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Photon mass as a probe to extra dimensions

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ABSTRACT

In this manuscript we show that the geometrical localization mechanism implies a four dimensional mass for the photon. The consistence of the model provides a mass given exactly by $m_\gamma = \sqrt{R}/4$ where R is the Ricci scalar. As a consequence, the cosmological photon has a mass related to the vacuum solution of the Einstein equation. At the present age of the universe we have a dS vacuum with $R = 4\Lambda$, where Lambda is a positive cosmological constant. With this we find that $m_\gamma \approx 2\times 10^{-69}$ kg, which is below the present experimental upper bounds, and such correction may be observed in the next years with more precise measurements. By considering the value of R inside some astrophysical sources and environments we find that the bound is also satisfied. The experimental verification of this mass, beyond pointing to the existence of extra dimensions, would imply in a fundamental change in cosmology, astrophysics and in particle physics.

Compact extra dimensions has first been considered by Kaluza and Klein in the 20's. In this model the only way to recover the four dimensional physics is by considering a small compact extra dimension. The scenario has changed in the 90's when Arkani et al. proposed that the hierarchy problem can be solved by a large extra dimensions [1,2]. In this model the metrics is factorable and the Einstein–Hilbert (EH) action becomes $\bar{M}_{pl}^3 V\int d^4x\sqrt{g(x)}R(x)$. Here $\bar{M}_{pl}$ is the higher dimensional Planck mass, and V is the volume of the compact space. Therefore, if a TeV scale for $\bar{M}_{pl}$ is considered we can get an effective four dimensional $M_{pl} = 10^{18}$ GeV if V is large. Soon later Randall–Sundrum proposed a model, with a non-factorable metric, in which the extra dimension can be in fact of infinity range [3,4]. The idea is that a strong decreasing metric can provide a finite integration even for an infinity range in the extra dimension, and a consistent gravity theory is obtained over the membrane. However, a drawback in the above models is that gauge fields do not confine, failing to provide a consistent four dimensional observable universe. More precisely, in a conformal coordinate the metrics is given by $G_{MN} = e^{2A(z)}g_{MN}(x)$, where $A(z)$ is the warp factor, z and x are the extra dimension and brane coordinates, respectively. Before considering an specific form for the energy momentum tensor some comments are important. The EH action is given by $S = 2M^3\int d^5x\sqrt{-G}R(x,z)$ and for the above metric we know that the determinant of the metric is given by $\sqrt{-G} = e^{5A}\sqrt{-g(x)}$ and for the Ricci scalar

$$R(x,z) = e^{-2A}(R(x) - 8A'' - 12A'^2). \tag{1}$$

With this we see that the action contains a term given by

$$S \supset 2\bar{M}^3\int dze^{3A}\int d^4x\sqrt{-g(x)}R(x), \tag{2}$$

and the four dimensional EH action is recovered if $\int dze^{3A}$ is finite and this also relates the Planck mass in five and four dimensions by $M^2 = \bar{M}^3\int dze^{3A}$.

In the RS setup we have a Minkowski vacuum with solution $A(z) = -\ln(k|z|+1)$. This is obtained by considering a cosmological constant and two branes, one with positive tension at the origin and one with negative tension at the location z_c. Also, by imposing a Minkowski vacuum in four dimensions, a fine tuning between the tensions and the cosmological constant is needed. The above relation is then given by

$$M^3 = \frac{\bar{M}^3}{k}\left(1 - \frac{1}{kz_c+1}\right). \tag{3}$$

Here an interesting issue about this result is that it is valid for $z_c \to \infty$, and the model becomes an alternative to compactification. Next, Randall and Sundrum showed that the graviton zero

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mode is bound to the brane. Despite the fact that the model localizes the graviton, it can not be applied to more complex gravitational scenarios since the RS brane is flat. For example, the model is not suitable for a cosmological description, since the most recent observations indicate the existence of a positive cosmological constant. Soon after the RS paper, models with a non-vanish constant scalar curvature emerged [5,6]. In particular for the dS brane, which describes the current expansion phase of the universe, the warp factor is convergent. Another advantage of these models is that no fine tuning between the brane tension and the bulk cosmological constant is needed. These models localize the gravity, but the localization of other fields is not guaranteed, in particular the gauge fields. For this case the action is given by

$$S_A = \frac{1}{4}\int d^5x\sqrt{-G}G^{MO}G^{NP}Y_{MN}Y_{OP} \tag{4}$$

where $Y_{MN} = \partial_M X_N - \partial_N X_M$. Just like for the gravity case, in order to obtain a well defined four dimensional action the integration over the extra dimension must be finite. This is attached by performing the separation of variables $X_\mu(x,z) = e^{-\frac{A}{2}}\psi(z)A_\mu(x)$ and the equations of motion (EOM) are given by

$$\partial_\mu\sqrt{g}F^{\mu\nu} = -m^2A^\nu, \tag{5}$$

with a Schrödinger like mass equation

$$\psi'' - V(z)\psi = m^2\psi. \tag{6}$$

The prime means a z derivative and $V(z)$ is given by

$$V = \frac{A''}{2} + \frac{A'^2}{4}. \tag{7}$$

With this, similarly to the gravity case, the five dimensional action contains the term

$$S_A = \frac{1}{4}\int \psi^2dz\int d^4x\sqrt{-g}g^{\mu\nu}g^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}, \tag{8}$$

and the problem of obtaining a well defined action is resumed to find a normalized solution with $\int\psi^2dz = 1$. However, differently from the gravity case the solution to the zero mode is not normalizable. This is easily found since the above potential provides the general analytical solution $\psi = e^{\frac{A}{2}}$ for the zero mode. This is very similar to the gravity case but the integral does not converge if asymptotically we have an AdS solution. In the search for a solution to the gauge field localization which does not include the addition of new degrees of freedom a new model has been proposed in a series of papers [7–10]. This model provides an analytical solution given by $\psi = e^A$, which is a square integrable solution to the mass equation valid for any warp factor recovering RS asymptotically. The basic ingredient is the addition of a new term to the action given by

$$S_I = -\frac{1}{32}\int d^5x\sqrt{-G}RG^{MN}X_MX_N. \tag{9}$$

In the first version of the geometrical localization mechanism [7], the original RS model was considered when $R(x) = 0$. Using Eq. (1) $R(x,z) = -e^{-2A}(8A'' + 12A'^2)$, and a massless photon is obtained over the brane at least in first approximation. Another interesting point about the interaction term is that it has no free parameters and this will become crucial for determining the photon mass. From now on we will consider the full theory and show that the gauge field is confined for arbitrary four dimensional background. The proof is very similar to the flat case, but we give it here for completeness. We must be careful since now we have $ds^2 = e^{2A(z)}(g_{\mu\nu}(x)dx^\mu dx^\nu + dz^2)$. The equations of motion are

$$\nabla_M(G^{MO}G^{NP}Y_{OP}) = -\frac{1}{16}RG^{NO}X_O, \tag{10}$$

leading to the condition $\nabla_N(RG^{NO}X_O) = 0$, or

$$e^{3A}\nabla_\mu(RX^\mu) = -\nabla_5(e^{3A}RX_5). \tag{11}$$

Since the gauge invariance is now broken by the interaction term we have to show that a transversal gauge invariant zero mode is localized. For this we must split the gauge field in longitudinal and transversal components and show that they decouple. As mentioned before, in the previous version of the mechanism R was independent of x and the derivative on the left side of the above identity did not act on it. However, we will see that this does not spoil the decoupling. First we split our field as $X^\mu = X^\mu_L + X^\mu_T$, where L stands for longitudinal and T stands for transversal with $X^\mu_T = (\delta^\mu_\nu - \nabla^\mu\frac{1}{\Box}\nabla_\nu)X^\nu$; $X^\mu_L = \nabla^\mu\frac{1}{\Box}\nabla_\nu X^\nu$. Now we define $X_5 = \Phi$ and the first thing we observe is that (11) will give us a relation between the scalar field and the longitudinal part of X^μ. We also need the following identities

$$Y^{5\mu} = \partial X^\mu_T + \partial X^\mu_L - \partial^\mu\Phi \equiv \partial X^\mu_T + Y^{5\mu}_L;$$
$$Y^{\mu5}_L = \nabla^\mu\frac{1}{\Box}\nabla_\nu Y^{\nu5}. \tag{12}$$

Now considering the EOM for $N = 5;\nu$ the set of equations are obtained:

$$\nabla_\mu Y^{\mu5} + \frac{1}{16}e^{2A}R\Phi = 0, \tag{13}$$

and

$$e^A\nabla_\mu Y^{\mu\nu} + \frac{1}{16}e^{3A}RX^\nu_T +$$
$$+\nabla_5\left(e^A\partial X^\nu_T\right) + \nabla_5(e^AY^{5\mu}_L) + \frac{1}{16}e^{3A}RX^\nu_L = 0. \tag{14}$$

Using (11), (12) and (13) we get

$$\nabla_5(e^AY^{\mu5}_L) = -\frac{1}{16}\nabla^\mu\frac{1}{\Box}\nabla_5(e^{3A}R\Phi) = -\frac{1}{16}e^{3A}RX^\nu_L, \tag{15}$$

and finally we can decouple the equation of motion for the transverse part of the gauge field for arbitrary R

$$e^A\nabla_\mu Y^{\mu\nu} + \partial\left(e^A\partial X^\nu_T\right) + \frac{1}{16}e^{3A}RX^\nu_T = 0. \tag{16}$$

Finally performing the same transformation as before, or, $X_\mu(x,z) = e^{-\frac{A}{2}}\psi(z)A_\mu(x)$ we get the following equations of motion

$$\nabla_\mu(F^{\mu\nu}) = -(m^2 + R(x))A^\nu, \tag{17}$$

with a Schrödinger like mass equation

$$\psi'' - V(z)\psi = m^2\psi, \tag{18}$$

but now with $V(z)$ given by

$$V = A'' + A'^2. \tag{19}$$

The solution $\psi = ce^A$ for the zero mode and our four dimensional action of the vector field is given by

$$S_A = \frac{1}{4}\int c^2e^{2A}dz\int d^4x\sqrt{-g(x)}g^{\mu\nu}g^{\sigma\delta}F_{\mu\sigma}F_{\nu\delta}. \tag{20}$$

Therefore the confining of the gauge field is reduced to the condition $\int c^2e^{2A}dz = 1$. It is important to point that for the warp factor given before the above integral is convergent for any range of the extra dimension and the four dimensional gauge action is

Table 1
Photon mass values.

Environments	$m_\gamma \lesssim$ (kg)	$m^0_\gamma \approx$ (kg)	$m^\Lambda_\gamma \approx$ (kg)
Neutron star core	–	8×10^{-48}	8×10^{-48}
Terrestrial ionosphere	4×10^{-49}	2×10^{-57}	2×10^{-57}
Jupiter magnetosphere	7×10^{-52}	10^{-65}	10^{-65}
Sun core	–	4×10^{-54}	4×10^{-54}
Sun magnetosphere	2×10^{-54}	2×10^{-72}	2×10^{-69}
Intergalactic medium	10^{-62}	2×10^{-75}	2×10^{-69}
Cosmological vacuum	–	0	2×10^{-69}

recovered for both RS models. Moreover, the above integral is also convergent for any warp factor that recovers the RS metrics at the boundaries. This is very powerful since the only condition is that we must consider an AdS five dimensional vacuum to obtain a well defined theory. However, beyond the above term we have the non-minimal coupling which will generate a four dimensional contribution given by

$$S_{int} = -\frac{1}{32}\int c^2 e^{2A} dz \int d^4x \sqrt{-g(x)} R(x) g^{\mu\nu} A_\mu A_\nu . \tag{21}$$

Interestingly if we have a convergent solution to the gauge field this term is necessarily confined and this spoils the four dimensional gauge invariance. This result is a testable prevision of the model. Note that, in the original RS model, the brane vacuum is flat with $R(x) = 0$ and the gauge invariance is recovered for the vacuum. However, when we go to the next order we find a necessary breaking of the gauge invariance throughout the above interaction. Since we have no free parameters, the exact value of this mass can be obtained, $m_\gamma = \sqrt{R}/4$. In the Table 1 we consider the value of the Ricci scalar inside some sources and we find that the mass obtained for the photon is within the experimental bounds. The more interesting and important case occurs when we consider that the four dimensional universe is not flat but has a vacuum energy and $R = 4\Lambda$, where lambda is a positive cosmological constant. With this we get that a cosmological photon propagating in the vacuum must have the exact mass given by $m_\gamma \approx 2 \times 10^{-69}$ kg. As far as we know this is the first model in which the mass of the photon is a not a supposition but a necessary ingredient. Moreover, since the mass is not a free parameter, this provides a testable prevision of the model. We should point that the above interaction, despite being very small, should have consequences for astrophysics and stellar evolution.

To obtain values for the photon mass, m_γ, according to our geometric model of gauge field localization, we must consider some cosmological and astrophysical environments in order to calculate R and then $m_\gamma = (\hbar/c)\sqrt{R}/4$, comparing such values with the respective constraints for the mass photon obtained from the current experimental and speculative inferences. To do this, we will suppose that the astrophysical medium is a perfect fluid with matter density ρ and pressure P, with the vacuum being filled with an energy density which comes from the cosmological constant Λ. Thus the Ricci scalar is in the rest frame given by

$$R = \frac{8\pi G}{c^4}(\rho c^2 - 3P) + 4\Lambda . \tag{22}$$

For example, at the center of a neutron star, with $\rho \approx 5.0 \times 10^{17}$ kg/m^3 and $P \approx 10^{32}$ Pa [11], we find $m_\gamma \approx 8 \times 10^{-48}$ kg. At the Sun core, $\rho \approx 1.5 \times 10^5$ kg/m^3 and $P \approx 2 \times 10^{16}$ Pa [12], yielding $m_\gamma \approx 4 \times 10^{-54}$ kg. Finally, in the vacuum, we have $R = 4\Lambda$, and then $m_\gamma \approx 2 \times 10^{-69}$ kg, for $\Lambda \approx 2 \times 10^{-52}/\mathrm{m}^2$ [13].

It is interesting to establish a comparison, shown in the Table 1, of these and other values based on the model with the upper bounds ($m_\gamma \lesssim$) coming from both experimental procedures and theoretical estimates, according to [14,15].

Note that m^Λ_γ and m^0_γ are the photon masses calculated from the model with and without cosmological constant. The photon masses associated to the solar magnetosphere and intergalactic medium were calculated according to the energy density of the magnetic fields present in these scenarios, of 10^{-10} T and 10^{-13} T [16], respectively. It is worth also notice that all the obtained values are far below those upper limits.

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