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Flavour-singlet g_A and the QCD sum rule incorporating instanton effects

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Abstract

We derive a QCD sum rule for the flavour-singlet axial coupling constant $g_A^{(0)}$ from a two-point correlation function of flavour-singlet axial vector currents in a one-nucleon state. In evaluating the correlation function by an operator product expansion we take into account the terms up to dimension 6. This correlation function receives an additional two-loop diagram which comes from an (anti-)instanton. If we do not include it, $g_A^{(0)}$ is estimated to be 0.8. However, the additional diagram due to instantons contributes negatively and reduces $g_A^{(0)}$ towards the experimental value.

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Axial coupling constants are defined by nucleon matrix elements of axial currents at zero momentum transfer. Since an axial current, $\bar{q}(x)\gamma_\mu\gamma_5 q(x)$, is a spin operator, the flavour-singlet axial coupling constant $g_A^{(0)}$ represents the fraction of the nucleon spin carried by quarks. In the naive parton model, $g_A^{(0)}$ is expected to be close to 1. However, an unexpected small value of $g_A^{(0)}$ was found from the EMC experiment, which implies the quarks contribute only a small fraction to the proton's spin. This has led to the so-called "spin crisis" and raised a number of questions of understanding the dynamics of the proton spin [1]. A number of subsequent experiments have been performed. The results are in the range $g_A^{(0)} = (0.28–0.41)$, see [2] for a recent review.

The investigations of $g_A^{(0)}$ by QCD sum rules have been done so far by the authors in Refs. [3,4]. Ioffe and Oganesian [3] derived a QCD sum rule for $g_A^{(0)}$ by considering a two-point correlation function of nucleon inter-

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polating fields in an external flavour-singlet axial-vector field. In their sum rule, $g_A^{(0)}$ is expressed by expectation values of QCD composite operators induced by the external field. An important contribution comes from dimension 3 term in the operator product expansion, which is related to the derivative of QCD topological susceptibility $\chi'(0)$. They found the lower limit of $g_A^{(0)}$ and $\chi'(0)$ from the self consistency of the sum rule: $g_A^{(0)} \geqslant 0.05$, $\chi'(0) \geqslant 1.6 \times 10^{-3}\ \mathrm{GeV}^2$.

The authors in Ref. [4] considered a three-point function of nucleon interpolating fields and the divergence of a flavour-singlet axial-vector current. They took into account chiral anomaly by using the anomaly relation from the very beginning. The form factor, $g_A^{(0)}(q^2)$, is related to the vacuum condensates of the quark–gluon composite operators through a double dispersion relation. To know $g_A^{(0)}(q^2)$ at $q^2 = 0$ one must evaluate the correlation function at zero momentum. Although the method to evaluate it is known [5], it involves large uncertainty.

Recently, we have proposed a new method to construct QCD sum rules for axial coupling constants from two-point correlation functions of axial-vector currents in a one-nucleon state [6]. With the method, the axial coupling constants are expressed in terms of the π–N and K–N sigma-terms and the moments of parton distributions. We have seen good agreement with experiment for the non-singlet constants, $g_A^{(3)}$ and $g_A^{(8)}$.

In this Letter, we extend the previous work to the case for $g_A^{(0)}$. For the calculation of $g_A^{(0)}$ we need to fully take into account the chiral anomaly. Since the origin of the chiral anomaly is considered to be instantons, one might suspect the anomalous suppression of $g_A^{(0)}$ is somehow related to instantons. We therefore evaluate their effects on $g_A^{(0)}$.

Following Ref. [6], we consider a correlation function of flavour-singlet axial-vector currents in a one-nucleon state:

$$\varPi^{(0)}_{\mu\nu}(q;P) = i\int d^4x\, \mathrm{e}^{iqx}\bigl\langle T\bigl[j^{(0)}_{\mu5}(x), j^{(0)}_{\nu5}(0)\bigr]\bigr\rangle_N, \tag{1}$$

where $q^\mu \equiv (\omega, \mathbf{q})$ and the nucleon matrix element is defined by $\langle\cdots\rangle_N \equiv (1/2)\sum_S[\langle N(PS)|\cdots|N(PS)\rangle - \langle\cdots\rangle_0\langle N(PS)|N(PS)\rangle]$, where $P^\mu \equiv (E, \mathbf{P})$ is the nucleon momentum ($P^2 = M^2$, M is the mass), S the nucleon spin, $\langle\cdots\rangle_0 \equiv \langle 0|\cdots|0\rangle$, and the one-nucleon state is normalized as $\langle N(PS)|N(P'S')\rangle = (2\pi)^3\delta^3(\mathbf{P}-\mathbf{P}')\delta_{SS'}$. The flavour-singlet axial-vector current is defined as

$$j^{(0)}_{\mu5}(x) = \bar u(x)\gamma_\mu\gamma_5 u(x) + \bar d(x)\gamma_\mu\gamma_5 d(x) + \bar s(x)\gamma_\mu\gamma_5 s(x), \tag{2}$$

where u, d and s are the up, down and strange quark fields, respectively.

We write a Lehmann representation of Eq. (1):

$$\varPi^{(0)}_{\mu\nu}(\omega, \mathbf{q}; P) = \int_{-\infty}^{\infty} d\omega' \frac{\rho^{(0)}_{\mu\nu}(\omega', \mathbf{q}; P)}{\omega - \omega'}, \tag{3}$$

where $\rho_{\mu\nu}$ is the spectral function. We derive a Borel sum rule from Eq. (3) for the even function part of Eq. (1) in ω, $\varPi^{(0)}_{\mu\nu}(\omega^2, \mathbf{q}; P)_{\mathrm{even}}$, as [6]

$$\hat B\bigl[\varPi^{(0)}_{\mu\nu}\bigl(\omega^2, \mathbf{q}; P\bigr)_{\mathrm{even}}\bigr] = -\int_{-\infty}^{\infty} d\omega'\, \omega' \exp\bigl(-\omega'^2/s\bigr)\rho^{(0)}_{\mu\nu}(\omega', \mathbf{q}; P), \tag{4}$$

where s denotes the square of the Borel mass and $\hat B$ the Borel transformation with respect to ω^2. In Eq. (4) the left-hand side is evaluated theoretically, which give rise to a Borel transformed QCD sum rule.

Let us now consider the physical content of the spectral function with the insertion of intermediate states between the currents. Here the lowest one is a one-nucleon state. The continuum state consists of η'-nucleon states, excited nucleon states and so on. There is an energy gap between the nucleon pole and the continuum threshold. The contribution of the one-nucleon state to the spectral function is expressed in terms of axial coupling constants, because the nucleon matrix element of $j^{(0)}_{\mu 5}$ is written as $\langle N(PS)|j^{(0)}_{\mu 5}(0)|N(P'S')\rangle = \bar{u}(PS)[g^{(0)}_A(q^2)\gamma_\mu\gamma_5 + h^{(0)}_A(q^2)q_\mu\gamma_5]u(P'S')$, where $u(PS)$ is a Dirac spinor and $q = P' - P$ [6]. The contribution of the continuum state becomes small in the Borel sum rule, since it is exponentially suppressed compared to that of the one-nucleon state because of the energy gap. Therefore, it is allowed to use a rough model of the continuum: the form of the continuum is approximated by the step function with the coefficient being the imaginary part of the asymptotic form of the correlation function in the OPE [8]. In the present case, however, the continuum contribution to the spectral function is absent within the approximation, because the perturbative part is subtracted from the definition of Eq. (1). This means that the continuum contribution may be very small at least in the high energy region. We therefore neglect the continuum contribution in this work.

Hereafter we consider the correlation function in the rest frame of the initial and final nucleon states and contract the Lorentz indices of the currents. Expanding the right-hand side of Eq. (4) in powers of $|\mathbf{q}|^2$, we find the coefficient of $|\mathbf{q}|^2$ is proportional to $|g^{(0)}_A(0)|^2$ [6]. $h^{(0)}_A(0)$ contributes to higher order terms since $h^{(0)}_A(q^2)$ has no singularity at $q^2 = 0$. From the first derivative of Eq. (4) with respect to $|\mathbf{q}|^2$ we obtain the desired QCD sum rule at $|\mathbf{q}|^2 = 0$:

$$\left.\frac{\partial \hat{B}[\Pi^{(0)}(\omega^2, \mathbf{q})_{\text{even}}]}{\partial |\mathbf{q}|^2}\right|_{|\mathbf{q}|^2=0} = -\frac{3}{M}\left|g^{(0)}_A\right|^2, \tag{5}$$

where $\Pi(\omega, \mathbf{q}) = \Pi_\mu{}^\mu(\omega, \mathbf{q}; M, \mathbf{0})$.

Let us now turn to the evaluation of $\Pi^{(0)}(q)$. $\Pi^{(0)}(q)$ consists of the following two terms:

$$\Pi^{(0)}(q) = \sum_{q=u,d,s} C(q)_q + \sum_{q,q'=u,d,s} D(q)_{qq'}, \tag{6}$$

where $C(q)_q$ is a connected or "one-loop" term which is given by

$$C(q)_q = -i\int d^4(x-y)\,\mathrm{e}^{iq(x-y)}\big\langle \mathrm{Tr}\big\{T\big[q(y)\bar{q}(x)\big]\gamma_\mu\gamma_5 T\big[q(x)\bar{q}(y)\big]\gamma^\mu\gamma_5\big\}\big\rangle_N \tag{7}$$

and $D(q)_{qq'}$ a disconnected or "two-loop" term:

$$D(q)_{qq'} = i\int d^4(x-y)\,\mathrm{e}^{iq(x-y)}\big\langle \mathrm{Tr}\big\{\gamma_\mu\gamma_5 T\big[q(x)\bar{q}(x)\big]\big\}\,\mathrm{Tr}\big\{\gamma^\mu\gamma_5 T\big[q'(y)\bar{q}'(y)\big]\big\}\big\rangle_N. \tag{8}$$

Note here that *only the correlator of flavour singlet currents receives the contributions of the "two-loop" terms*. Indeed, for the correlation function of iso-vector currents, $j^{(3)}_{\mu 5} = (1/2)(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$, "two-loop" terms cancel with each other if we neglect the differences of quark masses:

$$\begin{aligned}\Pi^{(3)}(q) &\equiv i\int d^4x\,\mathrm{e}^{iqx}\big\langle T\big[j^{(3)}_{\mu 5}(x), j^{(3)}_{\nu 5}(0)\big]\big\rangle_N \\ &= (1/2)^2\big[C_u(q) + C_d(q) + D(q)_{uu} - 2D(q)_{ud} + D(q)_{dd}\big] \\ &= (1/2)^2\big[C_u(q) + C_d(q)\big].\end{aligned} \tag{9}$$

Similarly, for the 8th component of flavour-octet currents, $j^{(8)}_{\mu 5} = (1/2\sqrt{3})(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s)$,

$$\begin{aligned}\Pi^{(8)}(q) &\equiv i\int d^4x\,\mathrm{e}^{iqx}\big\langle T\big[j^{(8)}_{\mu 5}(x), j^{(8)}_{\nu 5}(0)\big]\big\rangle_N \\ &= (1/2\sqrt{3})^2\big[C_u(q) + C_d(q) + 4C_s(q)\end{aligned}$$

$$+ D(q)_{uu} + D(q)_{dd} + 4D(q)_{ss} + 2D(q)_{ud} - 4D(q)_{us} - 4D(q)_{ds}\big]$$
$$= (1/2\sqrt{3})^2\big[C_u(q) + C_d(q) + 4C_s(q)\big]. \tag{10}$$

We evaluate Eqs. (7) and (8) by a standard operator product expansion (OPE). Let us first consider the "one-loop" terms. In the OPE, operators of the leading terms are of dimension 4. We take into account the terms up to dimension 6. The result is in the following:

$$\begin{aligned} C(q)_q = {} & \frac{10}{q^2} m_q \langle \bar{q}q\rangle_N - \frac{1}{2q^2}\left\langle \frac{\alpha_s}{\pi} G^2\right\rangle_N - \frac{8q^\mu q^\nu}{q^4} i\langle \bar{q}\mathcal{S}\gamma_\mu D_\nu q\rangle_N \\ & - \frac{22\pi\alpha_s}{3q^4}\big\langle \bar{q}\gamma^\mu\lambda^a q\big(\bar{u}\gamma_\mu\lambda^a u + \bar{d}\gamma_\mu\lambda^a d + \bar{s}\gamma_\mu\lambda^a s\big)\big\rangle_N \\ & + \frac{10\pi\alpha_s q^\mu q^\nu}{q^6}\big\langle \mathcal{S}\big(\bar{q}\gamma_\mu\lambda^a q\big)\big(\bar{u}\gamma_\nu\lambda^a u + \bar{d}\gamma_\nu\lambda^a d + \bar{s}\gamma_\nu\lambda^a s\big)\big\rangle_N \\ & + \frac{32 q^\mu q^\nu q^\lambda q^\sigma}{q^8} i\langle \bar{q}\mathcal{S}\gamma_\mu D_\nu D_\lambda D_\sigma q\rangle_N, \end{aligned} \tag{11}$$

where D_μ's are covariant derivatives, $G^2 \equiv G^a_{\mu\nu}G^{a\mu\nu}$, and $\mathcal{S}$ denotes a symbol which makes the operators symmetric and traceless with respect to the Lorentz indices.

We now discuss about the nucleon matrix elements in Eq. (11). It is known well that $m_q\langle \bar{q}q\rangle_N$ is related to the π–N or K–N sigma-term as $(m_u + m_d)(\langle \bar{u}u\rangle_N + \langle \bar{d}d\rangle_N) = 2\Sigma_{\pi N}$ and $(m_s + m_u)(\langle \bar{s}s\rangle_N + \langle \bar{u}u\rangle_N) = 2\Sigma_{KN}$. $\langle(\alpha_s/\pi)G^2\rangle_N$ is expressed by the nucleon mass and $m_q\langle \bar{q}q\rangle_N$ through the QCD trace anomaly: $\langle(\alpha_s/\pi)G^2\rangle_N = -(8/9)(M - \sum_{q=u,d,s} m_q\langle \bar{q}q\rangle_N)$. The matrix elements which contain covariant derivatives are related to the parton distributions as $\langle \mathcal{S}\bar{q}\gamma_{\mu_1}D_{\mu_2}\cdots D_{\mu_n}q(\mu^2)\rangle_N = (-i)^{n-1}A^q_n(\mu^2)T_{\mu_1\mu_2\ldots\mu_n}$, where $A_n(\mu^2)$ is the nth moment of the parton distributions at scale μ^2, and $T_{\mu_1\mu_2\ldots\mu_n} = \mathcal{S}[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}]$. For the matrix elements of four quark operators, we apply the factorization hypothesis [8]: the matrix elements are factorized by assuming that the contribution from one nucleon state dominates in the intermediate states: $\langle \mathcal{O}_1\mathcal{O}_2\rangle_N \approx \langle \mathcal{O}_1\rangle_N\langle \mathcal{O}_2\rangle_0 + \langle \mathcal{O}_1\rangle_0\langle \mathcal{O}_2\rangle_N$. We apply this hypothesis to the following type of the nucleon matrix elements, which appear in Eq. (11): $\langle \bar{q}_f\gamma_\mu\lambda^a q_f\bar{q}_{f'}\gamma_\nu\lambda^a q_{f'}\rangle_N = -(8/9)g_{\mu\nu}\langle \bar{q}_f q_f\rangle_0\langle \bar{q}_f q_f\rangle_N\delta_{f,f'}$, where f and f' are flavor indices.

Let us next consider the "two-loop term", Eq. (8). There is no contribution from this term as long as we do not account for higher-dimensional terms (larger than dimension 6) in the OPE and perturbative corrections. However, "two-loop term" receives a contribution arising from instantons. We evaluate it under the "dilute instanton gas approximation" (DIGA) [12]. Namely we assume there exists only one instanton or anti-instanton in vacuum. We have two reasons why this approximation is expected to be valid. The first is that since we use the framework of QCD sum rules we are interested only in the short distance behavior of the correlation function. The second is that instantons in vacuum is sufficiently dilute [12]. Indeed, it is known that the density of instantons is about 1 fm^{-4} and the critical size of an (anti-)instanton ρ_c is $\rho_c \simeq 0.3$ fm. The value of ρ_c is significantly smaller than the typical separation between instantons.

In order to evaluate Eq. (8), we first consider the correlation function in nuclear matter with its baryon number density being ρ_B. Then Eq. (8) is obtained as the first derivative of the correlator in nuclear matter with respect to ρ_B, because, in general, expanding a vacuum expectation value of an operator $\mathcal{O}$ at finite baryon number density in powers of ρ_B the coefficient of the linear term is nothing but the nucleon matrix element: $\langle \mathcal{O}\rangle_{\rho_B} = \langle \mathcal{O}\rangle_0 + \rho_B\langle \mathcal{O}\rangle_N + \cdots$. Thus Eq. (8) can be written as

$$D(Q)_{qq'} = \left[\frac{\partial}{\partial\rho_B}\int d^4(x-y)\,\mathrm{e}^{iQ(x-y)}\big\langle \mathrm{Tr}\big\{\gamma_\mu\gamma_5 T\big[q(x)\bar{q}(x)\big]\big\}\,\mathrm{Tr}\big\{\gamma^\mu\gamma_5 T\big[q'(y)\bar{q}'(y)\big]\big\}\big\rangle_{\rho_B}\right]_{\rho_B=0}, \tag{12}$$

where Q is an Euclidean momentum defined by $Q^2 = -q^2$ and $\langle\cdots\rangle$ means that averaging is performed over all gauge configurations with the weight function $\exp(-S)$, where S is the Euclidean action. Under DIGA Eq. (12)

becomes [15]

$$D(Q)_{qq'} = \int d\rho \left[\frac{\partial}{\partial \rho_B} n(\rho, \rho_B) \right]_{\rho_B=0} \int d^4x\, e^{iQx}\, \mathrm{Tr}\big[\gamma_\mu \gamma_5 S_q(x,x)\big] \int d^4y\, e^{-iQy}\, \mathrm{Tr}\big[\gamma_\nu \gamma_5 S_{q'}(y,y)\big], \tag{13}$$

where $S_q(x,y)$ is a quark (q) propagator in the field of an (anti-)instanton. We performed integrations over the center of an (anti-)instanton z and instanton size ρ with a weight $n(\rho,\rho_B)$. $n(\rho,\rho_B)$ is the tunneling rate at finite baryon number density.

$S_q(x,y)$ is expressed as follows:

$$S_q(x,y) = \sum_\lambda \frac{\psi_\lambda(x)\psi_\lambda^\dagger(y)}{\lambda + i m_q}, \tag{14}$$

where $\psi_\lambda(x)$ is an eigen function of Dirac operator with the eigen value λ: $\not{D}\psi_\lambda(x) = \lambda\psi_\lambda(x)$. Then the dominant contribution comes from the zero-mode, $\psi_0(x)$. In Eq. (13), however, quarks do not propagate in zero-mode states. The reason is very simple. A zero-mode changes its chirality in passing through an instanton (see Fig. 1(a)). On the other hand, quarks created by an axial current have same chirality (Fig. 1(b)). So zero-modes are not allowed in Eq. (13) and only non-zero-modes contribute. Non-zero-mode propagator $S_q^{\mathrm{NZM}}(x,y)$, in which all non-zero-modes are summed up,

$$S_q^{\mathrm{NZM}}(x,y) \equiv \sum_{\lambda\neq 0} \frac{\psi_\lambda(x)\psi_\lambda^\dagger(y)}{\lambda + i m_q}, \tag{15}$$

satisfies the equation

$$(\not{D} + i m_q) S_q^{\mathrm{NZM}}(x,y) = \delta^{(4)}(x-y) - \psi_0(x)\psi_0^\dagger(y). \tag{16}$$

Subtracting the zero-mode contribution in the right-hand side, the remaining $S_q^{\mathrm{NZM}}(x,y)$ is ensured to be orthogonal to the zero-mode [16]. The solution of this equation, in general, has the following form [16]:

$$S_q^{\mathrm{NZM}}(x,y) = (\overrightarrow{\not{D}}_x - i m_q)\Delta(x,y)\frac{1+\gamma_5}{2} + \Delta(x,y)\overleftarrow{\not{D}}_y \frac{1-\gamma_5}{2}, \tag{17}$$

where $\overrightarrow{D}_\mu$ and $\overleftarrow{D}_\mu$ are covariant derivatives: $\overrightarrow{D}_\mu = \overrightarrow{\partial}_\mu - i\frac{\tau^a}{2}A_\mu^a$ and $\overleftarrow{D}_\mu = -\overleftarrow{\partial}_\mu - i\frac{\tau^a}{2}A_\mu^a$, with τ^a's are Pauli matrices and A_μ^a being an (anti-)instanton solution:

$$A_\mu^a(x) = 2\frac{x_\nu}{x^2}\frac{\bar\eta_{a\mu\nu}\rho^2}{x^2+\rho^2}. \tag{18}$$

Eq. (18) is the solution in the singular gauge. Hereafter we work in this gauge. An anti-instanton solution is obtained by replacing $\bar\eta_{a\mu\nu}$ to $\eta_{a\mu\nu}$. $\eta_{a\mu\nu}$ and $\bar\eta_{a\mu\nu}$ are the 't Hooft symbols [12]. $\Delta(x,y)$ is the propagator of a scalar field which satisfies the equation

$$\big(-D^2 + m_q^2\big)\Delta(x,y) = \delta^{(4)}(x-y). \tag{19}$$

It is known that this equation is solved by

$$\Delta(x,y) = \frac{1}{4\pi^2(x-y)^2}\left(1+\frac{\rho^2}{x^2}\right)^{-1/2}\left(1+\frac{\rho^2}{y^2}\right)^{-1/2}\left(1+\rho^2\frac{\tau^-\cdot x\,\tau^+\cdot y}{x^2y^2}\right), \tag{20}$$

where $\tau_\mu^\pm \equiv (\vec\tau, \mp i)$ [16,17]. Here we have neglected $\mathcal{O}(m_q^2)$ terms. The propagator in the field of an anti-instanton is obtained by interchanging τ^+ and τ^-.

The calculation of $\mathrm{Tr}[\gamma_\mu\gamma_5 S_q^{\mathrm{NZM}}(x,x)]$ in Eq. (13) needs some care, since $S_q^{\mathrm{NZM}}(x,y)$ is not well defined at $x\to y$. This is because $S_q^{\mathrm{NZM}}(x,y)$ at $x\neq y$ is not gauge invariant. The trace should be defined as limit of an gauge invariant expression in which a path ordered product is inserted:

$$\mathrm{Tr}\big[\gamma_\mu\gamma_5 S_q^{\mathrm{NZM}}(x,x)\big]=\lim_{\epsilon\to 0}\mathrm{Tr}\left\{\gamma_\mu\gamma_5 S_q^{\mathrm{NZM}}(x-\epsilon/2,x+\epsilon/2)\exp\left[i\int_{x-\epsilon/2}^{x+\epsilon/2}\frac{\tau^a}{2}A_\nu^a(z)\,dz_\nu\right]\right\}, \tag{21}$$

where the right-hand side must be averaged over all the direction of the four vector ϵ_μ. As a result we obtain

$$\mathrm{Tr}\big[\gamma_\mu\gamma_5 S_q^{\mathrm{NZM}}(x,x)\big]=\frac{1}{4\pi^2}\frac{\partial}{\partial x_\mu}\frac{2\rho^2}{(\rho^2+x^2)^2}. \tag{22}$$

The Fourier transform of this equation is given by

$$\int d^4x\, \mathrm{e}^{iQx}\,\mathrm{Tr}\big[\gamma_\mu\gamma_5 S(x,x)\big]=-i\rho^2 Q_\mu K_0(Q\rho). \tag{23}$$

Here $K_0(z)$ is an 0th modified Bessel function. Then Eq. (13) reads

$$D_{qq'}(Q)=\int d\rho\left[\frac{\partial}{\partial\rho_B}n(\rho,\rho_B)\right]_{\rho_B=0}Q^2\rho^4K_0(Q\rho)^2. \tag{24}$$

An important quantity in Eq. (24) is the tunneling rate, $n(\rho,\rho_B)$. In normal vacuum, where vacuum condensation does not exist, the tunneling rate at one-loop was first given in Ref. [13]. After the pioneering work, it is now available in two-loop renormalization group invariant form [14],

$$n(\rho)=n_0(\rho)\prod_{q=u,d,s}(m_q\rho)(\rho\mu_0)^{n_f\gamma_0\frac{\alpha_{\overline{\mathrm{MS}}}(\mu_0)}{4\pi}}, \tag{25}$$

where $n_0(\rho)$ is that for quarkless theory:

$$n_0(\rho)=\frac{d_{\overline{\mathrm{MS}}}}{\rho^5}\left(\frac{2\pi}{\alpha_{\overline{\mathrm{MS}}}(\mu_0)}\right)^{2N_c}\exp\left(-\frac{2\pi}{\alpha_{\overline{\mathrm{MS}}}(\mu_0)}\right)(\rho\mu_0)^{\beta_0+(\beta_1-4N_c\beta_0)\frac{\alpha_{\overline{\mathrm{MS}}}(\mu_0)}{4\pi}}, \tag{26}$$

$$d_{\overline{\mathrm{MS}}}=\frac{2\mathrm{e}^{5/6}}{\pi^2(N_c-1)!(N_c-2)!}\exp(-1.511374N_c+0.291746n_f), \tag{27}$$

$$\beta_0=\frac{11}{3}N_c-\frac{2}{3}n_f, \tag{28}$$

$$\beta_1=\frac{34}{3}N_c^2-\left(\frac{13}{3}N_c-\frac{1}{N_c}\right)n_f, \tag{29}$$

$$\gamma_0=3\frac{N_c^2-1}{N_c}. \tag{30}$$

In Eqs. (25)–(30), μ_0 is some arbitrary normalization point, $N_c=3$ and $n_f=3$ are the numbers of color and flavor.

In physical vacuum, where vacuum condensation exists, according to Shifman et al. [18], the current quark mass in Eq. (25) is substituted by a dynamical one. Shifman et al. considered the tunneling rate for small size instantons. Then it can be expanded in powers of ρ. The coefficient of each term of the expansion should be a quantity characterizing the vacuum structure. They found that the tunneling rate is written as a vacuum expectation value of an "effective Lagrangian", which has form analogous to a standard operator product expansion. The Lagrangian is as follows:

$$\begin{aligned}\Delta\mathcal{L} = {} & n_0(\rho)(\rho\mu_0)^{n_f\gamma_0\frac{\alpha_{\overline{\mathrm{MS}}}(\mu_0)}{4\pi}}\Bigg\{\prod_{q=u,d,s}\left(m_q\rho-\frac{4}{3}\pi^2\rho^3\bar{q}\Gamma q\right)\\ & +\frac{3}{32}\left(\frac{4}{3}\pi^2\rho^3\right)^2\left[\left(\bar{u}\Gamma^a u\bar{d}\Gamma^a d-\frac{4}{3}\bar{u}\Gamma^a_{\mu\nu}u\bar{d}\Gamma^a_{\mu\nu}d\right)\left(m_s\rho-\frac{4}{3}\pi^2\rho^3\bar{s}\Gamma s\right)\right.\\ & \left.+\frac{9}{40}\cdot\frac{4}{3}\pi^2\rho^3 d_{abc}\bar{u}\Gamma^a_{\mu\nu}u\bar{d}\Gamma^b_{\mu\nu}d\bar{s}\Gamma^c s+(2\text{ permutations})\right]\\ & +\frac{9}{320}\left(\frac{4}{3}\pi^2\rho^3\right)^3 d_{abc}\bar{u}\Gamma^a u\bar{d}\Gamma^b d\bar{s}\Gamma^c s+\frac{9}{256}i\left(\frac{4}{3}\pi^2\rho^3\right)^3 f_{abc}\bar{u}\Gamma^a_{\mu\nu}u\bar{d}\Gamma^b_{\nu\gamma}d\bar{s}\Gamma^c_{\gamma\mu}s\Bigg\},\end{aligned} \tag{31}$$

where $\Gamma=(1-\gamma_5)/2$, $\Gamma^a=(1-\gamma_5)/2\cdot(\lambda^a/2)$, $\Gamma^a_{\mu\nu}=\sigma_{\mu\nu}(1-\gamma_5)/2\cdot(\lambda^a/2)$ and f_{abc} and d_{abc} are SU(3) symbols. For anti-instanton replace $1-\gamma_5$ to $1+\gamma_5$. Then the tunneling rate at finite density is given by the average of Eq. (31) over the ground state of nuclear matter:

$$n(\rho,\rho_B)=\langle\Delta\mathcal{L}\rangle_{\rho_B}. \tag{32}$$

The expectation values of multi quark operators in Eq. (32) are evaluated by applying factorization hypothesis. In this approximation, all the terms in Eq. (31) containing γ_5, $\sigma_{\mu\nu}$, $\lambda^a/2$ drop off [18]. As a result, Eq. (32) is reduced to the same form as Eq. (25) but with the current quark mass replaced by the "effective mass":

$$n(\rho,\rho_B)=n_0(\rho)(\rho\mu_0)^{n_f\gamma_0\frac{\alpha_{\overline{\mathrm{MS}}}(\mu_0)}{4\pi}}\prod_{q=u,d,s}m_q^*(\rho,\rho_B)\rho, \tag{33}$$

where the "effective mass" is defined by

$$m_q^*(\rho,\rho_B)=m_q-\frac{2}{3}\pi^2\rho^2\langle\bar{q}q\rangle_{\rho_B}. \tag{34}$$

In order to know the derivative of Eq. (33) with respect to ρ_B in Eq. (24), we must know the ρ_B dependence of $\langle\bar{q}q\rangle_{\rho_B}$ in the effective quark mass. $\langle\bar{q}q\rangle_{\rho_B}$ is expanded in powers of ρ_B as $\langle\bar{q}q\rangle_{\rho_B}=\langle\bar{q}q\rangle_0+\frac{\Sigma_{\pi N}}{m_u+m_d}\rho_B+\cdots$ for $q=u,d$ and $\langle\bar{s}s\rangle_{\rho_B}=\langle\bar{s}s\rangle_0+y\frac{\Sigma_{\pi N}}{m_u+m_d}\rho_B+\cdots$ [10], where $y=2\langle\bar{s}s\rangle_N/(\langle\bar{u}u\rangle_N+\langle\bar{d}d\rangle_N)$ is the strangeness content of the nucleon. Then we obtain the derivative of Eq. (33) with respect to ρ_B as

$$\begin{aligned}\left[\frac{\partial}{\partial\rho_B}n(\rho,\rho_B)\right]_{\rho_B=0} = {} & n_0(\rho)(\rho\mu_0)^{n_f\gamma_0\frac{\alpha_{\overline{\mathrm{MS}}}(\mu_0)}{4\pi}}\frac{-2\pi^2\rho^5}{3}\frac{\Sigma_{\pi N}}{m_u+m_d}\\ & \times\left[m_u^*(\rho,0)m_s^*(\rho,0)+m_d^*(\rho,0)m_s^*(\rho,0)+ym_u^*(\rho,0)m_d^*(\rho,0)\right].\end{aligned} \tag{35}$$

Now we have all the ingredients for deriving the QCD sum rule. Collecting all the terms in Eq. (6), namely, the one-loop term Eq. (11) and the two-loop term Eq. (24) with Eq. (35), and substituting Eq. (6) into Eq. (5), we obtain the QCD sum rule for $g_A^{(0)}$ as follows:

$$\begin{aligned}\left|g_A^{(0)}\right|^2 = {} & -\frac{M}{3}\Bigg\{\frac{\Sigma_{\pi N}}{s}\left[\frac{28}{3}\left(1-\frac{m_s}{m_u+m_d}\right)\right]+\frac{\Sigma_{KN}}{s}\left[\frac{56}{3}\frac{m_s}{m_s+m_u}\right]\\ & +\frac{M}{s}\left[\frac{2}{3}-7\left(A_2^u(\mu^2)+A_2^d(\mu^2)+A_2^s(\mu^2)\right)\right]\\ & -\frac{4\pi\alpha_s\langle\bar{q}q\rangle_0}{s^2}\left[\frac{352}{27}\frac{\Sigma_{\pi N}}{m_u+m_d}\right]-\frac{4\pi\alpha_s\langle\bar{s}s\rangle_0}{s^2}\left[\frac{176}{27}\left(\frac{2\Sigma_{KN}}{m_s+m_u}-\frac{\Sigma_{\pi N}}{m_u+m_d}\right)\right]\\ & +\frac{15M^3}{s^2}\left[A_4^u(\mu^2)+A_4^d(\mu^2)+A_4^s(\mu^2)\right]+I(s)\Bigg\},\end{aligned} \tag{36}$$

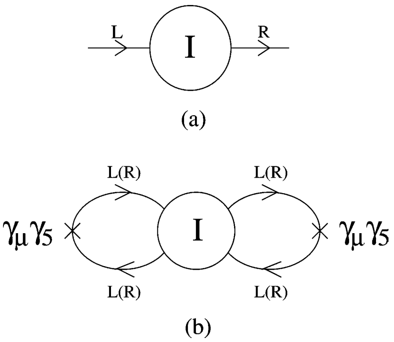

Fig. 1. (a) Zero-modes propagating through an instanton. The solid lines correspond to zero-modes with the chirality left-handed (L) and right-handed (R). An instanton is shown as an open circle with “I”. A zero-mode changes its chirality when passing through an instanton. (b) The diagram corresponding to the two-loop term, Eq. (8). Quarks created by an axial current have same chirality. So quarks do not propagate in an zero-mode state, which changes its chirality in passing through an instanton.

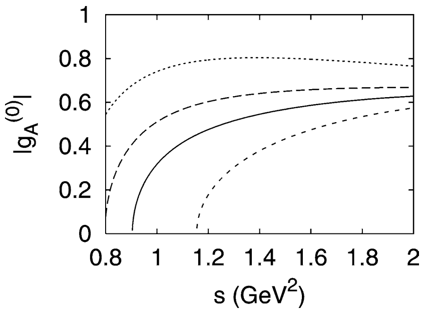

Fig. 2. The square of the Borel mass, s, dependence of $|g_A^{(0)}|$ in Eq. (36) with and without the instanton contribution. The upper curve (long-dashed line) corresponds to that without the instanton contribution. The lower three curves show those with the instanton contribution for different choices of ρ_c; The solid line corresponds to $\rho_c = 0.3$ fm, the dotted line to $\rho_c = 0.29$ fm and the short-dashed line to $\rho_c = 0.31$ fm.

where $\langle\bar{q}q\rangle_0 \equiv \langle\bar{u}u\rangle_0 = \langle\bar{d}d\rangle_0$. In Eq. (36) we assume $m_u = m_d$. $I(s)$ is the instanton contribution which is given by

$$I(s) = 9\int_0^{\rho_c} d\rho \left[\frac{\partial}{\partial\rho_B} n(\rho, \rho_B)\right]_{\rho_B=0} \rho^4 \int_0^{\infty} dt \left(s - \rho^2\cosh^2 t \cdot s^2\right)\exp\left(-\rho^2\cosh^2 t \cdot s\right). \tag{37}$$

Here the integration over ρ has been cut off at the critical size ρ_c. We note that the value of ρ_c estimated by Shuryak is close to the upper boundary for the validity of the expansion Eq. (31) [18].

We show in Fig. 1 the square of the Borel mass, s, dependence of $|g_A^{(0)}|$ in Eq. (36). In plotting the curve in Fig. 1, we used the following values of the constants in the right-hand side in Eq. (36). The π–N sigma-term is taken from Ref. [9], which are $\Sigma_{\pi N} = 45$ MeV. The quark masses are taken to be $m_u = m_d = 7$ MeV, $m_s = 110$ MeV [7]. Using the above values and the ratio $y = 2\langle\bar{s}s\rangle_N/(\langle\bar{u}u\rangle_N + \langle\bar{d}d\rangle_N) = 0.2$ given in Ref. [9], we can calculate the K–N sigma-term averaged over the iso-spin states and the result is $\Sigma_{KN} = 226$ MeV. We calculated the moments of parton distributions adopting the LO scheme in Ref. [11]: $A_2^u(1\ \mathrm{GeV}^2) + A_2^d(1\ \mathrm{GeV}^2) = 1.1$, $A_4^u(1\ \mathrm{GeV}^2) + A_4^d(1\ \mathrm{GeV}^2) = 0.13$, $A_2^s(1\ \mathrm{GeV}^2) = 0.03$, $A_4^s(1\ \mathrm{GeV}^2) = 0.002$. The vacuum condensates are taken from Ref. [7], which are $\langle\bar{q}q\rangle_0 = (-225\ \mathrm{MeV})^3$ and $\langle\bar{s}s\rangle_0 = 0.8\langle\bar{q}q\rangle_0$. The normalization point μ_0 in Eq. (33) was taken to be 1 GeV, which is the relevant scale for QCD sum rules. For the critical size of an (anti-)instanton we used the value $\rho_c = 0.3$ fm. The upper curve in Fig. 2 corresponds to $g_A^{(0)}$ in Eq. (36) but without the instanton contribution $I(s)$. We see that the curve is well stabilized. In the stabilized region $g_A^{(0)} \simeq 0.8$. This is consistent with the well-known fact that $g_A^{(0)}$ is about 30% suppressed due to relativistic effect compared with the naive quark model's expectation, but is much larger than the experimental value $g_A^{(0)} = (0.28$–$0.41)$. The lower three curves correspond to $g_A^{(0)}$ including the instanton contribution $I(s)$ with different choices of ρ_c. We see the Borel curve is extremely sensitive to ρ_c and not stabilized. Therefore we cannot predict the value of $g_A^{(0)}$. However, we can say that apparently instantons tend to lower $g_A^{(0)}$ compared with that without the instanton contribution.

In summary, we have derived a QCD sum rule for $g_A^{(0)}$ from a two-point correlation function of flavour singlet axial-vector currents in one-nucleon state. In deriving the sum rule, we evaluated the correlation function by an OPE up to dimension 6. We have also took into account an additional contribution arising from an (anti-)instanton and evaluated it under DIGA. When we do not include the instanton contribution, $g_A^{(0)}$ is not so suppressed as the experimental value and is about 0.8. Including the instanton contribution $g_A^{(0)}$ tends to be suppressed compared with the result when we do not include the instanton contribution. Recently, Schäfer and Zetocha [19] have computed the axial coupling constants of the nucleon using numerical simulations of the instanton liquid. They found the isovector axial coupling constant is $g_A^{(3)} = 1.28$, in good agreement with experiment, while flavour singlet coupling is $g_A^{(0)} = 0.77$. $g_A^{(0)}$ comes from a connected part and OZI violating disconnected part of the three-point correlation function. Taking into account only the connected part they found $g_A^{(0)} = 0.79$, while the disconnected part is very small, $g_A^{(0)}(\text{dis}) = -(0.02 \pm 0.02)$. It would be interesting if we can clarify the relation between the present result obtained by adding a single instanton contribution to the OPE-based QCD sum rules and that by the instanton liquid model.

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