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A D-brane inspired $U(3)_C \times U(3)_L \times U(3)_R$ model

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Abstract

Motivated by D-brane scenarios, we consider a non-supersymmetric model based on the gauge symmetry $U(3)_C \times U(3)_L \times U(3)_R$ which is equivalent to the $SU(3)^3$ "trinification" model supplemented by three $U(1)$'s. Two $U(1)$ combinations are anomalous while the third $U(1)_{\mathcal{Z}'}$ is anomaly free and contributes to the hypercharge generator. This hypercharge embedding corresponds to $\sin^2\theta_W = \frac{6}{19}$ in the case of full gauge coupling unification. The $U(3)^3$ symmetry is broken down to the Standard Model by vevs of two $(1, 3, \bar{3})$-scalar multiplets supplemented by two Higgs fields in $(1, 3, 1)$ and $(1, 1, 3)$ representations. The latter break $U(1)_{\mathcal{Z}'}$ and provide heavy masses to the extra lepton doublets. Fermions belong to $(3, \bar{3}, 1) + (\bar{3}, 1, 3) + (1, 3, \bar{3})$ representations as in the trinification model. The model predicts a natural quark–lepton hierarchy, since quark masses are obtained from tree-level couplings, while charged leptons receive masses from fourth order Yukawa terms, as a consequence of the extra Abelian symmetries. Light Majorana neutrino masses are obtained through a see-saw type mechanism operative at the $SU(3)_R$ breaking scale of the order $M_R \geqslant 10^9$ GeV.

1. Introduction

Extended objects of the non-perturbative sector of string theory, the so-called D-branes [1], appear to be a promising framework for model building. Intersecting D-branes in particular, can provide chiral fermions and gauge symmetries which contain the Standard Model spectrum and the $SU(3) \times SU(2) \times U(1)$ symmetry as a subgroup and thus D-brane models appear to be natural candidates for phenomenological explorations. During the last years, particular supersymmetric or non-supersymmetric models have been proposed [2–6], based on various D-brane configurations, which exhibit a number of interesting properties. A remarkable feature is that a low unification scale can be possible, since the four-dimensional gauge couplings depend on the volume of extra dimensions. In this case, one can solve the hierarchy problem without supersymmetry. Further, there exist anomalous $U(1)$ symmetries whose anomalies are cancelled by a generalized Green–Schwarz mechanism; a linear combination of these $U(1)$'s remains anomaly free and plays a significant role in particular phenomenological explorations.

In the present work, we propose a model based on the gauge symmetry $U(3)^3$ which can arise in a D-brane construction. This symmetry contains as a subgroup the $SU(3)_C \times SU(3)_L \times SU(3)_R$ symmetry, (trinification model) which has been proposed long time ago [7,8] and subsequently explored in a non-supersymmetric [9], or supersymmetric [10–12] context. It has also been explored as a subgroup of the E_6 symmetry in field theory [13] or in the context of strings [14,15]. In this Letter, we restrict to the non-supersymmetric case, as the problem of supersymmetry breaking may be resolved at the D-brane level [16].

In the D-brane analogue of the trinification model all fermions are accommodated in the $(3, \bar{3}, 1) + (\bar{3}, 1, 3) + (1, 3, \bar{3})$ representations charged under three additional anomalous $U(1)$'s. One linear combination of these $U(1)$ symmetries—hereafter $U(1)_{\mathcal{Z}'}$—is anomaly-free and can serve as a hypercharge component, leading to very interesting phenomenological implications. Two Higgs fields $\mathcal{H}_a$ $(a = 1, 2)$ in the representation $(1, 3, \bar{3})$ (which is the same one accommodating the lepton fields) are needed to break the original symmetry down to the

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SM. In the D-brane construction, in addition, two Higgs fields $\mathcal{H}_{\mathcal{L}} = (1, 3, 1)$ and $\mathcal{H}_{\mathcal{R}} = (1, 1, 3)$ may also appear in the spectrum. When these Higgs fields obtain vevs, they break $U(1)'_{\mathcal{Z}}$ and at the same time provide heavy masses to a pair of the extra lepton doublets.

Quark masses arise from tree-level couplings of the Yukawa potential. The same coupling supports with a heavy mass an extra color triplet. Due to the additional $U(1)$ symmetries, Yukawa couplings for leptons are not allowed at tree-level, however, they arise already at fourth order giving thus a natural explanation to quark–lepton hierarchy. Further, higher-order invariants for Yukawa mass terms appear at even powers of the expansion parameter $\langle \mathcal{H}\mathcal{H}^\dagger\rangle/M_S^2 \leqslant 10^{-1}$ (where M_S is the string scale) ensuring thus the validity of the perturbation theory in this model. The $U(3)^3$ model retains also all the interesting features of the trinification model. Among them, the Higgs doublets and colored fields are in different representations, therefore, no doublet–triplet splitting is required.

The Letter is organized as follows. In Section 2 we give a description of the $U(3)^3$ model motivated by D-brane scenarios and present the fermion and Higgs spectrum. We discuss the mixed anomaly cancellation and we identify the anomaly-free $U(1)$ combination which contributes to the hypercharge generator. We further discuss the gauge coupling evolution and determine the range of the string scale as well as the $SU(3)_L \times SU(3)_R$ intermediate breaking scale. In Section 3 we calculate the Yukawa potential and show that a quark–lepton hierarchy arises, while all extra colored triplets and doublets become massive at a high scale. We also discuss the implications of the model for the neutrino masses. In Section 4 we present our conclusions.

2. Description of the model

The model proposed here can be considered as a D-brane analogue of the trinification $SU(3)^3$ model proposed in [7,8]. The minimal gauge symmetry obtained from D-branes which has as a subgroup the $SU(3)^3$ model is $U(3)^3$. Without going into details, we give here a brief description how such a symmetry could arise in the context as a D-brane construction. The basic ingredient is the brane stack, i.e., a certain number of parallel, almost coincident D-branes. A single D-brane carries a $U(1)$ gauge symmetry which is the result of the reduction of the ten-dimensional Yang–Mills theory. A stack of N parallel branes gives rise to a $U(N)$ gauge group.

We thus consider three stacks of D-branes, each stack containing 3 parallel almost coincident branes giving rise to the gauge symmetry

$$U(3)_C \times U(3)_L \times U(3)_R.$$

The first $U(3)$ is related to $SU(3)$ color, the second involves the weak $SU(2)_L$ and the third is related to a possible intermediate $SU(2)_R$ gauge group. Since $U(3)$ is equivalent to $SU(3) \times U(1)$ our D-brane construction contains also three extra $U(1)$ Abelian symmetries. The $U(1)_C$ symmetry obtained from the color $U(3)_C$ is related to the baryon number [2] which survives at low energies as a global symmetry. There are two additional Abelian factors originating from $U(3)_L$, $U(3)_R$ so the $U(3)^3$ symmetry can be equivalently written

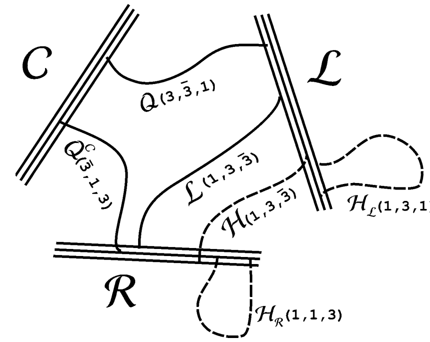

Fig. 1. Schematic representation of a $U(3)_C \times U(3)_L \times U(3)_R$ D-brane configuration and the matter fields of the model.

$$SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_C \times U(1)_L \times U(1)_R. \quad (1)$$

In the D-brane context, matter fields appear as open strings having both their ends attached to some of the brane stacks. For example, strings with both ends attached on two different 3-brane stacks belong to the $(3, \bar{3})$ multiplets of the corresponding gauge group factors. The possible representations which arise in this scenario should be appropriate to accommodate the standard model particles and Higgs fields. As such candidates we choose the open strings that appear in Fig. 1. Under the decomposition (1) these lead to the following matter representations:

$$\mathcal{Q} = (3, \bar{3}, 1)_{(+1,-1,0)}, \quad (2)$$

$$\mathcal{Q}^c = (\bar{3}, 1, 3)_{(-1,0,+1)}, \quad (3)$$

$$\mathcal{L} = (1, 3, \bar{3})_{(0,+1,-1)}. \quad (4)$$

We adopt a notation where the three first numbers refer to the color, left and right $SU(3)$ gauge groups, while the three indices correspond to the three $U(1)_{C,L,R}$ symmetries, respectively. It turns out that these three representations are sufficient to accommodate all the fermions of the Standard Model. In particular, the representation (2) includes the quark left-handed doublets and an additional colored triplet with quantum numbers as those of the down quark, while representation (3) contains the right-handed partners of (2). Finally (4) involves the lepton doublet, the right-handed electron and its corresponding neutrino, two additional $SU(2)_L$ doublets and another neutral state, called neutreto [7]. For a single family, we write the following assignment:

$$(3, \bar{3}, 1) = \begin{pmatrix} u_r & d_r & g_r \\ u_g & d_g & g_g \\ u_b & d_b & g_b \end{pmatrix}, \qquad (\bar{3}, 1, 3) = \begin{pmatrix} u_r^c & u_g^c & u_b^c \\ d_r^c & d_g^c & d_b^c \\ g_r^c & g_g^c & g_b^c \end{pmatrix},$$
$$(1, 3, \bar{3}) = \begin{pmatrix} E^{c0} & E^- & e \\ E^{c+} & E^0 & \nu \\ e^c & \nu^{c+} & \nu^{c-} \end{pmatrix}. \quad (5)$$

In addition, (4) may also accommodate the Higgs multiplets responsible for the symmetry breaking down to the Standard Model

$$\mathcal{H}_a = (1, 3, \bar{3})_{(0,+1,-1)}. \quad (6)$$

According to [7], two Higgs replicas ($\alpha = 1, 2$) are required in order to obtain both symmetry breaking to SM and non-trivial quark mixing.

In the D-brane construction additional matter can arise from strings with both ends attached on the same brane stack. In particular, as we will see in the next sections, the following scalar fields are required in the present model in order to eliminate additional Z' bosons

$$\mathcal{H}_{\mathcal{L}} = (1, 3, 1)_{(0,-2,0)}, \tag{7}$$

$$\mathcal{H}_{\mathcal{R}} = (1, 1, 3)_{(0,0,-2)}. \tag{8}$$

These representations arise from strings having both their ends on left and right brane-stacks, respectively (see Fig. 1).

Employing the usual hypercharge embedding[1]

$$Y = -\frac{1}{6} X_{L'} + \frac{1}{3} X_{R'} \tag{9}$$

(where $X_{L'}$ and $X_{R'}$ represent the $U(1)_{L'}$ and $U(1)_{R'}$ generators, respectively), the transformations of the fermion fields under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\Omega$ are as follows (here we have suppressed the $U(1)_{C,L,R}$ indices):

$$\begin{aligned}
\mathcal{Q} &= q\left(3, 2; \tfrac{1}{6}, 0\right) + g\left(3, 1; -\tfrac{1}{3}, 0\right), \\
\mathcal{Q}^c &= d^c\left(\bar{3}, 1; \tfrac{1}{3}, 1\right) + u^c\left(\bar{3}, 1; -\tfrac{2}{3}, 0\right) + g^c\left(\bar{3}, 1; \tfrac{1}{3}, -1\right), \\
\mathcal{L} &= \ell^+\left(1, 2; -\tfrac{1}{2}, 1\right) + \ell^-\left(1, 2; -\tfrac{1}{2}, -1\right) + \ell^c\left(1, 2; +\tfrac{1}{2}, 0\right) \\
&\quad + \nu^{c+}(1, 1; 0, 1) + \nu^{c-}(1, 1; 0, -1) + e^c(1, 1; 1, 0).
\end{aligned} \tag{10}$$

Similarly, for the scalars we have

$$\begin{aligned}
\mathcal{H}_a &= (1, 3, \bar{3}) \\
&= h_a^{d+}\left(1, 2; -\tfrac{1}{2}, 1\right) + h_a^{d-}\left(1, 2; -\tfrac{1}{2}, -1\right) \\
&\quad + h_a^u\left(1, 2; \tfrac{1}{2}, 0\right) + e^c_{Ha}(1, 1; 1, 0) + \nu^{c+}_{Ha}(1, 1; 0, 1) \\
&\quad + \nu^{c-}_{Ha}(1, 1; 0, -1), \quad a = 1, 2, \ldots.
\end{aligned} \tag{11}$$

For the present work, we choose the following vevs:

$$\mathcal{H}_1 \to \langle h_1^u \rangle = u_1,\ \langle h_1^{d-} \rangle = u_2,\ \langle \nu^{c+}_{H,1} \rangle = U, \tag{12}$$

$$\begin{aligned}
\mathcal{H}_2 \to\ & \langle h_2^u \rangle = v_1,\ \langle h_2^{d-} \rangle = v_2,\ \langle h_2^{d+} \rangle = v_3, \\
& \langle \nu^{c-}_{H2} \rangle = V_1,\ \langle \nu^{c+}_{H2} \rangle = V_2.
\end{aligned} \tag{13}$$

The vacuum expectation values U, V_1, V_2 are taken to be of the order of the $SU(3)_R$ breaking scale. Any of them, breaks the $SU(3)^3$ to an $SU(2) \times SU(2) \times U(1)$ residual symmetry, whereas two of them, namely, U and V_1, suffice to break the symmetry down to $SU(2) \times U(1)$ Standard Model gauge group. The vevs $u_{1,2}, v_{1,2,3}$ related to the $SU(2)_L$ Higgs doublets, should be taken of the order of the electroweak scale. In the sequel we shall assume for simplicity that $V_i \sim U \sim M_R$.

One of the characteristics of string derived models is the appearance of anomalous $U(1)$ symmetries. Unlike the heterotic string case where only one Abelian factor is anomalous, in type I theory, many anomalous Abelian factors can be present and their cancellation is achieved through a generalized Green–Schwarz mechanism [17] which utilizes the axion fields of the Ramond–Ramond sector [18,19], providing masses to the corresponding anomalous gauge bosons.

In the model under consideration, the mixed anomalies of the non-Abelian $SU(3)^3$ gauge part with the Abelian $U(1)_{C,L,R}$ factors are proportional to $\mathcal{A} \sim \operatorname{Tr} Q_I T_J^2$ where $T_J = \{SU(3)_C, SU(3)_L, SU(3)_R\}$ and $Q_I = \{U(1)_C, U(1)_L, U(1)_R\}$ with

$$\mathcal{A} = \begin{pmatrix} 0 & +1 & -1 \\ -1 & 0 & 1 \\ +1 & -1 & 0 \end{pmatrix}. \tag{14}$$

It is easy to see that there is only one anomaly-free $U(1)$ combination, namely,

$$U(1)_{\mathcal{Z}'} = U(1)_C + U(1)_L + U(1)_R. \tag{15}$$

All states represented from strings having their ends attached on two different brane stacks, i.e., $\mathcal{Q}, \mathcal{Q}^c, \mathcal{L}$ and $\mathcal{H}$ in (10) and (11), have zero "charge" under $\mathcal{Z}'$. States represented by strings having both their ends attached to the same brane stack, as is the case of $\mathcal{H}_{\mathcal{L}}$ and $\mathcal{H}_{\mathcal{R}}$, are "charged" under $U(1)_{\mathcal{Z}'}$. Under the standard hypercharge definition, $\mathcal{H}_{\mathcal{L}}$, $\mathcal{H}_{\mathcal{R}}$ are fractionally charged. The standard hypercharge (9) is embedded in $SU(3)_L \times SU(3)_R$, however, it could also include the anomaly-free $U(1)_{\mathcal{Z}'}$, so that $Y' = Y + x\mathcal{Z}'$. This is possible since, as explained above, the fermion and standard Higgs multiplets carry zero $U(1)_{\mathcal{Z}'}$ charge, therefore, their hypercharge is not affected. On the contrary, the fractionally charged states $\mathcal{H}_{\mathcal{L},\mathcal{R}}$ will receive a $U(1)_{\mathcal{Z}'}$-contribution in their hypercharge. Choosing an appropriate value for the coefficient x, the representations $\mathcal{H}_{\mathcal{L}} = (1, 3, 1)$ and $\mathcal{H}_{\mathcal{R}} = (1, 1, 3)$ obtain integral charges like those of the standard model Higgs and lepton fields. In particular, the embedding

$$Y' = Y + \frac{1}{6}\mathcal{Z}' \equiv -\frac{1}{6} X_L + \frac{1}{3} X_R + \frac{1}{6}\mathcal{Z}' \tag{16}$$

leaves all the representations containing the SM spectrum unchanged, while for the $\mathcal{H}_{\mathcal{L},\mathcal{R}}$ scalar fields it yields [2]

$$\mathcal{H}_{\mathcal{L}} = (1, 3, 1) = \hat{h}_L^+\left(1, 2; -\tfrac{1}{2}, 0\right) + \hat{\nu}_{\mathcal{H}_{\mathcal{L}}}(1, 1; 1, 0), \tag{17}$$

[1] The decomposition of the $SU(3)^3$ representations with respect to $SU(3)_C \times SU(3)_L \times SU(3)_R \supseteq SU(3)_C \times [SU(2)_L \times U(1)_{L'}] \times [U(1)_{R'} \times U(1)_\Omega]$ are:

$$(3, \bar{3}, 1) \to (3, 2; -1, 0, 0) + (3, 1; 2, 0, 0),$$

$$(\bar{3}, 1, 3) \to (\bar{3}, 1; 0, 1, 1) + (\bar{3}, 1; 0, 1, -1) + (\bar{3}, 1; 0, -2, 0),$$

$$\begin{aligned}
(1, 3, \bar{3}) \to\ & (1, 2; 1, -1, 1) + (1, 1; -2, -1, 1) + (1, 2; 1, -1, -1) \\
& + (1, 1; -2, -1, -1) + (1, 2; 1, 2, 0) + (1, 1; -2, 2, 0).
\end{aligned}$$

[2] The transformations of the fields $\mathcal{H}_{\mathcal{L},\mathcal{R}}$, under $SU(3)_C \times [SU(2)_L \times U(1)_{L'}] \times [U(1)_{R'} \times U(1)_\Omega]$ and $U(1)_{C,L,R}$, are:

$$\mathcal{H}_{\mathcal{L}} = \hat{h}_L(1, 2, 1; +1, 0, 0)_{(0,-2,0)} + \hat{\nu}_{\mathcal{H}_{\mathcal{L}}}(1, 1, 1; -2, 0, 0)_{(0,-2,0)},$$

$$\begin{aligned}
\mathcal{H}_{\mathcal{R}} = \ & \hat{\nu}^+_{\mathcal{H}_{\mathcal{R}}}(1, 1, 1; -2, -1, +1)_{(0,0,-2)} \\
& + \hat{\nu}^-_{\mathcal{H}_{\mathcal{R}}}(1, 1, 1; -2, -1, -1)_{(0,0,-2)} \\
& + \hat{e}^0_{\mathcal{H}_{\mathcal{R}}}(1, 1, 1; -2, 2, 0)_{(0,0,-2)}.
\end{aligned}$$

$$\mathcal{H}_{\mathcal{R}} = (1,1,3) = \hat{e}^c_H(1,1;1,0) + \hat{\nu}^{c+}_{\mathcal{H}_{\mathcal{R}}}(1,1;0,1) + \hat{\nu}^{c-}_{\mathcal{H}_{\mathcal{R}}}(1,1;0,-1), \tag{18}$$

where, an in the case of the representations in (10), the transformation properties and the quantum numbers of $\mathcal{H}_{\mathcal{L},\mathcal{R}}$ are written here with respect to the symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\Omega$. Thus, under (16) the multiplet $\mathcal{H}_{\mathcal{L}}$ contains a standard Higgs doublet $\hat{h}_L$ and a neutral singlet $\hat{\nu}_{\mathcal{H}_{\mathcal{L}}}$. The $\mathcal{H}_{\mathcal{R}}$ representation is decomposed into a charged singlet $\hat{e}_{\mathcal{H}_{\mathcal{R}}}$ and the two neutral components $\hat{\nu}^+_{\mathcal{H}_{\mathcal{R}}}$, $\hat{\nu}^-_{\mathcal{H}_{\mathcal{R}}}$ which will play a crucial role to the formation of heavy mass terms for the additional lepton doublets and the breaking of the extra $U(1)_{\mathcal{Z}'}$. Indeed, since $\mathcal{H}_{1,2}$ Higgs fields do not carry any charge under $U(1)_{\mathcal{Z}'}$, the latter remains unbroken. Thus, to break this remnant Abelian factor, we need to assume non-zero vevs for the $\mathcal{H}_{\mathcal{L}}$ and/or $\mathcal{H}_{\mathcal{R}}$ field.

2.1. *The string and the weak angle*

In a D-brane realization of the proposed model, the three $U(3)$ gauge factors originate from 3-brane stacks that span different directions of the higher-dimensional space. As a consequence, the corresponding gauge couplings $\alpha_{C,L,R}$ are not necessarily equal at the string scale M_S. This is a general properly of Type I string constructions where the volume enters the relation between the string and the gauge couplings, in contrast to the heterotic string case. However, in certain constructions, at least two D-brane stacks can be superposed and the associated couplings are equal [2]. In this scenario the low energy data together with the gauge coupling running can be used to determine the string scale M_S [20]. In this context, we examine three different cases (i) $\alpha_L = \alpha_R \equiv a$, (ii) $\alpha_C = \alpha_L \equiv a$ and (iii) $\alpha_C = \alpha_R \equiv a$ at M_S which correspond to superposing the left with the right, the color with the left and the color with the right $U(3)$ brane stacks.

The reduction of the $SU(3)^3 \times U(1)^3$ to the SM is in general associated with three different scales corresponding to the $SU(3)_R$, $SU(3)_L$ and $U(1)_{\mathcal{Z}'}$ symmetry breaking. We will assume here for simplicity that the $SU(3)_{L,R}$ and $U(1)_{\mathcal{Z}'}$ symmetries break simultaneously at a common scale M_R, hence the model is characterized only by two large scales, the string/brane scale M_S, and the scale M_R.[3] Clearly, the M_R scale cannot be higher than M_S, i.e., $M_R \leqslant M_S$, and the equality holds if the $SU(3)_R \times SU(3)_L$ symmetry breaks directly at M_S. In order to determine the range of M_S, M_R, we use as inputs the low energy data for α_3, α_{em} and $\sin^2\theta_W$ and perform a one-loop renormalization group analysis. Taking into account the $U(1)$ factor normalizations, the hypercharge embedding (16), (15), implies for $\mu \geqslant M_R$

$$\frac{1}{\alpha_Y} = \frac{1}{2}\frac{1}{\alpha_L} + \frac{3}{2}\frac{1}{\alpha_R} + \frac{1}{6}\frac{1}{\alpha_C}. \tag{19}$$

[3] For a detailed analysis see [21].

As a consequence of (19) the weak angle at the string scale is given by

$$\sin^2\theta_W = \frac{6}{9\left(1+\frac{\alpha_L}{\alpha_R}\right)+\frac{\alpha_L}{\alpha_C}}. \tag{20}$$

Thus, for equality of all gauge couplings $\alpha_C = \alpha_L = \alpha_R$ at M_S, we obtain $\sin^2\theta_W = \frac{6}{19}$.

The one loop renormalization group equations are:

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi}\ln\frac{\mu}{M_Z},$$

where $i = 2, 3, Y$, for $M_Z \leqslant \mu \leqslant M_R$, and

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_R) - \frac{b'_j}{2\pi}\ln\frac{\mu}{M_Z},$$

where $j = C, L, R, Z'$, for $M_R \leqslant \mu \leqslant M_S$. The two Higgs SM beta functions are: $b_3 = -7$, $b_2 = -3$, $b_Y = 7$ and the $SU(3)^3$ beta functions are: $b'_C = -5$, $b'_L = b'_R = -\frac{59}{12} + \frac{n_{\hat{H}}}{4}$, where $n_{\hat{H}}$ the number of the Higgs fields $\mathcal{H}_{\mathcal{L},\mathcal{R}}$ which in our case is taken to be $n_{\hat{H}} = 2$. Solving the RGEs for the three cases mentioned above we obtain M_R and M_S as a function of the common coupling a. The results are presented in Fig. 2. The curves extend from the point $M_S = M_R$ to the Planck scale.

The case in the right part of the graph corresponds to $\alpha_L = \alpha_R = a$. We observe that in this case, the M_R scale remains constant $M_R \sim 1.7 \times 10^9$ GeV, i.e., it is independent of the common gauge coupling a. The second case (in the middle of the graph) corresponds to the case $\alpha_L = \alpha_C = \alpha$. The identification of M_S, M_R scales occurs at the unification point $M_S = M_R \approx 2.3 \times 10^{16}$ GeV. Finally, for $\alpha_R = \alpha_C$, we obtain $M_R = M_S \approx 2.3 \times 10^{11}$ GeV. The bounds on the M_S, M_R scales for the three cases under consideration are summarized in Table 1.

We finally consider the case of unification of all couplings $\alpha_C = \alpha_R = \alpha_L$ at M_S. Using the renormalization group equations and the fact that the spectrum of the model implies $b_L = b_R$ we find that the M_R scale does not depend on $b_{L,R,C}$ beta functions and can be expressed only in terms of the low energy

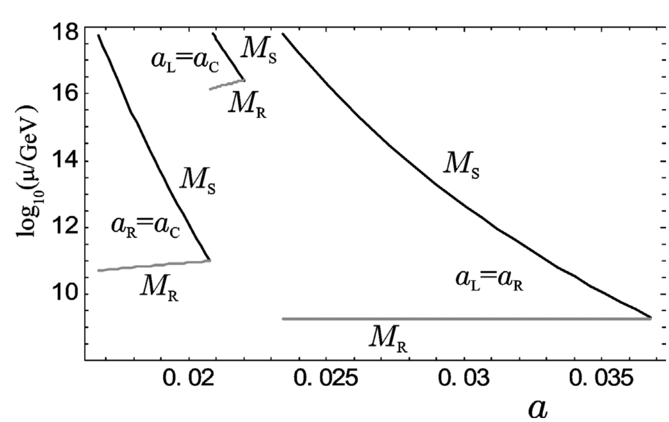

Fig. 2. The string scale M_S, and $SU(3)_R$ breaking scale M_R as functions of the common coupling a for (i) $\alpha_L = \alpha_R = a$, (ii) $\alpha_L = \alpha_C = a$ and (iii) $\alpha_C = \alpha_R = a$. In all cases, M_S is truncated at 10^{18} GeV. In case (i) the M_R scale is constant $M_R \approx 1.7 \times 10^9$ GeV. In the remaining two cases, we find that M_R lowers as M_S attains higher values.

Table 1
Upper and lower bounds for $SU(3)_R$ breaking scale (M_R) and the corresponding string scale (M_S) for the three cases $a_L = a_C$, $a_R = a_C$ and $a_L = a_R$

Model	M_R/GeV	M_S/GeV
$a_L = a_R$	1.7×10^9	$> 1.7 \times 10^9$
$a_L = a_C$	$< 2.3 \times 10^{16}$	$> 2.3 \times 10^{16}$
$a_C = a_R$	$< 2.3 \times 10^{11}$	$> 2.3 \times 10^{11}$

parameters as follows:

$$M_R = M_Z \exp\left[\frac{\frac{6}{a_Y} - \frac{12}{a_2} - \frac{1}{a_3}}{6b_Y - 12b_2 - b_3}\right] \approx 1.7 \times 10^9 \text{ GeV}. \tag{21}$$

Thus, M_R, at least in the one loop approximation, is independent of the physics at the string scale. On the contrary, M_S, as expected, strongly depends on the $SU(3)^3$ beta functions and turns out to be rather high for the minimal content. Demanding $M_S \sim 4 \times 10^{17}$ GeV implies that the beta-functions $b_{L,R}$ should be at least $b_{L,R} \geqslant -3/2$. Such values for the $b_{L,R}$ beta functions are obtained for a large number of Higgs fields and other matter multiplets which are usually present in a string spectrum.

3. Yukawa couplings and fermion masses

We turn now to the fermion mass problem. In contrast to the $SU(3)^3$ model [7,8] in our $U(3)^3$ construction the tree level Yukawa potential consists of a single fermion mass term. This is due to the existence of the additional $U(1)_{C,L,R}$ symmetries which eliminate other possible mass terms. Indeed, only the coupling

$$\lambda^{ij}_{Q,a} \mathcal{Q}_i \mathcal{Q}^c_j H_a, \quad i, j = 1, 2, 3, \quad \alpha = 1, 2, \tag{22}$$

is allowed at tree-level, providing up and down quark masses as well masses for the extra triplets. More precisely, for the Higgs breaking pattern described in (12), (13) the following mass terms arise from (22) for the up quarks

$$m^{ij}_{uu^c} u_i u^c_j = \left(\lambda^{ij}_{Q,1} u_1 + \lambda^{ij}_{Q,2} v_1\right) u_i u^c_j. \tag{23}$$

For the down-type quarks d_i, d^c_j, g_i, g^c_j, we obtain a 6×6 mass matrix in flavor space, of the form

$$m_{dg} = \begin{array}{c c} & \begin{array}{cc} d & \qquad g \end{array} \\ \begin{array}{c} d^c \\ g^c \end{array} & \begin{pmatrix} \lambda^{ij}_{Q,1} u_2 + \lambda^{ij}_{Q,2} v_2 & \lambda^{ij}_{Q,2} V_1 \\ \lambda^{ij}_{Q,2} v_3 & \lambda^{ij}_{Q,1} U + \lambda^{ij}_{Q,2} V_2 \end{pmatrix} \end{array}. \tag{24}$$

As seen from (24) the m_{dd^c} and the m_{dg^c} 3×3 submatrices are of the order of the electroweak scale, whilst M_{dg^c}, M_{gg^c} are of the order M_R. The diagonalization of the full non-symmetric mass matrix (24) results to light masses for the down quarks and masses of the order M_R for the extra states. Few comments are in order. Realistic quark mixing [7] implies the necessity of two replicas of Higgs representations. Indeed, checking the structure of the mass terms in (22) and (24), we observe that if a single Higgs field is present, the up and down quark mass matrices are proportional $\lambda^{ij}_{Q,1} u_1 \propto \lambda^{ij}_{Q,1} u_2$ resulting to absence of KM mixing.[4] Note further that we may set $v_3 = 0$ which leads to $m_{dg^c} = 0$. Moreover, assuming $V_2 = 0$, there is no substantial change on the matrix. If both $v_3 = 0$ and $V_1 = 0$, then d and g quarks decouple completely.

We turn now our attention to the lepton mass matrices. Due to the three extra $U(1)$ factors, tree-level lepton masses are not allowed. In particular, the $U(1)_{C,L,R}$ 'charges' do not allow for a coupling of the form $\mathcal{L}\mathcal{L}\mathcal{H}_i$. The lowest order allowed leptonic mass term arises at fourth order

$$\frac{f^{ab}_{ij}}{M} \mathcal{H}^\dagger_a \mathcal{H}^\dagger_b \mathcal{L}_i \mathcal{L}_j. \tag{25}$$

This term provides electroweak scale masses for the charged leptons as well as Dirac and Majorana neutrino masses. For the charged leptons, considering all possible combinations of the two Higgs doublets, i.e., $\mathcal{H}_1\mathcal{H}_1$, $\mathcal{H}_1\mathcal{H}_2$ and $\mathcal{H}_2\mathcal{H}_2$, we obtain the mass terms

$$\begin{aligned} \frac{f^{ab}_{ij}}{M} \mathcal{H}^\dagger_a \mathcal{H}^\dagger_b \mathcal{L}_i \mathcal{L}_j &\to \frac{f^{ab}_{ij}}{M} \left(\langle h^{u\,*}_a\rangle\langle \nu^{c+*}_{Hb}\rangle \ell^+_i e^c_j + \langle h^{u\,*}\rangle\langle \nu^{c-*}_{Hb}\rangle \ell^-_i e^c_j\right) \\ &\to \left(\alpha_{ij}\ell^+_i + \beta_{ij}\ell^-_i\right) e^c_j, \end{aligned} \tag{26}$$

where

$$\begin{aligned} \alpha_{ij} &= \rho\left(f^{11}_{ij} u_1 + f^{21}_{ij} v_1\right) + \sigma\left(f^{22}_{ij} v_1 + f^{12}_{ij} u_1\right), \\ \beta_{ij} &= \xi\left(f^{22}_{ij} v_1 + f^{21}_{ij} u_1\right) \end{aligned} \tag{27}$$

and $\rho = U/M$, $\sigma = V_2/M$, $\xi = V_1/M$.

Thus, up to this point, we find that the linear combination $\alpha_{ij}\ell^+_i + \beta_{ij}\ell^-_i$ defines the light left-handed lepton doublet ℓ_i which couples to the right-handed electron e^c_j. Clearly, there is an orthogonal to the above linear combination which, together with the anti-lepton doublet ℓ^c remain massless at this level. It would be necessary to obtain a heavy mass for this remaining pair of doublets. Surprisingly, the additional Higgs fields $\mathcal{H}_{\mathcal{L},\mathcal{R}}$ introduced in order to break $U(1)_{\mathcal{Z}'}$ residual symmetry generate heavy masses for these extra doublets. The Yukawa couplings involving the Higgs fields $\mathcal{H}_\mathcal{L}$, $\mathcal{H}_\mathcal{R}$ are of the form

$$\begin{aligned} \frac{\zeta_{ij}}{M} \mathcal{H}_\mathcal{L} \mathcal{H}^\dagger_\mathcal{R} \mathcal{L}_i \mathcal{L}_j &\to \langle \hat{h}^+_L\rangle\left(\hat{\alpha}_{ij}\ell^+_i + \hat{\beta}_{ij}\ell^-_i\right) e^c_j \\ &\quad + \langle \hat{\nu}_{\mathcal{H}_\mathcal{L}}\rangle\left(\hat{\alpha}_{ij}\ell^+_i + \hat{\beta}_{ij}\ell^-_i\right)\ell^c_j, \end{aligned} \tag{28}$$

with

$$\hat{\alpha}_{ij} = \zeta_{ij} \frac{\langle \hat{\nu}^{c+*}_{\mathcal{H}_\mathcal{R}}\rangle}{M}, \qquad \hat{\beta}_{ij} = \zeta_{ij} \frac{\langle \hat{\nu}^{c-*}_{\mathcal{H}_\mathcal{R}}\rangle}{M}.$$

In the above coupling, without loss of generality, we may assume $\langle \hat{h}^+_L\rangle = 0$. The remaining vevs $\langle \hat{\nu}_{\mathcal{H}_\mathcal{L}}\rangle$, $\langle \hat{\nu}^{c+*}_{\mathcal{H}_\mathcal{R}}\rangle$, $\langle \hat{\nu}^{c-*}_{\mathcal{H}_\mathcal{R}}\rangle$ should be taken of the order M_R[5] so that the heavy linear combination for leptons $\hat{\ell}_i = \hat{\alpha}_{ij}\ell^+_i + \hat{\beta}_{ij}\ell^-_i$ couples to the antilepton doublet ℓ^c to form a massive state of the order M_R.

4 String/brane models can provide a different solution to this problem, as the flavor matrix may depend on geometric quantities [22].

5 These vevs have far reaching consequences to the symmetry breaking, since now one dispenses with the use of a second Higgs $\mathcal{H} = (1, 3, \bar{3})$ to break $U(3)^3$ down to SM [21].

We now turn our attention to the neutral lepton states. Suppressing the order one Yukawa coefficients (f_{ij}^{ab}), the neutrino mass matrix in the basis ℓ^+, ℓ^-, ν^{c+}, ν^{c-}, ℓ^c is

$$M_\nu \sim \frac{1}{M_S}\begin{pmatrix} m_W^2 & m_W^2 & m_W M_R & M_R^2 & M_R^2 \\ m_W^2 & m_W M_R & m_W M_R & m_W M_R & M_R^2 \\ m_W M_R & m_W M_R & M_R^2 & M_R^2 & 0 \\ m_W M_R & m_W M_R & M_R^2 & M_R^2 & 0 \\ M_R^2 & M_R^2 & 0 & 0 & 0 \end{pmatrix},$$

where we have assumed for simplicity common vevs $u_i = v_i = M_W$, $U = V_j = M_R$, where $i, j = 1, 2$. This is a see-saw type mass matrix. Three light neutrino species receive see-saw masses of the order m_W^2/M_S while the remaining states receive heavy masses of the order M_R^2/M_S. To obtain a light neutrino spectrum at the range of eV, the scale M_S should be of the order $M_S \sim 10^{13\text{–}15}$ GeV. Interestingly, this is in accordance with the findings of the RGE analysis in Section 3. In particular, M_S is found within the bounds of the cases $\alpha_L = \alpha_R$ and $\alpha_C = \alpha_R$ shown in Fig. 2. It is further compatible with the effective gravity scale in theories with large extra dimensions obtained in the context of Type I string models [3].

4. Conclusions

Inspired by D-brane scenarios, in this work, we analyzed a non-supersymmetric $U(3)_C \times U(3)_L \times U(3)_R$ model which is equivalent to the standard $SU(3)_C \times SU(3)_L \times SU(3)_R$ "trinification" gauge group supplemented by three $U(1)_{C,L,R}$ factors. The three additional Abelian factors have mixed anomalies with $SU(3)_{C,L,R}$ generators. These anomalies can be cancelled in the context of a D-brane derived model by a generalized Green–Schwartz mechanism. The single anomaly free combination $U(1)_{\mathcal{Z}'} = U(1)_C + U(1)_L + U(1)_R$ contributes to the hypercharge.

The Standard Model fermions, represented by strings attached to two different brane-stacks, belong to $(3, \bar{3}, 1) + (\bar{3}, 1, 3) + (1, 3, \bar{3})$ representations as in the case of the "trinification" model. Two Higgs fields in $(1, 3, \bar{3})$ (in the same representation as the lepton fields) and two in $(1, 3, 1)$ and $(1, 1, 3)$ representation can acquire vevs that completely break the gauge symmetry to the SM. They also provide masses to all additional matter fields. The model is characterized by a natural quark–lepton hierarchy since quark masses are obtained from tree-level couplings, while, due to the extra $U(1)$ symmetries, charged leptons are allowed to receive masses from fourth order Yukawa terms. Light Majorana neutrino masses are obtained through a see-saw type mechanism operative at the scale M_R which turns out to be $M_R \geqslant 10^9$ GeV.

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