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# A Maximum Curvature Method for Predicting Transonic Buffet Onset Based on Steady Aerodynamic Parameters

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## Abstract

It is difficult to predict the buffet onset by kink analysis in steady aerodynamic parameter curves such as the lift coefficient curves, pitching moment coefficient curves and trailing edge pressure coefficient curves at a certain flow conditions. Thus, a maximum curvature method is proposed which is based on the aerodynamic parameter data, to find the buffet onset point at which the curve curvature achieves an extreme. The method is to fit a fourth order polynomial using the aerodynamic parameter data calculated by steady N-S equation. Subsequently, curvature analysis is carried on with the fitted curve and the stationary point in a particular interval of the curve is obtained. And this point is regarded as the buffet onset point. This method can be applied to predict the transonic buffet onset for airfoils based on steady aerodynamic parameters and good agreement is achieved with the comparison of predicted results and wind tunnel test data.

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*Keywords:* Buffet onset; Transonic buffet; Curve fitting; Aerodynamic parameters

## Nomenclature

$\alpha$	the incidence angle of airfoil (degree)
$\alpha_B$	buffet onset incidence angle (degree)
$c$	chord length(m)
$C_L$	airfoil lift coefficient of airfoil
$C_M$	pitch moment coefficient of airfoil
$C_p$	airfoil trailing edge pressure coefficient of airfoil

## 1. Introduction

The buffet envelope forms one of the main constraints in setting up the low-to-intermediate and high-speed in route performance capabilities of transport airplane. The buffet is defined as excitation imposed on the airplane due to separated airflow. The buffeting have a strong influence on airplane aerodynamic performance, even the intensity buffeting may give rise to structure fatigue damage or even catastrophic failure. Buffet can appear in different forms and have different causes, e.g., low-speed buffet due to flow separation when approaching the low-speed stall; buffet due to lift dumper or speed brake extension; Buffet due to local flow separation caused by inadequate design of geometry details in particular at the rear end of the airplane; and transonic buffet due to flow separation caused by strong shock waves.

The transonic buffet is a typical aerodynamic phenomenon that results in a large-scale self-sustained motion of shock over the surface of the airfoil. The onset of this phenomenon is not connected with any fluid/structure interaction, although

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it is unavoidable that some structure deformation and vibration may arise. In transonic airplane design, the transonic buffet should be avoided in its transonic cruising and it is necessary to demonstrated that no excessive vibration will be induced when enter the buffeting boundary undersignedly. This can be guaranteed by a accurate prediction of buffet onset boundary.

To predict the buffet onset boundary, numerical simulation and model test approaches can be adopted, and several wind-tunnel test methods have been developed. Pearcey and Holder [1] showed experimentally that the distinct slope change (or kink) in the lift curve coincided with buffet onset as measured with a strain gauge on a two-dimensional airfoil. However, In some flow conditions, the kink phenomenon of the aerodynamic parameter curves such as lift curve are not clear at relatively. Moreover, wind-tunnel test methods are not suitable for the first step in design of transonic airplane due to high expenditure of time and cost. For this reason, to predict the buffet onset theoretically, several CFD based numerical simulation methods have been developed instead of wind-tunnel tests. Michael and Daniella [2] employed URANS (Unsteady Reynolds-Averaged Navier-Stokes Simulation) method to compute buffet onset of NACA0012 airfoil. The buffet onset was computed by statically increasing the incidence angle until unsteady oscillations occur. The computed buffet boundary shows good agreement for all Mach numbers except Mach 0.8. However, the URANS method is time consuming and difficulty to be applied to engineering practice, especially for the three-dimensional full configuration of airplane model. Injae Chung [3] use steady RANS (Reynolds-Averaged Navier-Stokes Simulation) method to estimate the buffet onset boundary of NACA0012 airfoil and suggested that the kink of the center of pressure versus incidence angle curve can be used as a reliable indicator of buffet onset of airfoil NACA0012 for the steady numerical method. Nevertheless, no methods were presented to deal with the aerodynamic parameter curves whose slope changes gradually near the buffet onset. The purpose of this paper is to propose a method to predict the buffet onset for airplane transonic buffet utilizing numerical simulated aerodynamic parameter curves. In this method the curve fitting is adopted for the aerodynamic parameter data calculated from a steady Navier-Stokes solver and then curvature analysis is conducted to find the stationary point of these curves and the stationary point corresponding to the maximum curvature can be identified as the buffet onset. The accuracy of present method is verified by comparison with the results of transonic buffet wind-tunnel tests for a NACA0012 airfoil.

## 2. Numerical method for NACA0012 airfoil transonic flow simulation

The two dimensional compressible Navier-Stokes equations can be expressed as

$$\frac{\partial}{\partial t} \iint_{\Omega} Q dv + \oint_{\partial\Omega} F \cdot n dS = \oint_{\partial\Omega} G \cdot n dS \quad (1)$$

where  $v$  is the cell volume,  $Q$  is the conservative variable vector,  $\Omega$  is the control volume and  $\partial\Omega$  denotes the boundary of control volume.  $Q$  is the solution vector.  $F$  denotes the convective flux vector and  $G$  denotes the viscous flux vector. The convective flux terms are finite differenced with FDS (Flux Difference Splitting) -Roe [4] upwind method. Second-order spatial accuracy is obtained by the MUSCL approach with the Minmod flux limiter. In the time direction the second-order fully implicit LUSGS (Low-Upper Symmetric Gauss-Seidel) scheme [5] with a sub-iteration time-stepping procedure is used.

The accuracy of the numerical predictions for buffet onset is dictated by both the accuracy of the numerical discretization scheme and the accuracy of the turbulence model. Researches [6] have revealed that the accuracy of the numerical calculations is mainly dominated by the accuracy of the turbulence model. Transonic flows are known to be extremely sensitive to turbulence modeling, because the exactly capturing for the shock location depends on the accurate prediction of the turbulent boundary layer.

There are three classes of models currently employed in computations of practically relevant aerodynamic flows: eddy-viscosity models, non-linear eddy-viscosity models and Reynolds-stress models. Among which the eddy-viscosity turbulence models have been the most widely applied group of models in the CFD of aeronautical engineering applications. The eddy-viscosity turbulence models were classified in general into three categories: they are algebraic models (a representative model is Baldwin and Lomax model), the one-equation models (a representative model is Spalart and Allmaras model) and the two-equation models (a representative model is  $k-\omega$  SST model).

The  $k-\omega$  based turbulence models represent more flow physics than algebraic models or one equation models and are practicable to boundary layer predominated flows. Menter [7] tested four  $k-\omega$  models ( $k-\omega$  SST,  $k-\omega$  BSL,  $k-\omega$  Wilcox,  $k-\omega$  2-L) against a number of transonic airfoil and wing flows. It was found that the SST model gives much better results than the other three models and generally predicts shock locations and separation locations in pretty good agreement with the experiment. So, the  $k-\omega$  SST turbulence model is chosen to compute the viscous term in the present study.

The  $k-\omega$  SST (shear stress transport) turbulence model [8] is

$$\frac{D\rho k}{Dt} = \tau_{ij} \frac{\partial u_i}{\partial x_j} - \beta^* \rho \omega k + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_k \mu_t) \frac{\partial k}{\partial x_j} \right] \quad (2)$$

$$\frac{D\rho \omega}{Dt} = \frac{\gamma}{\nu_t} \tau_{ij} \frac{\partial u_i}{\partial x_j} - \beta \rho \omega^2 + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_\omega \mu_t) \frac{\partial \omega}{\partial x_j} \right] + 2(1 - F_1) \rho \sigma_{\omega 2} \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} \quad (3)$$

The term in the left-hand sides of Eq. (2) and Eq. (3), i.e.,  $D/Dt = \partial/\partial t + u_i \partial/\partial x_i$  is the Lagrangian derivative. And the constants of set  $\phi$  of the k- $\omega$  SST model is calculated from the constants  $\phi_1$ ,  $\phi_2$ ,

$$\phi = F_1 \phi_1 + (1 - F_1) \phi_2 \quad (4)$$

the constants of set  $\phi_1$  are

$$\sigma_{k1} = 0.85, \sigma_{\omega 1} = 0.5, \beta_1 = 0.075, a_1 = 0.31, \beta^* = 0.09, k = 0.41, \gamma_1 = \beta_1 / \beta^* - \sigma_{\omega 1} k^2 / \sqrt{\beta^*}.$$

the constants of set  $\phi_2$  are

$$\sigma_{k2} = 1.0, \sigma_{\omega 2} = 0.856, \beta_2 = 0.0828, \beta^* = 0.09, k = 0.41, \gamma_2 = \beta_2 / \beta^* - \sigma_{\omega 2} k^2 / \sqrt{\beta^*}.$$

with the following definitions

$$F_1 = \tanh(\arg_1^4) \quad \arg_1 = \min \left[ \max \left( \frac{\sqrt{k}}{0.09\omega y}, \frac{500\nu}{y^2\omega} \right); \frac{4\rho\sigma_{\omega 2}k}{CD_{k\omega}y^2} \right]$$

where  $y$  is the distance to the next surface and  $CD_{k\omega}$  is the positive portion of the cross-diffusion term of Eq. (3)

$$CD_{k\omega} = \max(2\rho\sigma_{\omega 2} \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}, 10^{-20})$$

and the eddy viscosity is defined as

$$\nu_t = \frac{a_1}{\max(a_1\omega; \Omega F_2)}$$

where  $\Omega$  is the absolute value of the vorticity.  $F_2$  is given by

$$F_2 = \tanh(\arg_2^2) \quad \arg_2 = \max(2 \frac{\sqrt{k}}{0.09\omega y}, \frac{500\nu}{y^2\omega})$$

The steady Navier-Stokes solver is performed on the conventional NACA0012 airfoil with the assumption of steady flow for the buffet onset. The flow conditions are: March number 0.775, incidence angle range 1~3 degree, and Reynolds number based on chord length  $6.0 \times 10^6$ . A local time stepping was used in the steady calculation to accelerate convergence to steady state. And a steady state was defined to be reached when the lift coefficient reached 0.2% of its final value with at least four orders of magnitude of residual reduction.

### 3. Mesh and Numerical Method Validation

In the present work, nominal conditions ( $Re=6 \times 10^6$ , incidence  $\alpha=2.05$  deg and  $Ma=0.775$ ) which were considered in the experiments in Ref [9] are adopted in the numerical simulation of the present work. To validate the grid convergence, three two dimensional grids have been evaluated. These basic two dimensional grids are of multi-block type. The coarse grid (Grid A) contains 52252 nodes, the fine grid (Grid B) contains 189922 nodes and the very fine grid (Grid C) contains 272888 points covering the computational domain. The far field conditions are imposed at 50 times the chord length away from the profile.

The pressure coefficient distribution obtained for the three grids with present numerical simulation based on the steady Navier-Stokes with SST turbulence model at incidence angle equal to 2.05 degree is compared with the experimental results [9] as shown in Fig. 1. The overall agreement is good for Grid A, Grid B and C, but Grid B and Grid C give the better agreement than Grid A with the experimental data in flow recovery region behind the shock and it shows that the difference between the results yielded by the two grids (Grid B and Grid C) is very small, so grid convergence is achieved. Therefore Grid B was selected in the present studies, as shown in Fig. 2.

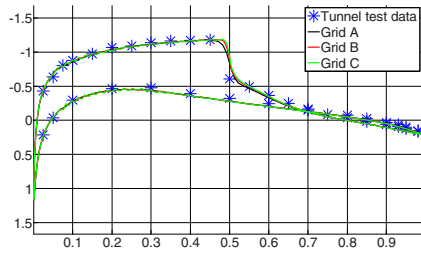


Fig. 1. Pressure coefficient compared with the experimental data

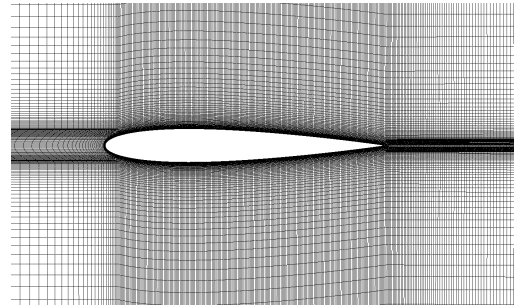


Fig. 2. Mesh around NACA0012 airfoil (Grid B)

**4. Maximum curvature method**

To determine the buffet onset using the numerically calculated aerodynamic parameters, two main steps are involved in the present method: The first step is to conduct curve fitting for the various aerodynamic parameters and obtain the polynomial expressions of these curves. The second step is to perform the curvature analysis of the fitted curves and determine the stationary point which is corresponding to the buffet onset.

*4.1. Curve fitting for aerodynamic parameters*

In the curve fitting for the aerodynamic parameters obtained by solving steady N-S equation using polynomial regression, the least-squares procedure can be readily extended to fit the data to a higher-order polynomial. And the fourth degree polynomial was adopted to match these data.

$$y = p_1x^4 + p_2x^3 + p_3x^2 + p_4x + p_5 + e \tag{6}$$

Where,  $p_1, p_2, p_3, p_4$  and  $p_5$  are polynomial coefficients.  $e$  is the fitting error or residual [10] which is the discrepancy between the true value of  $y$  and the approximate value. The standard error is formulated as

$$S_{y/x} = \sqrt{\frac{S_r}{n - (m + 1)}} \tag{7}$$

Where  $m$  is the order of polynomial,  $n$  is the total number of data points. The sum of the residuals is:

$$S_r = \sum_{i=1}^n (y_i - p_1x_i^4 - p_2x_i^3 - p_3x_i^2 - p_4x_i - p_5)^2 \tag{8}$$

The polynomial coefficients of curve fitting for different aerodynamic parameters and their standard errors are listed in Table 1. The fitted curves for the aerodynamic parameters are shown in Fig. 3, Fig. 4 and Fig. 5. In addition, the pitching moment is referenced at the leading edge of the airfoil.

Table 1 Polynomial coefficients and standard errors of fitting curves

Polynomial Coefficients	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$S_{y/x}$
Aerodynamic Parameters						
Lift coefficients	0.011903	-0.097362	0.23759	-0.051974	0.098503	0.0012373
Pitching moment coefficients	-0.00554	0.045678	-0.11783	0.071247	-0.046395	0.00053699
Pressure coefficients at trailing edge	0.012216	-0.099866	0.26074	-0.27775	0.27868	0.0012296

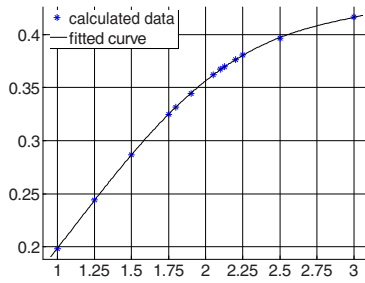


Fig. 3 The lift coefficient curve

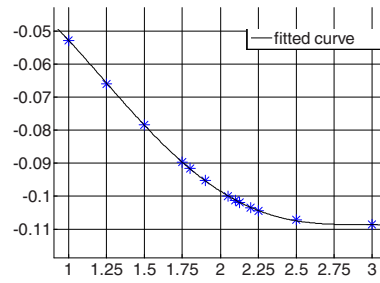


Fig.4 The pitching moment coefficient curve

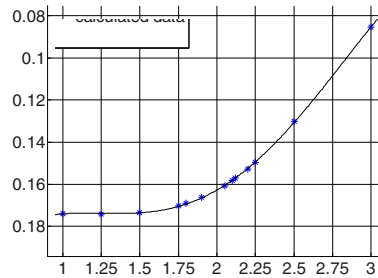


Fig. 5 The trailing edge(x/c=0.95) pressure coefficient curve

4.2. Curvature analysis of fitted curve

Curvature is a measure of how sharply a curve is turning as it is traversed, and curvature is then defined as the magnitude of the change rate of rotating angle  $\theta$  with respect to the arc length  $S$ . That is

$$k = \left| \frac{\Delta\theta}{\Delta S} \right| \tag{9}$$

From this definition, in order to obtain a convenient expression when the equation of the curve is expressed in Cartesian form, i.e.  $y = f(x)$ , the curvature formulation can be expressed as

$$K = \frac{|y''|}{(1 + y'^2)^{3/2}} \tag{10}$$

Now using the Eq. (6) and Eq. (10), we can obtain the curvature of the fitted curve

$$K(x) = \frac{|12p_1x^2 + 6p_2x + 2p_3|}{(1 + (4p_1x^3 + 3p_2x^2 + 2p_3x + p_4)^2)^{3/2}} \tag{11}$$

To find the extrema of the continuous function  $K(x)$ , the corresponding closed interval  $[a, b]$  should be determined at first, and then the stationary point of  $K(x)$  in  $[a, b]$  is found by  $dK/dx = 0$ , that is

$$\frac{\text{sign}(M)N}{(H^2 + 1)^{\frac{3}{2}}} - \frac{3|M|MH}{(H^2 + 1)^{\frac{5}{2}}} = 0 \tag{12}$$

Where  $M = 12p_1x^2 + 6p_2x + 2p_3$ ,  $N = 6p_2 + 24p_1x$ ,  $H = 4p_1x^3 + 3p_2x^2 + 2p_3x + p_4$ ;  $\text{sign}(M)$  is a symbol function, it equals

- 1 if  $M$  is greater than zero;
- 0 if the  $M$  equals zero;
- 1 if the  $M$  is less than zero.

By solving Eq. (12) the stationary point  $x$  correspond to the extreme of the function  $K(x)$  is obtained. When the above curvature analysis is executed for the fitted curves of aerodynamic parameters, the stationary point  $x$  is the buffet onset incidence to be predicted.

## 5. Buffet onset predicted by the Maximum curvature method

To predict the transonic buffet onset for the NACA0012 airfoil from the calculated aerodynamic parameters, the maximum curvature method based on the various aerodynamic parameter data is examined.

As incidence angle is increased beyond the lift curve linear range, the shock begins to move towards upstream and become strong enough to cause the separation bubble. When the separation bubble extends rapidly from the shock towards the trailing edge, the slope of curve is greatly reduced. This is defined as buffet onset [11,12]. However, the distinct slope changes are not shown clearly at some flow conditions, as shown in Fig.3. So it is difficult to predict the buffet onset by kink analysis for lift coefficient curve at a certain flow conditions. Thus the maximum curvature method described above is used to find the buffet onset angle. The closed interval for incidence angle is determined as [1.5, 2.5] by observing these curves. The predicted buffet onset angle given by the present method based on the lift coefficient curve is 2.06 degree, as shown in Fig.6.

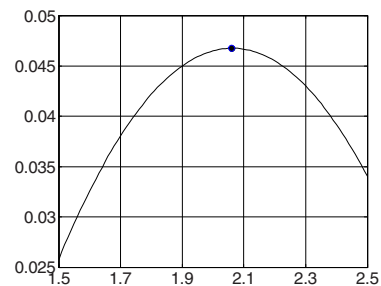
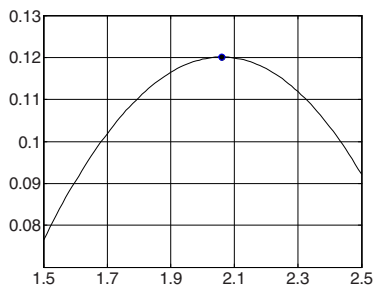


Fig. 6. Predicted buffet onset using the lift coefficient curve

Fig. 7. Predicted buffet onset using the pitching moment coefficient curve

As an alternative criterion, the incidence angle corresponding to the maximum curvature of the pitching moment coefficient curve is also adopted. As shown in Fig. 4, the slope of pitching moment coefficient curve changes gradually near the buffet onset, so it is difficult to determine the buffet onset by kink analysis. Fig.7 shows that the predicted buffet onset angle given by the present method based on the pitching moment coefficients is 2.06 degree, which is the same as that obtained by the lift coefficient curve.

Another commonly used buffet onset indicator is the divergence of pressure measured near the trailing edge, which means that the pressure will have a sudden increasing at the buffet onset angle. For the pressure coefficient curve at trailing edge ( $x/c=0.95$ ) as shown in Fig.5, the buffet onset angle can be determined as 2.04 degree using the maximum curvature method as shown in Fig. 8.

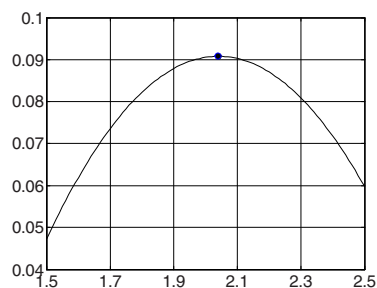


Fig. 8. Predicted buffet onset using the trailing edge pressure coefficient curve

## 6 Comparison with experimental results

The predicted results of transonic buffet onset for NACA0012 airfoil in the present study are shown in Table 2 along with that obtained by wind-tunnel test. It can be seen that the predicted transonic buffet onset by the present method is very



close to the experimental ones with a very small underestimated error. It may be caused by the difference between the shock locations of experiments and numerical predictions.

Table 2 Comparison of buffet onset

Method	Tunnel test results	Maximum curvature	Relative error(%)
Parameter curves			
Lift curve		$\alpha_B=2.06^\circ$	1.9
Pitching moment curve	$\alpha_B=2.1^\circ$	$\alpha_B=2.06^\circ$	1.9
Trailing edge pressure curve		$\alpha_B=2.04^\circ$	2.8

## 5 Conclusions

To improve the accuracy of transonic buffet onset prediction for airplane using numerical simulation approach at a certain flow conditions, a new method is proposed to determine the buffet onset using maximum curvature of the fitted curve of steady aerodynamic parameters. The aerodynamic parameters are extracted from data obtained by a steady Navier-Stokes Solver. This method is applied for the transonic buffet onset prediction for a NACA0012 airfoil and the predicted results using lift coefficient curve, pitching moment coefficient curve and trailing edge pressure coefficient curve, respectively, are compared with the buffet onset obtained by wind-tunnel test and good agreement is achieved. It can be suggested that the proposed method is applicable to the determination of transonic buffet onset for airplane.

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