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# AN $O(N^2)$ METHOD FOR COMPUTING THE EIGENSYSTEM OF $N \times N$ SYMMETRIC TRIDIAGONAL MATRICES BY THE DIVIDE-AND-CONQUER APPROACH

by DORON GILL* and EITAN TADMOR[6]

The $QR$ algorithm computes the eigenvalues of $N \times N$ symmetric tridiagonal (ST) matrices with $O(N^2)$ operations. The cost of computing the eigenvectors of such matrices is $O(N^3)$ operations. The additional order of magnitude required to compute these eigenvectors is typical of sequential algorithms.

Recently, a parallel divide-and-conquer algorithm was introduced [1, 2] for computing the spectral decomposition of ST matrices. A sequential implementation of this algorithm requires the same number of operations. Namely, the eigenvalues which coincide with the roots of the so-called secular equation [3] are computed at the cost of no more than $O(N^2)$ sequential operations; to compute the associated eigenvectors necessitates, as before, $O(N^3)$ sequential operations.

Here, we propose an efficient method derived from the DC algorithm, which computes the eigensystem of ST matrices with only $O(N^2)$ sequential operations. The method employs linear three-term recurrence relations which successively compute the *rows* of the eigenvector matrix. The coefficients of these relations depend on the already computed eigenvalues, and the method hinges on the observation that the initial first two rows for the recurrence relations emerge naturally from the DC computation of these eigenvalues. Thus, the input data for the recurrence relations depend solely on the $O(N^2)$ operations for the DC calculation of the eigenvalues. Together with the additional $O(N^2)$ operations required to carry out these relations, we end up with a most efficient method to compute the whole eigensystem of ST matrices.

Due to the sensitivity of the three-term recurrence relations, their input data should be provided with high accuracy. To achieve this, we employ an improved root finder—interesting for its own sake—in order to solve the

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secular equation mentioned above. Numerical examples which demonstrate the efficiency as well as the limitations of the proposed method are presented.

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## OPTIMIZATION OF FUNCTIONALS OF HURWITZ POLYNOMIALS

by I. NAVOT[7]

Hurwitz polynomials are intimately related to positive definite quadratic forms by several connections, e.g. the Routh-Hurwitz problem [1, 2], Hermite's symmetric-matrix form of Hurwitz's test [3–5], Lyapunov's second method [6, 2, 7], and Markov's stability criterion [2, 4]. (For interrelations see [2, 4].) Though the conditions governing these relations are generally both necessary and sufficient, in practice they have almost exclusively been applied unilaterally, i.e., the positive-definiteness of a quadratic form is invoked to imply the Hurwitzness of the polynomial under consideration (and with it the asymptotic stability of the system matrix of which the polynomial considered is the characteristic one). We consider the inverse situation in the following sense: After settling satisfactorily the fulfilment of the necessary conditions for the existence of an optimum in a constrained optimization problem involving Hurwitz polynomials, the decision on the existence of an optimum depends (in most cases where the second-order Taylor expansion in the vicinity of the stationary point exists and is sufficient to settle the issue) on the definiteness of a certain quadratic form. The Hurwitzness of the polynomials involved in the optimization problem is then invoked to imply the definiteness of the quadratic form concerned.

Summarily, we propose the following: Given a problem where it is required to find a Hurwitz polynomial $H(p)$ such that a certain functional of $H(p)$ is an optimum. Let $H_{\mathrm{opt}}(p)$ satisfy the necessary conditions for an

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