain composite matrix. Then we can have the iterative prediction process as follows.

For time step $k + h - 1$ (where $h$ is the predictive time horizon), the mean and variance of MRVM output are written as $m(X_{k+h-1})$ and $v(X_{k+h-1})$, where $X_{k+h-1}$ is the input vector of MRVM at time step $k + h - 1$. Then the distribution of MRVM input vector $X_{k+h}$ for time step $k + h$ can be written as

$$
(X_{k+h})_{l \times 1} \sim N((M_{k+h})_{l \times 1}, (S_{k+h})_{l \times l})
$$

(4)

Where $M_{k+h}$ and $S_{k+h}$ are the mean and variance of MRVM input at time step $k + h$, here function $N$ stands for Gaussian distribution. We proceed with $X_{k+h}$ as the MRVM input, then the distribution of one step ahead prediction $y_{k+h}$ at time step $k + h$ can be written as

$$
(y_{k+h})_{l \times 1} \sim N((m(X_{k+h}))_{l \times 1}, (v(X_{k+h}))_{l \times l})
$$

(5)

The iterative relationship between MRVM input mean $M_{k+h}$ and previous MRVM output can be written as

$$
(M_{k+h})_{l \times 1} = \left[ (m(X_{k+h-1}))_{l \times 1} (\bar{M}_{k+h-1})_{l \times (d-1)} \right]^{T}
$$

(6)

Where $\bar{M}_{k+h-1}$ is the residual of $M_{k+h-1}$ at time step $k + h - 1$, which excludes elements relevant to the oldest time step in $M_{k+h-1}$. The iterative relationship between MRVM input variance $S_{k+h}$ and previous MRVM output can be written as

$$
(S_{k+h})_{l \times l} = \left[ (v(X_{k+h-1}))_{l \times l} (\text{cov}(y_{k+h-1}, \bar{X}_{k+h-1}))_{l \times (l-1)} (\bar{S}_{k+h-1})_{l \times (l-1)} \right]
$$

(7)

Where $\bar{S}_{k+h-1}$ is the residual of $S_{k+h-1}$ at time step $k + h - 1$, which excludes elements relevant to the oldest time step in $S_{k+h-1}$. $\text{cov}(y_{k+h-1}, \bar{X}_{k+h-1})$ is the covariance between MRVM output $y_{k+h-1}$ and random vector $\bar{X}_{k+h-1}$. While $\bar{X}_{k+h-1}$ is the residual of $X_{k+h-1}$ which excludes elements relevant to the oldest time step in $X_{k+h-1}$. Based on matrix partition technique, $\text{cov}(y_{k+h-1}, \bar{X}_{k+h-1})$ can then be rewritten as
\[
\text{cov}(y_{k+h-1}, X_{k+h-1}) = \\
\begin{bmatrix}
\text{cov}(y_{k+h-1}, y_{k+h-2}) \\
\text{cov}(y_{k+h-1}, y_{k+h-3}) \\
\vdots \\
\text{cov}(y_{k+h-1}, y_{k+d})
\end{bmatrix}
\tag{8}
\]

For above process, mean \( m(X_{k+h-1}) \) and variance \( v(X_{k+h-1}) \) of MRVM output and covariance \( \text{cov}(y_{k+h-1}, X_{k+h-1}) \) are the three key variables needed to fulfill this iterative calculation. For the calculation of \( m(X_{k+h-1}) \) and \( v(X_{k+h-1}) \), based on Monte Carlo sampling technique, we can draw \( N \) independent sampling points from random variable \( X_{k+h-1} \). Then \( m(X_{k+h-1}) \) and \( v(X_{k+h-1}) \) can be approximated by

\[
(m(X_{k+h-1}))_{i=1} = \int y_{k+h-1} p(X_{k+h-1}) dX_{k+h-1} = \sum_{i=1}^{N} m(X_{k+h-1}^i) W_{k+h-1}^i
\]

\[
(v(X_{k+h-1}))_{i=1} = \int [(y_{k+h-1} - m(X_{k+h-1})) \times (y_{k+h-1} - m(X_{k+h-1}))^T] \times p(X_{k+h-1}) dX_{k+h-1}
= \sum_{i=1}^{N} W_{k+h-1}^i \times [v(X_{k+h-1}^i) + (m(X_{k+h-1}^i) - m(X_{k+h-1})) \times (m(X_{k+h-1}^i) - m(X_{k+h-1}))^T]
\tag{9}
\]

Where \( p(X_{k+h-1}) \) is the probability density of MRVM input. \( X_{k+h-1}^i \) is the sampling point of \( X_{k+h-1} \), \( W_{k+h-1}^i \) is the relevant weight. \( N \) is the total number of sampling points. \( m(X_{k+h-1}^i) \) and \( v(X_{k+h-1}^i) \) are the mean and variance of MRVM output with \( X_{k+h-1}^i \) as its input. \( m(X_{k+h-1}^i) \) and \( v(X_{k+h-1}^i) \) are only decided by MRVM parameters.

For the calculation of \( \text{cov}(y_{k+h-1}, X_{k+h-1}) \), we can first calculate \( \text{cov}(y_{k+h-1}, X_{k+h-1}) \) and then remove parts relevant to the oldest time step from it. \( \text{cov}(y_{k+h-1}, X_{k+h-1}) \) can be written as

\[
(\text{cov}(y_{k+h-1}, X_{k+h-1}))_{i=1} = (E(X_{k+h-1} y_{k+h-1}^T))_{i=1} - E(X_{k+h-1})_{i=1} (E(y_{k+h-1}^T))_{i=1}
\tag{10}
\]

Where according to Monte Carlo sampling technique \( E(X_{k+h-1} y_{k+h-1}^T) \), \( E(X_{k+h-1}) \) and \( E(y_{k+h-1}) \) can be approximated as

\[
(E(X_{k+h-1} y_{k+h-1}^T))_{i=1} = \int X_{k+h-1} y_{k+h-1}^T p(X_{k+h-1}) dX_{k+h-1} = \sum_{i=1}^{N} X_{k+h-1}^i m(X_{k+h-1}) W_{k+h-1}^i
\]

\[
(E(X_{k+h-1}))_{i=1} = \int X_{k+h-1} p(X_{k+h-1}) dX_{k+h-1} = \sum_{i=1}^{N} X_{k+h-1}^i W_{k+h-1}^i
\]

\[
E(y_{k+h-1})_{i=1} = \int y_{k+h-1} p(X_{k+h-1}) dX_{k+h-1} = \sum_{i=1}^{N} m(X_{k+h-1}^i) W_{k+h-1}^i
\tag{11}
\]

Compared to traditional algorithm in [6] whose kernel must be Gaussian in order to calculate certain integrals analytically which are approximated here by Monte Carlo sampling, above iterative process don’t need to specify what kind of kernel function to be used which can greatly extend its practical application area.
5. Prognostic Decision Making

Based on above predictive distribution results of system fault features and combined with Monte Carlo sampling technique and some prior knowledge, prognostic indexes used to assist prognostic decision making can be induced such as predictive uncertainty bound and system fault probability.

Assume we can draw \( N \) independent sampling points from the predictive distribution of system fault feature, i.e. \( \{y^i_k\}_{i=1}^{N} \), where marker \( k \) denotes the relevant time step of predictive results. Then the predictive system fault probability at time step \( k \) can be written as

\[
Pr_{f} \approx \sum_{i=1}^{N} \frac{1}{N} \times I^i_k,
\]

where

\[
I^i_k = \begin{cases} 
1 & y^i_k \in \zeta \\
0 & y^i_k \not\in \zeta 
\end{cases}
\]

(12)

Where \( \zeta \) is a preset fault space (prior knowledge about system fault). And predictive uncertainty bound (upper and lower bound) can be represented by the max and min value of there sampling points at each time steps.

6. Simulation Experiment

DTS200, a three-vessel water tank system produced by Amira Automation Corporation in Germany, is studied here to verify the proposed fault prognostic algorithm. Detailed information about this model can be found in [10] and three water levels are chosen as the major fault features. The whole process of the simulation experiment is as follows. At the initial stage, system is at normal state and three water levels are nearly unchanged. Once the system start to fail, the water level \( h_i \) which affected by the degradation of \( a z \) will change gradually. Finally, when certain \( h_i \) exceed the preset threshold, water tank system is considered to be failure.

In this simulation, the proposed algorithm is compared with three traditional algorithms. Single point prediction which measures the similarity between predictive expectation and the real value of feature time series are compared to demonstrate the performance of each algorithm concerned. The prognostic criteria are mean absolute percentage error (MAPE), root man squared percentage error (RMSPE) in [11]. The algorithms compared here are 1) LSSVM; 2) Single dimensional RVM in [12], named RVM1; 3) Single dimensional RVM in [6], named RVM2; 4) Proposed MRVM with Gaussian kernel, named MRVM(G); 5) Proposed MRVM with polynomial kernel, named MRVM(P).

As feature time series in the simulation are three dimensional vectors, for all above single dimensional algorithms, they need to construct predictive model for each dimension of feature series. Simulation configuration is as follows: processor speed 2.4GHz; RAM 2GB; Matlab/Simulink software environment. All these algorithms are tested repeatedly and the mean values of criterion are listed in the following table.

As we can see in the table, the performance of proposed algorithm is much better than traditional ones. And at the same time, the performance of MRVM with different kernel function (Gaussian and polynomial) is nearly the same which demonstrates that through Monte Carlo sampling, the proposed algorithm can overcome the constraint of kernel selection problem in RVM2.

Fig 1 is the predictive results of three water levels at time step 140. Fig 2 is its relevant fault probability derived from predictive results of each water level. As shown in Fig 1, the predictive results of three water levels are consistent with their real future trend and the predictive uncertainty bound can cover their real future trends. The predictive uncertainty bound area remains relatively small which means proposed algorithm can achieve a relatively low predictive uncertainty. In Fig 2, fault probability derived
from h1 and h3 are nearly zero in the whole simulation. While fault probability derived from h2 can well represent the fault degradation of water tank system because water level h2 will exceed preset threshold in the simulation at certain time step which means system is failure.

Table 1. Comparison of different algorithms’ predictive results

<table>
<thead>
<tr>
<th></th>
<th>MAPE</th>
<th>RMSPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSSVM</td>
<td>5.100</td>
<td>5.582</td>
</tr>
<tr>
<td>RVM1</td>
<td>3.654</td>
<td>4.819</td>
</tr>
<tr>
<td>RVM2</td>
<td>3.231</td>
<td>3.662</td>
</tr>
<tr>
<td>MRVM(G)</td>
<td>1.767</td>
<td>2.344</td>
</tr>
<tr>
<td>MRVM(P)</td>
<td>1.693</td>
<td>2.234</td>
</tr>
</tbody>
</table>

Fig. 1. (a) h1 predictive results at time 140; (b) h2 predictive results at time 140; (c) h3 predictive results at time 140

Fig. 2. (a) fault probability derived from predictive results of h1; (b) fault probability derived from predictive results of h2; (c) fault probability derived from predictive results of h3
Fig 3 is a part of predictive results of system fault probability. We can see that as current time step is getting closer to the point when system failure occurs, the fault probability predictive results becomes more and more accurate. This is consistent with the results of theoretical analysis.

![Fig. 3. part of predictive results of system fault probability](image)

7. Conclusion

To cope with prediction uncertainty and multiple fault features in fault prognostic applications, a new fault prognostic algorithm based on MRVM is proposed. Uncertainty management is taken full consideration in the overall prognostic process by adopting Monte Carlo sampling and matrix partitioning technique. Prognostic outputs have the form of probability distribution which is more suitable for prognostic application. The major creative points are:

1) Under certain preconditions, fault prognostics is transformed to a multivariate time series prediction problem. MRVM is firstly introduced to fault prognostic application, and a new algorithm based on MRVM is proposed.

2) Through matrix partitioning and Monte Carlo sampling, the algorithms extend the time series iterative multi-step ahead prediction to make it suitable for fault prognostic application with multiple fault features.

3) Some key variables in time series iterative prediction are computed approximately by Monte Carlo sampling which effectively solves the problem that certain integrals can’t have analytical solutions and at the same time overcome the constraint of kernel selection problem in traditional algorithm.

References


