



The production, spectrum and evolution of cosmic strings in brane inflation

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Abstract

Brane inflation in superstring theory predicts that cosmic strings (but not domain walls or monopoles) are produced towards the end of the inflationary epoch. Here, we discuss the production, the spectrum and the evolution of such cosmic strings, properties that differentiate them from those coming from an abelian Higgs model. As D-branes in extra dimensions, some type of cosmic strings will dissolve rapidly in spacetime, while the stable ones appear with a spectrum of cosmic string tensions. Moreover, the presence of the extra dimensions reduces the interaction rate of the cosmic strings in some scenarios, resulting in an order of magnitude enhancement of the number/energy density of the cosmic string network when compared to the field theory case.

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1. Introduction

The cosmic microwave background (CMB) data [1,2] strongly supports the inflationary universe scenario [3] to be the explanation of the origin of the big bang. However, the origin of the inflaton and its potential is not well understood—a paradigm in search of a model.

Recently, the brane world scenario suggested by superstring theory was proposed, where the standard model of the strong and electroweak interactions are open string (brane) modes while the graviton and the radions are closed string (bulk) modes. In a generic

brane world scenario, there are three types of light scalar modes: (1) bulk modes like radions (i.e., the sizes/shape of the compactified dimensions) and the dilaton (i.e., the coupling), (2) brane positions (or relative positions) and (3) tachyonic modes which are present on non-BPS branes or branes that are not BPS relative to each other [4]. In general, the bulk modes have gravitational strength couplings (so too weak to reheat the universe at the end of inflation) and so are not good inflaton candidates. Neither are the tachyonic modes, which roll down the potential too fast for inflation. This leaves the relative brane positions (i.e., brane separation) as candidates for inflation. So, natural in the brane world is the brane inflation scenario [5], in which the inflaton is an open string mode identified with an inter-brane separation, while the inflaton potential emerges from the exchange

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of closed string modes between branes; the latter is the dual of the one-loop partition function of the open string spectrum, a property well-studied in string theory. This interaction is gravitational strength, resulting in a very weak (that is, relatively flat) potential, ideally tailored for inflation.

The scenario is simplest when the radion and the dilaton (bulk) modes are assumed to be stabilized by some unknown non-perturbative bulk dynamics at the onset of inflation. Since the inflaton is a brane mode, and the inflaton potential is dictated by the brane mode spectrum, it is reasonable to assume that the inflaton potential is insensitive to the details of the bulk dynamics.

Brane inflation has been shown to be very robust (see e.g., [6–9]). The inflaton potential is essentially dictated by the gravitational attractive (and the Ramond–Ramond) interaction between branes. As the branes move towards each other, slow-roll inflation takes place. This yields an almost scale-invariant power spectrum for the density perturbation. As they reach a distance around the string scale, the inflaton potential becomes relatively steep so that the slow-roll condition breaks down. Inflation ends when branes collide and heat the universe [10], which is the origin of the big bang. Towards the end of the brane inflationary epoch in the brane world, tachyon fields appear. As a tachyon rolls down its potential, defects are formed (see, e.g., [11]). Due to properties of the superstring theory and the cosmological conditions, only cosmic strings (but not domain walls or monopoles) are copiously produced during the brane collision [8, 12]. These cosmic strings are Dp -branes with $p - 1$ dimensions compactified. The CMB radiation data fixes the superstring scale to be close to the grand unified (GUT) scale, which then determines the cosmic string tensions, which turn out to have values that are compatible with today's observation, but may be tested in the near future.

In field theory, one may also devise an abelian Higgs like model around the GUT scale to produce cosmic strings towards the end of inflation in which the cosmic string tension is essentially a free parameter. Although such a model may not be as well motivated as brane inflation, it is a possibility, so we aim to find signatures that distinguish cosmic strings in brane inflation from those coming from an abelian Higgs model.

In this Letter, we explore more closely the production of cosmic strings after inflation, the properties of the cosmic strings, in particular their tensions and stability, and finally their evolution to an eventual network. In summary, we find that the final outcome depends crucially on the quantitative details of a particular brane inflationary scenario being contemplated. In some scenarios, the cosmic strings produced via the Kibble mechanism may dissolve quickly. It is likely that their dissolution (which can happen soon after (re)heating) leads to the thermal production of lower-dimensional branes as cosmic strings. This is very likely if the (re)heating process is efficient [10], since the (re)heat temperature is comparable to the superstring scale. In other scenarios, they will evolve to a cosmic string network. In this case, the general properties of the resulting cosmic string network is likely to be quite different from that arising from field theory. The cosmic strings appear as defects of the tachyon condensation and can be $D1$ -branes or Dp -branes wrapping a $(p - 1)$ -dimensional compact manifold. They yield a spectrum of cosmic string tensions including Kaluza–Klein modes. Moreover, due to the presence of the compactified dimensions the interaction rate of the cosmic strings in some scenarios decreases, and when compared to the case in ordinary field theory, the result is an increase by orders of magnitude in the number density of the cosmic string network in our universe.

2. A variety of brane inflationary scenarios

The brane inflationary scenarios we are interested in have the string scale close to the GUT scale so we consider only brane world models which are supersymmetric (post-inflation) at the GUT scale. (Supersymmetry is expected to be broken at the TeV scale, which is negligible for the physics we are interested in here.) In the 10-dimensional superstring theory, the cosmic strings in our 4-dimensional spacetime shall be D -branes with one spatial dimension lying along the 3 large spatial dimensions representing our universe. Hence, we seek to enumerate the possible stable configurations of branes of different dimensionality in 10 dimensions, compactified on a six-manifold. To be specific, let us consider a typical Type IIB orientifold model compactified on $(T^2 \times T^2 \times T^2)/\mathbb{Z}_N$ or some of

its variations (see, for instance, [13]). The model has $\mathcal{N} = 1$ spacetime supersymmetry. Although we shall focus the discussion on Type IIB orientifolds, the underlying picture is clearly more general. We seek to categorize stable configurations of branes which remain after inflation, and give rise to stable cosmic strings in the universe. By “stable” we mean that some fraction of the cosmic strings produced is required to persist until at least the epoch of big-bang nucleosynthesis in order for observable effects to be generated.

In supersymmetric Type IIB string theory with branes and orientifold planes, it is well known that only odd (spacial) dimensional branes are stable. The conditions for stable brane configurations are simple given the compactification manifold; branes must differ by only 0, 4 or 8 in dimension, and branes of the same dimension can be angled at right angles in two orthogonal directions [14,15]. In the generic case where the second homotopy class of the compactification manifold is $\pi_2 = \mathbb{Z}^3$, branes will be stable when wrapping 2-cycles in the compact manifold. From these conditions, we formulate Table 1 of branes from which we shall build models of post-inflation cosmology.

In cosmological situations, branes which are non-BPS relative to the others can be present. Generally the non-BPS configurations will decay, and the decay products of many are well known. For instance, a Dp – $D(p-2)$ brane combination will form a bound state of a Dp -brane with an appropriate amount of “magnetic” flux [16]. This process is best understood as the delocalisation or “smearing out” of the $D(p-2)$ -brane

within the Dp -brane. This process in the Dp – $D(p-2)$ brane system is described by the presence of a tachyon field, an open string that stretches between them. This tachyon condenses as the $D(p-2)$ -brane decays and leads to a singular “magnetic” flux on the Dp -brane; this “magnetic” flux then spreads out across the Dp -brane and diminishes, leaving the total flux conserved. In an uncompactified theory, the residual “magnetic” field strength then vanishes. Since the tachyon in the Dp – $D(p-2)$ brane combination is a complex scalar field inside the $D(p-2)$ -brane world volume, its rolling/condensation allows the formation of $D(p-4)$ -branes as defects. (The actual formation/production of $D(p-4)$ -branes may require the dissolution of a $D(p-2)$ – $\overline{D}(p-2)$ pair inside the Dp -brane.)

Another important set of non-BPS brane configurations which will be generated in early universe brane-world cosmology are branes of the same dimension oriented at general angles, which will also decay into branes with magnetic flux, as described above. There are also special cases of non-BPS configurations which will not decay; between a $D3_3$ - and $D5_1$ -brane (or its T-dual equivalents, for instance, a $D1$ - and $D7$ -brane) there is a repulsive force as seen in the total interbrane potential, which includes all gravitational and RR forces, between a Dp - and a Dp' -brane ($p' < p$) (in terms of the separation distance r when $r \gg M_s^{-1}$)

$$V(r) \sim -\frac{4 - (p - p' + 2a)}{r^{p-7+a}}, \quad (1)$$

where a is the number of directions in which the branes are orthogonal [15]. This potential also makes clear that there is no force between the BPS configurations of branes described above—those which differ in dimension by 4 and those of the same dimension which are angled in two orthogonal directions.

Brane world models of inflation require brane–antibrane pairs (or branes oriented at non-BPS angles) [6–9]; the inflaton field is described by the separation between the branes, and its potential can be organized to give slow roll inflation. To describe the Standard Model, we demand a chiral post-inflation brane-world, which requires that the branes which form our universe are angled in some dimension; sets of $D5_1$ - and $D5_2$ -branes will give a stable chiral low energy effective theory, for instance.

After the compactification to 4-dimensional spacetime, the Planck mass $M_P = (8\pi G)^{-1/2} = 2.4 \times$

Table 1

Stable configurations of D-branes. The labels on the D-branes indicate which of the three 2-cycles they wrap in the compactification dimensions and an empty spot indicates no wrapping/presence. For simplicity the cosmic strings are placed along the 1-direction

Stable branes	Dimension					
	01	23	45	67	89	
D9	✓	✓	✓	✓	✓	} $\mathbb{R}^{3,1}$ branes
D5 ₁	✓	✓	✓			
D5 ₂	✓	✓		✓		
D5 ₃	✓	✓			✓	
D5 _{1,2}	✓		✓	✓		} cosmic strings
D5 _{1,3}	✓		✓		✓	
D5 _{2,3}	✓			✓	✓	
D1 ₀	✓					

10^{18} GeV is given by

$$g_s^2 M_p^2 = \frac{1}{\pi} M_s^2 (M_s r_1)^2 (M_s r_2)^2 (M_s r_3)^2, \quad (2)$$

where M_s is the superstring scale and the compactification volumes (of the (45)-, (67)- and (89)-directions) are $V_i = l_i^2 = (2\pi r_i)^2$ for $i = 1, 2$ and 3 , respectively. Here, $M_s r_i \geq 1$. The string coupling g_s should be large enough for non-perturbative dynamics to stabilize the radion and the dilaton modes (but not too large that a dual version of the model has a weak coupling). We expect the string coupling generically to be $g_s \gtrsim 1$. To obtain a theory with a weakly coupled sector in the low-energy effective field theory (i.e., the standard model of strong and electroweak interactions with weak gauge coupling constant), it then seems necessary to have the brane-world picture [18]. Suppose the $D5_1$ -branes contain the standard model open string modes, then

$$g_s \simeq \alpha_{\text{GUT}} (M_s r_1)^2, \quad (3)$$

where $\alpha_{\text{GUT}} \simeq 1/25$ is the standard model coupling at the GUT scale, which is close to the superstring scale M_s . This implies that $(M_s r_1)^2 \sim 30$. If some standard model modes come from $D5_2$ -branes, or from open strings stretching between $D5_1$ - and $D5_2$ -branes, then $(M_s r_2)^2 \sim 30$.

In the early universe, additional branes (and antibranes) may be present. Additional branes must come in pairs of brane–antibrane (or at angles), so that the total (conserved) RR charge in the compactified volume remains zero. Any even-dimensional D-branes are non-BPS and so decay rapidly. The Hubble constant during inflation is roughly

$$H^2 \simeq M_s^4 / M_p^2. \quad (4)$$

In Table 2, we catalogue the various brane–antibrane pairs (provided that they are separated far enough apart) which can inflate the 4-dimensional Minkowski brane-world volume of $D5_1$ - and $D5_2$ -branes. Towards the end of inflation, a tachyon field appears and its rolling allows the production of defects. A priori, the defects (only cosmic strings here) which are allowable under the rules of K-theory [19] may be produced immediately after inflation, when the tachyon field starts rolling down. Following Eq. (2) and Eq. (4), we see that the Hubble size $1/H$ during this epoch is much

Table 2

Various inflatons and the cosmic strings to which they decay for a brane world built of sets of $D5_1$ - and $D5_2$ -branes. The cosmic string types allowed are determined by K-theoretic analysis of the non-BPS systems. Since the Hubble size is greater than the compactification radii, the Kibble mechanism is capable of producing only defects localized in the three large spacial dimensions. Cosmic strings can be thermally produced if unstable states are able to persist until reheating

Inflaton	Inflation possible	Cosmic string types		
		Allowed	Kibble	Thermal
$D(9-\bar{9})$	×			
$D(7-\bar{7})_{1,2}$	✓	$1_0, 3_1, 3_2, 5_{1,2}$	$5_{1,2}$	–
$D(7-\bar{7})_{1,3}$	✓	$1_0, 3_1, 3_3, 5_{1,3}$	$5_{1,3}$	–
$D(5-\bar{5})_1$	✓	$1_0, 3_1$	3_1	1_0
$D(5-\bar{5})_3$	✓	$1_0, 3_3$	3_3	–
$D(3-\bar{3})_0$	✓	1_0	1_0	–
$D(1-\bar{1})$	×			

bigger than any of the compactification radii,

$$H^{-1} \gg r_i. \quad (5)$$

This means that the Kibble mechanism is capable of producing only defects with vortex winding in the three large spacial dimensions. The cosmological production of these defects towards the end of inflation are referred to as “Kibble” in Table 2. During this epoch, the universe is essentially cold and so no thermal production of any defect is possible. Generically, codimension-one non-BPS defects may also be produced. However, these decay rapidly and will be ignored here. If a non-trivial 3-cycle is present in the orientifold model, a $D5$ -brane wrapping such a cycle can appear as a domain wall, while a $D3$ -brane wrapping it can appear as a monopole. However, such a 3-cycle will be in the (468) (or an equivalent) directions. Since none of the $Dp-\bar{D}p$ pair that can generate inflation wrap all these 3 directions, such defects are not produced.

Let us now elaborate on the various possibilities listed in Table 2 (below, a $Dp-\bar{D}p$ pair includes the case of a stack of Dp -branes separated from a stack of $\bar{D}p$ -branes):

- $D9-\bar{D}9$ pair. In this case, the tachyon field is always present and the annihilation happens rapidly. Also since the branes are coincident, there is no inflaton.

- $D1-\bar{D}1$ pair. Since they do not span the 3 uncompactified dimensions, they do not provide the necessary inflation. In the presence of inflation (generated by other pairs), a density of these $D1$ -branes will be inflated away.
- $(D3-\bar{D}3)_0$ pair. They span the 3 uncompactified dimensions and move towards each other inside the volume of the 6 compactified dimensions during inflation. (The conservation of the total zero RR charge prevents them from becoming parallel and so BPS with respect to each other.) At the end of inflation, their collision heats the universe and yields $D1_0$ -branes as vortex-like solitons. These $D1_0$ -branes appear as cosmic strings. They form a gas of $D1_0$ -branes (at all possible orientations in the 3-dimensional uncompactified space). The $D3_0$ -branes are unstable in the presence of the $D5_1$ - and $D5_2$ -branes. It is possible that during inflation, the $D3_0$ -brane can simply move towards a $D5$ -brane and then dissolve into it. The $\bar{D}3_0$ -brane can either hit the same $D5$ -brane ending inflation, producing $D1_0$ -branes as cosmic strings, or it can collide with another $D5$ -brane. This $D5$ -brane shall no longer be BPS with respect to the other $D5$ -branes and more inflation may result from their interactions. Towards the end of inflation these $D5$ -branes collide with the BPS $D5$ -branes. $D1_0$ -branes are expected to be produced as defects in this scenario.
- $D5_1-\bar{D}5_1$ pair. This $D5_1$ -brane is indistinguishable from the other $D5_1$ -branes that are present. They span the 3 uncompactified dimensions and move towards each other inside the volume of the 4 compactified dimensions (i.e., (6789)) during inflation. Towards the end of inflation, a tachyon field appears and its rolling produces $D3_1$ -branes as cosmic strings. However, such $D3_1$ -branes are unstable and eventually a tachyon field (an open string mode between the $D3$ - and the $D5$ -branes) will emerge. Its rolling signifies the dissolution of the $D3$ -brane into the $D5_1$ -branes. Generically, by the time these $D3$ -branes start dissolving, (re)heating of the universe should have taken place, so the tachyon rolling can thermally produce $D1_0$ -branes as cosmic strings.
- $D5_3-\bar{D}5_3$ pair. They may generate inflation directly, and being mutually BPS with the $D5_1$ - and $D5_2$ -branes shall not be subject to more compli-

cated interactions. After inflation, $D3_3$ -branes as cosmic strings will be produced. Although they are not BPS with respect to the $D5$ -branes, the interaction is repulsive (with $p = 5$, $p' = 3$ and $a = 2$ in Eq. (1)), so we expect them to move away from the $D5_1$ -branes in the (67) directions (to the anti-podal point) and from the $D5_2$ -branes in the (45) directions. This way, these $D3_3$ -branes shall mostly survive and evolve into a cosmic string network. However, some of the $D3_3$ -branes will scatter with the $D5$ -branes in the thermal bath. This may also result in the production of some $D1_0$ -branes as cosmic strings.

- $D7_{1,3}-\bar{D}7_{1,3}$ pair. To provide the needed inflation, these pairs wrap 4 of the 6 compactified dimensions and move towards each other in the remaining 2 compactified dimensions during the inflationary epoch. Their collision heats the universe and yields $D5_{1,3}$ -branes as cosmic strings. The $D5$ -branes that wrap only 2 of the 4 wrapped dimensions of the $D7$ -branes may appear to simply span all 3 uncompactified dimensions. However, the production of these objects is severely suppressed since the Hubble size is much bigger than the typical compactification sizes. While the tachyon is falling down, the universe is still cold, so no thermal production is possible either. As a result, only $D5_{1,3}$ -branes that appear as cosmic strings are produced. It is possible for the $D5_1$ -branes to dissolve into magnetic flux on the $D7$ -brane during inflation. After the annihilation of the $D7_{1,3}-\bar{D}7_{1,3}$ pair, this flux shall reemerge as $D5$ -branes, together with any additional $D5$ -branes solitons as cosmic strings.
- $D7_{1,2}-\bar{D}7_{1,2}$ pair. This case is similar to the above case, except both sets of $D5$ -branes may dissolve into the $D7$ pair during inflation.

We have considered only the IIB theory with two sets of $D5$ -branes. Under T-duality, the branes become $D9-D5$ -branes, or $D7-D3$ -branes in a IIB orientifold theory, with corresponding descriptions. Generalizing the above analysis to the branes-at-angle scenario [7] should be interesting. It is also possible to describe similar inflationary models with cosmic strings in Type IIA theory, in which even-dimensional branes are stable. In this case, one simply adds additional

brane–antibrane pairs to the $\mathcal{N} = 1$ spacetime supersymmetric IIA orientifold models [17]. It will be interesting to consider the brane inflationary scenario in M theory and the Horava–Witten model. In general, we see that the brane inflationary scenario includes numerous possibilities, each with its own intriguing features and consequences.

Although not necessary, we may consider the early universe starting as a gas of branes (see, for example, [20]). The presence of the orientifold planes fixes the total RR charge. After all but one pair of D-brane–antibrane (that span the 3 large dimensions) have annihilated, we end with an early universe that is the starting point of the above discussion. In this picture, it is hard to predict which set of D-brane–antibrane should be last standing.

3. The spectrum of the cosmic strings

The cosmic string tension μ is estimated for a number of brane inflationary scenarios [8,12]. The value μ is quite sensitive to the specific scenario. Here we give an order of magnitude sketch.

For all brane separation smaller than the compactification size, the D– \bar{D} potential is too steep for enough e-folding. When the brane and the antibrane is far apart in the compactified volume, the images of the brane exert attractive forces on the antibrane, so that at the antipodal point the force is exactly zero. In the cubic compactification, this results in a potential $V(\phi) = B - \hat{\lambda}\phi^4$, where ϕ measures the distance from the antipodal point [6]. The density perturbation generated by the quantum fluctuation of the inflaton field is [6,8]

$$\delta_H \simeq \frac{8}{5\pi^2} \frac{N_e^{3/2}}{M_{Pl\perp}}. \tag{6}$$

Using COBE’s value $\delta_H \simeq 1.9 \times 10^{-5}$ [1],

$$M_{Pl\perp} \simeq 3 \times 10^6. \tag{7}$$

This still leaves M_s unfixed. To estimate M_s and the cosmic string tension μ , let us consider a couple of scenarios. Consider D5₁– $\bar{D}5_1$ brane inflation. With $(M_s r_1)^2 \sim 30$ and $r_2 = r_3 = r_\perp$, Eq. (2) and Eq. (7) then imply that $M_s \sim 10^{14}$ GeV. If the cosmic strings are D1-branes, the cosmic string tension μ_1 is simply

the D1-brane tension τ_1 :

$$\mu_1 = \tau_1 = \frac{M_s^2}{(2\pi g_s)}. \tag{8}$$

This implies that $G\mu \simeq 6 \times 10^{-12}$. Now the D1-brane may have discrete momenta in the compactified dimensions. These Kaluza–Klein modes give a spectrum of the cosmic string tension,

$$\mu \rightarrow \mu + e_1/r_1^2 + e_2/r_2^2 + e_3/r_3^2 \tag{9}$$

where e_i ($i = 1, 2, 3$) are respectively the discrete eigenvalues of the Laplacians on the (45), (67) and (89) compactification cycles. To get an order of magnitude estimate, we find that the lowest excitation raises the tension by about a few percent.

For D7_{1,2}– $\bar{D}7_{1,2}$ pair inflation, and $(M_s r_1)^2 \simeq (M_s r_2)^2 \sim 30$, we have $r_3 = r_\perp$. In this case, $M_s \sim 4 \times 10^{14}$ GeV, with D5_{1,2}-branes as cosmic strings. Noting that a D p -brane has tension $\tau_p = M_s^{p+1}/(2\pi)^p g_s$, the tension of such cosmic strings is

$$\mu_5 = \frac{(M_s r_1)^2 (M_s r_2)^2 M_s^2}{2\pi g_s}. \tag{10}$$

This yields $G\mu \sim 10^{-8}$. This tension is bigger than that of D1-branes. Depending on the particular inflationary scenario, this value may vary by an order of magnitude. For D– \bar{D} inflation, we have roughly [12]

$$10^{-7} \geq G\mu \geq 10^{-12}. \tag{11}$$

Higher values of $G\mu$ are possible for the branes-at-small-angle scenario.

The interesting feature of this type of cosmic strings is that there is a spectrum of cosmic string tension. The branes can wrap the compactified (4567) dimensions more than once. This gives

$$\mu \sim nw\mu_5, \tag{12}$$

where n is the defect winding number (i.e., the vorticity) and w is the wrapping number (i.e., the number of times it wraps the compactified volume) inside the brane, so nw is equivalent to the number of cosmic strings. Moreover, there can be “momentum” (Kaluza–Klein) excitations of the branes propagating in these compactified directions. All these result in quite an intricate spectrum of cosmic string tensions. For $n = w = 1$, we have:

$$\mu \sim \mu_5 \left(1 + \frac{p_1}{(M_s r_1)^2} + \frac{p_2}{(M_s r_2)^2} \right) + \frac{e_3}{r_3^2}, \tag{13}$$

where p_1 and p_2 are discrete “momentum” excitation modes depending on the geometry of the (45) and the (67) directions. Using $(M_s r_1)^2 \sim (M_s r_2)^2 \sim 30$, we see that each momentum excitation typically raises the cosmic string tension roughly by a few percent.

We see that the cosmic string tension can have a rich spectrum. This is very different from the field theory case, where the cosmic string always appear with the same tension, up to the vorticity number n .

4. Evolution of the cosmic string network

To see the impact of the extra dimensions on the cosmic string network evolution, let us use the simple one-scale model for the evolution of the cosmic string network [21]. The energy in the cosmic strings is much smaller than the energy in the radiation (or in the matter at later time). Let $L(t)$ be the characteristic length scale of the string network. The energy density of the cosmic string network is given by

$$\rho \simeq \frac{E}{L^3} \simeq \frac{\mu L}{L^3} \simeq \frac{\mu}{L^2}, \quad (14)$$

where E is the energy of the cosmic string network per characteristic volume. String self-intersections typically break off a loop, which then decays (e.g., via gravitational waves). String intercommutations generate cusps and kinks, which also decay rapidly. So the change in energy is given by

$$\Delta E = \Delta E_{\text{expansion}} - \Delta E_{\text{interaction}}. \quad (15)$$

Now, the cosmic string energy in an expanding universe $E = \rho V_0 a^3$ where the constant V_0 is the reference volume, and $a(t)$ is the cosmic scale factor. The number of interaction per unit volume per unit time is $\lambda(v/L)/L^3$ where v is a typical peculiar velocity and λ measures the probability of string intersections. Assuming slow-moving strings (to simplify the analysis), and substituting these quantities into Eq. (15), we obtain the equation governing the evolution of the energy density:

$$\dot{\rho} = -2\frac{\dot{a}}{a}\rho - \lambda\frac{\rho}{L}. \quad (16)$$

Here, $H = \dot{a}/a$ is the Hubble constant. Substituting the ansatz for $L(t) = \gamma(t)t$ in Eq. (16), we obtain

the following equation for $\gamma(t)$ during the radiation-dominated era:

$$\dot{\gamma} = -\frac{1}{2t}(\gamma - \lambda). \quad (17)$$

This equation has a stable fixed point at $\gamma(t) = \lambda$. We see from this solution that the characteristic length scale of the string network tends asymptotically towards the horizon size, $L \simeq \lambda t$.

As a check, we see that in the absence of string interactions (that is, $\lambda = 0$), $\gamma \sim \sqrt{t}$ so $\rho \sim a^{-2}$, as expected. In the presence of cosmic string interactions (that is, $\lambda \neq 0$), the asymptotic (late time) energy density of the cosmic string is given by

$$\rho = \frac{\mu}{\lambda^2 t^2} \sim \begin{cases} \mu/(\lambda^2 a^4), & \text{radiation-dominated era,} \\ \mu/(\lambda^2 a^3), & \text{matter-dominated era.} \end{cases} \quad (18)$$

Suppose λ_0 is the interaction strength for field theory models. The cosmic strings live in $4 + d_\perp$ dimensions and are localized in the d_\perp compact dimensions. The effect of the extra dimensions is to reduce the collision (self-intersection) probability of the cosmic strings. The simplest way to model this effect is to change the efficiency with which loops are formed by the long cosmic strings, so $\lambda < \lambda_0$. So the number density of the scaling cosmic string network is enhanced by a factor of

$$(\lambda_0/\lambda)^2 \geq 1.$$

Generically, we expect this to be a large enhancement. Since the extra dimensions are stabilized, ρ still scales like radiation during the radiation-dominated epoch (and it scales like matter during the matter-dominated epoch). The resulting cosmic string network then yields a scaling energy density ρ

$$\frac{\rho}{\rho_r} \simeq \Gamma G\mu \simeq \left(\frac{\lambda_0}{\lambda}\right)^2 \beta G\mu, \quad (19)$$

where ρ_r is the energy density of radiation during the radiation-dominated epoch (or of matter during the matter-dominated epoch). In field theory models, $\Gamma = \beta$. Numerical simulations [22] give $\beta \sim 6$.

Let us give an order-of-magnitude estimate of the effect of the extra dimensions on the cosmic string collision probability, namely, the ratio λ_0/λ .

Consider two points of two different cosmic strings or of the same cosmic string that coincide in the 4-dimensional spacetime. In 4-dimensional field theory, they are touching. The probability of this happening is dictated by λ_0 . In the brane world, they may still be separated in the extra dimensions. We like to estimate the likeliness of them actually touching (which then allows intercommuting or the pinching off of a loop).

Consider the compact directions where these two points (of cosmic strings) appear as points (that is, they are not wrapping these compact directions). In the case of $D5_3\text{--}\bar{D}5_3$ pair inflation, the repulsive force from the $D5_1$ -branes will push the $D3_3$ -branes into a corner in the (45) directions, while the repulsive force from the $D5_2$ -branes will push the $D3_3$ -branes into a corner in the (67) directions. As a result, all the $D3_3$ -branes end up at a corner in the (4567) directions. In this case, the extra dimensions should have little or no effect on their interaction, that is

$$\lambda_0/\lambda \sim 1 \quad \text{for } D3_3\text{-branes.} \quad (20)$$

In other scenarios, they are free to roam in the compact directions. If two cosmic strings coincide in the 4-dimensional spacetime, and in the compactified directions in which they are pointlike they are separated by a distance comparable to the superstring scale $1/M_s$, a tachyon field appears and the rolling of this tachyon field has a time scale around the superstring scale. So, we expect them to interact. Consider the scenario where the $D5_{1,2}$ -branes are cosmic strings. If they are randomly placed, the likeliness of them coming within that distance in the compact (89) directions is given by $\lambda_0/\lambda \simeq (M_s r_3)^2$. Now let us take the cosmic string interaction into account. Since the cosmic string appears as points and interact via an attractive Coulomb type potential in the extra dimensions (which becomes important only when the separation between them is relatively small). Let us get an estimate of this enhancement of the probability of interaction. The scattering cross-section of the two string points interaction in transverse dimensions via an attractive potential $V(r) = -A/r^{d_\perp-2}$ is given by

$$\sigma = \Omega_{d_\perp-1} r_{\text{capture}}^{d_\perp-1}, \quad (21)$$

where $\Omega_{d_\perp-1}$ is the volume of the unit $(d_\perp - 1)$ -sphere. The capture radius r_{capture} is comparable to the superstring scale $1/M_s$, so the likelihood of two string

points within that distance becomes

$$\frac{\lambda_0}{\lambda} \simeq \frac{r_3}{r_{\text{capture}}} \simeq M_s r_3 \sim 10. \quad (22)$$

Note that, generically, larger $G\mu$ gives less enhancement. The reason is: the total volume of the compactified dimensions is fixed by the value of G ; larger tension comes from brane-wrapping over larger compactified volume, which implies smaller volume for the cosmic strings to avoid each other, so smaller Γ . This means $\Gamma G\mu$ (the cosmic string density) is relatively insensitive compared to either μ or Γ alone. If observations give a bound $\rho/\rho_r < 10^{-5}$, the values of $G\mu$ appearing in the branes-at-small-angle scenario may seem too large. However, the production of cosmic strings in this scenario are more localized around the brane intersection, implying a smaller production as well as a smaller enhancement in Γ . Clearly, a careful estimate of λ_0/λ and Γ in that case will be very important.

The important message here is that the cosmic string network continues to have a scaling solution, and the enhancement in its energy density due to the extra dimensions can be very large. For a given μ , this will yield a very different cosmic string energy density than that in the field theory case. Measuring μ and ρ separately will be valuable.

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