Initial/Boundary Value Problems for the Semidiscrete Boltzmann Equation: Analysis by Adomian's Decomposition Method

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This paper provides a solution technique based on Adomian's decomposition method for a large class of initial/boundary value problems for the semidiscrete Boltzmann equation: a partial differential–integral equation of a semilinear type, in the kinetic theory of gases. The paper also proposes a use of the decomposition method as an algorithm for the continuous approximation of the solutions in a discretized time-space domain.

1. INTRODUCTION

The semidiscrete Boltzmann equation, proposed by Cabannes, is a mathematical model which provides, in terms of a partial differential–integral semilinear equation, a useful substitute in kinetic theory to the full semilinear Boltzmann equation [1]. Cabannes' model has been generalized to the description of a multicomponent gas mixture [2]. Further generalization can be found in [3] after a detailed analysis of the collision kinetics.

The model, mathematically described in the next section, defines the time and space evolution in the plane of the one-particle distribution function for a dilute gas. According to the model, all particles have the same velocity modulus, but all directions in the plane are allowed. In spite of the fact that the semidiscrete equation can be regarded as a "relatively easier" model with respect to the full Boltzmann equation, the analysis of the solutions of the initial/boundary value problems presents mathematical
difficulties quite similar to those of the full Boltzmann equation. And, in particular, a proof of the global existence for the solutions to the initial value problem in unbounded domains has been supplied only recently for small initial data, which decay to zero at infinity [4]. On the other hand, the analysis of the boundary value problem for this equation can be considered an open problem.

This paper deals with initial and boundary value problems, in one dimension, for the semidiscrete Boltzmann equation by means of the "inverse operator" or decomposition method" proposed by Adomian for partial differential equations [5–7]. This method has been developed in a number of papers referenced in [5–7], where the main results and relevant aspects of the method itself are collected.

The second section of this paper gives a detailed description of the mathematical model and related problems. The third section shows how the inverse operator method can supply a fixed point formulation to all problems defined in Section 2 and provides a preliminary analysis of the mathematical properties of such operator forms. The fourth section reports on the actual solution of the equations using a suitable application of the decomposition method [5–7]. There the use of the decomposition method provides an algorithm for the continuous approximation into discretized finite elements of the space-time domain. An application and a discussion follow.

2. MATHEMATICAL MODEL AND FORMULATION OF THE PROBLEMS

We consider the semidiscrete Boltzmann equation in one space dimension

\[ N = N(\theta, x; t): \frac{\partial N}{\partial t} + c \cos \theta \frac{\partial N}{\partial x} = Q(N, N) - NR(N), \]  

where the "gain" and "loss" terms, \( Q(N, N) \) and \( NR(N) \), respectively, are defined by

\[ Q(N, N)(x, t) = \frac{2cS}{\pi} \int_0^\pi N(\varphi, x, t) N(\varphi + \pi, x, t) \, d\varphi \]  

\[ NR(N)(\theta, x, t) = 2cSN(\theta, x, t) N(\theta + \pi, x, t) \]  

and, where the whole nonlinear collisional operator is indicated, in the sequel, by

\[ J(N, N)(\theta, x, t) = Q(N, N)(\theta, x, t) - NR(N)(\theta, x, t). \]
Here $S$ is the cross sectional area of the gas particle and $c$ is the velocity modulus of the particle's velocity vector $c$ forming an angle with respect to the $x$-axis.

As already mentioned, Eq. (1) defines a suitable mathematical model which describes the space-time evolution of the distribution function $N(\theta, x, t)$ of the one-particle number density of a dilute, monoatomic gas. Once $N$ has been obtained by solving Eq. (1) with given initial/boundary values, the relevant macroscopic quantities can be recovered via moments of $N$. For instance, the local number density $n = n(x, t)$ and mass velocity $U = U(x, t)$ in the $x$-direction are

$$n = n(x, t) = 2 \int_0^\pi N(\theta, x, t) \cos \theta \, d\theta$$

$$U = U(x, t) = \left[ \frac{2c}{n(x, t)} \int_0^\pi \cos \theta N(\theta, x, t) \, d\theta \right] 1.$$  

We refer to Cabannes' paper for further details of the model and to papers [2, 3] for its generalization to gas mixtures and space description. In addition, the present analysis is limited to the one-dimensional situation, but generalization to space geometries is immediate and is left to the reader.

In particular, the following problems will be considered:

(a) **Initial value problem in unbounded domains.** The gas particle system is located in an unbounded domain and Eq. (1) is subject to the initial condition

$$\forall x \in \mathbb{R}: N(\theta, x; t = 0).$$

As a particular case, we can consider the spatially homogeneous equation, i.e., Eq. (1) with $\partial/\partial x = 0$. In this case the initial condition is simply a function of $\theta$.

(b) **Half-space initial/boundary value problem with deterministic boundary conditions at $x = 0$.** The gas particle system is confined in the half-space $x \geq 0$ and the initial and "deterministic" boundary conditions are

$$\forall x \in \mathbb{R}_+, \quad N(\theta, x; t = 0)$$

$$\forall t \in \mathbb{R}_+, \quad x = 0: N(\theta, t; x = 0).$$

(c) **Half-space initial/boundary value problem with stochastic boundary conditions at $x = 0$.** The problem is analogous to the one considered in (b), but the boundary conditions are expressed by a linear stochastic operator $\mathcal{B}$, which maps the distribution function $N_i$ of the particles hitting the wall,
for which \( i \cdot e \leq 0 \), into one of the particles leaving the wall, for which \( i \cdot e > 0 \), where \( i \) is the outward-pointing normal to the wall. Such a map can be written in the general form

\[
N_r(\theta, t; x = 0) = \mathcal{B}N_i(\theta, t; x = 0)
\]

\[
= \frac{|e_r \cdot i|}{|e_r \cdot i|} \int_{e_r \cdot i \leq 0} N_i(\theta, t; x = 0) P(\theta_i \rightarrow \theta_2) d\theta_i,
\]

(9)

where the subscripts \( i \) and \( r \) indicate, respectively, \( e \cdot i \leq 0 \) and \( e \cdot i > 0 \). \( P \) defines, in Eq. (9), the probability density that a particle hitting the wall along the direction \( \theta_i \) is reemitted along \( \theta_r \).

The analysis developed in the next section is carried out under the hypothesis that a solution of the above problem exists and is unique, in a suitable function space, at least in some space-time interval.

The above point will be further discussed later. However, as already mentioned, a local existence theorem and a global existence theorem are available for the initial value problem [4].

3. THE INVERSE OPERATOR FORMULATION

The goal of this section is a fixed point formulation of Eq. (1) for the three mathematical problems described in the preceding section. In general, such a formulation will be written as

\[
N(\theta, x, t) = \mathcal{W}N(\theta, x, t),
\]

(10)

where "\( \mathcal{W} \)" is a suitable operator to be determined by the application of Adomian's inverse operator method.

Consider then the space \( B_T \) of the functions of \((\theta, x, t)\), continuous in \( \theta \in [0, 2\pi) \), 2\( \pi \)-periodically continued with respect to \( \theta \), and once differentiable with respect to \( t \) and \( x \). The norm is defined by

\[
\| N \| = \sup_{(\theta, x, t) \in [0, 2\pi) \times \mathbb{R} \times \mathbb{R}^+} \max (|N|, |\partial N/\partial t|, |\partial N/\partial x|).
\]

(11)

The inverse operator formulation requires that the following conditions hold:

*The initial and boundary conditions joined to Eq. (1) are such that there exists a time interval \([0, T] \subseteq \mathbb{R}_+\) and a domain \( D \) of the variable \( x \), \( D \subseteq \mathbb{R} \), wherein the solution \( N(\theta, x, t) \) exists unique and positive defined in the considered Banach space: \( 0 \leq N(\theta, x, t) \in B_T \).*
The result of [4] provides sufficient conditions to assure existence and uniqueness of the solution of the Cauchy problem in the space of continuous bounded functions. Differentiability conditions can be obtained upon suitable assumptions on the differentiability of the initial data. Analogous methods can be applied in order to obtain existence proofs, local in space and time, for the boundary value problems (a) and (b). On the other hand, the question of global existence for the boundary value problem is still open.

Using Adomian's method [5–7], we define the differential operators

$$L_t = \partial / \partial t, \quad L_x = \partial / \partial x$$

for which the inverse operators, $\forall N \in \mathbb{B}_T$, exist in the form

$$L_t^{-1} N = \int_0^t N(\theta, x, s) \, ds, \quad L_x^{-1} N = \int_0^x N(\theta, s, t) \, ds.$$  \hfill(13)

If now the $\pi$-shifted function $N(\theta + \pi, \cdot)$ is indicated by $M$, i.e.,

$$M(\theta, x, t) = N(\theta + \pi, x, t),$$

Eq. (1) can be rewritten as the system

$$L_t N(\theta, x, t) + c \cos \theta L_x N(\theta, x, t) = J(N, M)(\theta, x, t)$$
$$L_t M(\theta, x, t) - c \cos \theta L_x M(\theta, x, t) = J(N, M)(\theta, x, t).$$

Then applying the inverse operators (13) separately to Eqs. (17) gives

$$N(\theta, x, t)$$
$$= N(\theta, x, 0) - c \cos \theta L_t^{-1} L_x N(\theta, x, t) + L_t^{-1} J(\theta, x, t)$$

$$M(\theta, x, t)$$
$$= M(\theta, x, 0) + c \cos \theta L_t^{-1} L_x M(\theta, x, t) + L_t^{-1} J(\theta, x, t).$$

After this step, summing the two expressions of $N$ and $M$ and dividing by 2 yields

$$N(\theta, x, t) = N_0(\theta, x, t) - \mathcal{L} N(\theta, x, t) + \mathcal{N}(N, M)(\theta, x, t)$$
$$M(\theta, x, t) = M_0(\theta, x, t) + \mathcal{L} M(\theta, x, t) + \mathcal{M}(N, M)(\theta, x, t).$$
where

\[ N_0 = \frac{1}{2} [N(\theta, x, t = 0) + N(\theta, x = x_0, t)] \]  
\[ M_0 = \frac{1}{2} [M(\theta, x, t = 0) + M(\theta, x = x_0, t)] \]

and where the linear integro-differential operator \( L \) is defined as

\[ L = L_1 + L_2 = \frac{1}{2} \left( c \cos \theta L^{-1}_x + \frac{1}{c \cos \theta} L^{-1}_x L \right). \]

In addition, the nonlinear terms \( \mathcal{N} \) and \( \mathcal{M} \) are defined by

\[ \mathcal{N}(N, M) = (L_1 + L_2) J(N, M)(\theta, x, t) \]  
\[ \mathcal{M}(N, M) = (L_1 - L_2) J(N, M)(\theta, x, t), \]

where

\[ L_1 = \frac{1}{2} L^{-1}_x, \quad L_2 = (\frac{1}{2} / c \cos \theta) L^{-1}_x. \]

Note that

\[ \mathcal{N}(N, M) + \mathcal{M}(N, M) = 2L_1 J(N, M); \]
\[ \mathcal{N}(N, M) - \mathcal{M}(N, M) = 2L_2 J(N, M). \]

Equations (17)–(21) define the previously mentioned operator \( \mathcal{U} \):

\[ \mathcal{U} = \{ \mathcal{U}_1, \mathcal{U}_2 \}^T; \quad \mathcal{U}_1 = N_0 - L + \mathcal{N}; \quad \mathcal{U}_2 = M_0 + L + \mathcal{M}. \]

Such a form can be considered a fixed point operator formulation of Eq. (1) for all problems described in Section 2. In fact, the initial and/or boundary conditions can be included in the terms \( N_0 \) and \( M_0 \). Of course, one has to realize that the proof that the operator \( \mathcal{U} \) maps functions \( N \in \mathbf{B}_T \) into \( \mathbf{B}_T \) has not been provided yet and that such a proof can be given only after suitable assumptions on the initial and/or boundary conditions. An additional useful step would consist in proving that \( \mathcal{U} \) may be a contractive operator from some closed convex subset of \( \mathbf{B}_T \) into itself.

A further useful linear operator form can be derived by Eqs. (16) on the basis of the fact that the right-hand term is the same for both equations. As a consequence,

\[ L_1(N - M)(\theta, x, t) + c \cos \theta L_2(N + M)(\theta, x, t) = 0. \]
now applying the Adomian method method results in

\[ N(\theta, x, t) - M(\theta, x, t) = N(\theta, x, 0) - M(\theta, x, 0) - 2\mathcal{L}_1(N + M)(\theta, x, t) \]  
\[ N(\theta, x, t) + M(\theta, x, t) = N(\theta, x_0, t) + M(\theta, x_0, t) - 2\mathcal{L}_2(N - M)(\theta, x, t) \]

and, by algebraic manipulations,

\[ N(\theta, x, t) = N^*(\theta, x, t) - \mathcal{L}_1(N + M)(\theta, x, t) - \mathcal{L}_2(N - M)(\theta, x, t) \]  
\[ N(\theta, x, t) = M^*(\theta, x, t) - \mathcal{L}_1(N + M)(\theta, x, t) + \mathcal{L}_2(N - M)(\theta, x, t) \]

where

\[ N^*_0 = \frac{1}{2}(N(\theta, x, 0) + N(\theta, x_0, t) + M(\theta, x_0, t) - M(\theta, x, 0)) \]  
\[ N^*_0 = \frac{1}{2}(N(\theta, x_0, t) - N(\theta, x, 0) + M(\theta, x_0, t) + M(\theta, x, 0)) \]

4. SOLVING PROBLEMS BY ADOMIAN'S DECOMPOSITION METHOD

The actual solution of problems by means of the decomposition method can be carried out following the guidelines reported in [6, 7]. Accordingly, the solutions \( N(\theta, x, t) \) and \( M(\theta, x, t) \) can be decomposed, for \( \lambda = 1 \), as follows:

\[ N(\theta, x, t) \cong \sum_{j=0}^{n} \lambda^j N^{(j)}(\theta, x, t) \]  
\[ M(\theta, x, t) \cong \sum_{j=0}^{n} \lambda^j M^{(j)}(\theta, x, t) \]

In addition, the product \( MN \), which defines the nonlinearity of Eq. (1), can be expressed by the standard Cauchy form for the product of series

\[ N(\theta, x, t) M(\theta, x, t) \cong \sum_{j=0}^{n} \lambda^j F^{(j)}(\theta, x, t) \]

where

\[ F^{(j)} = F^{(j)}(N_1^{(j)}, M_1^{(j)}, ..., N_j^{(j)}, M_j^{(j)}) = \sum_{k=0}^{j} N_{j-k}(\theta, x, t) M_k(\theta, x, t) \]
Then a $\lambda$-decomposition of the nonlinear term $J$ is defined by

$$J(\theta, x, t) \equiv \sum_{\delta=0}^{h} \lambda^{\delta} J^{(\delta)}(N^{(0)}, M^{(0)}, \ldots, N^{(\delta)}, M^{(\delta)})(\theta, x, t), \quad (31)$$

where

$$J^{(\delta)} = \frac{2cS}{\pi} \int_{0}^{\pi} F^{(\delta)}(\phi, x, t) \, d\phi - 2cSF^{(\delta)}(\theta, x, t). \quad (32)$$

The decomposition indicated in Eqs. (28)-(32) can now be substituted into Eq. (17) and the terms with the same power of $\lambda$ can be equated; that is,

$$\sum_{\delta=0}^{n} \lambda^{\delta} N^{(\delta)}(\theta, x, t) = N_{0}(\theta, x, t) - \lambda L_{0} \sum_{\delta=0}^{n} \lambda^{\delta} N^{(\delta)}(\theta, x, t) + \lambda(L_{1} + L_{2}) \sum_{\delta=0}^{n} \lambda^{\delta} J^{(\delta)}(\theta, x, t) \quad (33a)$$

$$\sum_{\delta=0}^{n} \lambda^{\delta} M^{(\delta)}(\theta, x, t) = M_{0}(\theta, x, t) + \lambda L_{0} \sum_{\delta=0}^{n} \lambda^{\delta} M^{(\delta)}(\theta, x, t) + \lambda(L_{1} - L_{2}) \sum_{\delta=0}^{n} \lambda^{\delta} J^{(\delta)}(\theta, x, t). \quad (33b)$$

Consequently, we finally define

$$N^{(0)} = N_{0} \quad (34a)$$
$$M^{(0)} = M_{0}$$

$$N^{(1)} = -L_{0} N^{(0)} + (L_{1} + L_{2}) J^{(0)}(N^{(0)}, M^{(0)}) \quad (34b)$$
$$M^{(1)} = L_{0} M^{(0)} + (L_{1} - L_{2}) J^{(0)}(N^{(0)}, M^{(0)})$$

$$\vdots$$

$$N^{(\delta)} = -L_{0} N^{(\delta-1)} + (L_{1} + L_{2}) J^{(\delta-1)}(N^{(0)}, M^{(0)}, \ldots, N^{(\delta-1)}, M^{(\delta-1)}) \quad (34c)$$
$$M^{(\delta)} = (L_{1} - L_{2}) J^{(\delta-1)}(N^{(0)}, M^{(0)}, \ldots, N^{(\delta-1)}, M^{(\delta-1)})$$

where, for simplicity, the arguments of the functions $N$ and $M$ have been dropped. The sequence (34) clearly indicates how each term of the decomposition is obtained from the preceding one by suitable application of the differential–integral operator and of the integral operators $L_{1}$ and $L_{2}$. This result can now be particularized for each of the three problems described in the second section:

(a) The initial value problem. We refer to the initial value problems described at the point (a) of Section 2. The spatially inhomogeneous case
presents the difficulty that one must know the value of the solution for \( x \) tending to \( -\infty \), which can be equal to zero in the case considered in \([4]\), and the additional difficulty that the operators \( L_x^{-1} \) have \(-\infty\) as a lower bound. Nevertheless, the problem can be treated by the sequence (34) with \( N_0 \) and \( M_0 \) defined as

\[
N_0 = \frac{1}{2}(N(\theta, x, 0) + N(\theta, t; x \to -\infty)) = \frac{1}{2}N(\theta, x, 0) \tag{35a}
\]

\[
M_0 = \frac{1}{2}(M(\theta, x, 0) + M(\theta, t; x \to -\infty)) = \frac{1}{2}M(\theta, x, 0). \tag{35b}
\]

provided that the initial conditions and solutions can be defined in the space of the functions which vanish at infinity (in space).

The spatially homogenous case does not present the above-mentioned difficulties and the operator form is simply

\[
N(t; \theta) = N_0(0, \theta) + \int_0^t J(N, M)(s; \theta) \, ds \tag{36a}
\]

\[
M(t; \theta) = M_0(0, \theta) + \int_0^t J(N, M)(s; \theta) \, ds. \tag{36b}
\]

The modifications of the sequence (34) in order to deal with problem (36) are immediate.

(b) Initial/boundary value problem with deterministic boundary conditions. The resolvent sequence (34) holds, in this case, when \( N_0 \) and \( M_0 \) are defined as

\[
N_0 = \frac{1}{2}(N(\theta, x, 0) + N(\theta, 0, t)), \quad M_0 = \frac{1}{2}(M(\theta, x, 0) + M(\theta, 0, t)). \tag{37}
\]

(c) Initial/stochastic boundary value problem. This problem is analogous to the one treated in the preceding point with the exception of the statement of the boundary conditions defined by the operator transform on the boundary indicated in Eq. (9). In this case we have to decompose \( N_0 \) and \( M_0 \) as

\[
N_0 = \frac{1}{2}N(\theta, x, 0) + \frac{1}{2}\lambda(N_i(\theta, 0, t) + \mathcal{B}N_i(\theta, 0, t)) \tag{38a}
\]

\[
M_0 = \frac{1}{2}M(\theta, x, 0) + \frac{1}{2}\lambda(M_i(\theta, 0, t) + \mathcal{B}M_i(\theta, 0, t)). \tag{38b}
\]

The sequence (34) is, consequently, modified as

\[
N^{(0)} = \frac{1}{2}N(\theta, x, 0) \tag{39a}
\]

\[
M^{(0)} = \frac{1}{2}M(\theta, x, 0) \tag{39b}
\]

\[
\vdots
\]

\[
N^{(j)} = \frac{1}{2}(N^{(j-1)} + \mathcal{B}N^{(j-1)}) - \mathcal{L}N^{(j-1)} + (L_1 + L_2)J^{(j-1)} \tag{40a}
\]

\[
M^{(j)} = \frac{1}{2}(M^{(j-1)} + \mathcal{B}M^{(j-1)}) + \mathcal{L}M^{(j-1)} + (L_1 - L_2)J^{(j-1)}. \tag{40b}
\]
Remark 1. The decomposition can be applied also to Eqs. (26). In this case the operator equation is linear and the following sequence is obtained:

\[ N^{(0)}(\theta, x, t) = N_0^*(\theta, x, t) \]  
\[ M^{(0)}(\theta, x, t) = M_0^*(\theta, x, t) \]  
\[ \vdots \]
\[ N^{(j)} = -L_1(N^{(j-1)} + M^{(j-1)}) - L_2(N^{(j-1)} - M^{(j-1)}) \]  
\[ M^{(j)} = -L_1(N^{(j-1)} + M^{(j-1)}) + L_2(N^{(j-1)} + M^{(j-1)}) \]

Remark 2. The difficulty in dealing with the spatially inhomogenous initial value problem can be overcome by replacing the operator form (23) derived in the third section of this paper, by the classical "mild" integral form [4], considering that Eq. (1) is of hyperbolic type with characteristic lines defined by the trajectory of the particles between two collisions. Then

\[ \Phi(\theta, X, t) = \Phi(\theta, x - c \cos \theta t; t = 0) + \int_0^t \int J(N, M)(\theta, x - c \cos \theta(t - s), s) ds. \]  
\[ (43) \]

Applying the decomposition can now be left to the reader. The operator characterizing Eq. (43) is now an integral and the problem of integration over space starting from minus infinity is overcome.

Remark 3. The sequences (34) and (41) or the one which can be derived from Eq. (43) can provide a solution of the operator Eq. (23) if at least the following two conditions are satisfied:

(i) The operators which appear in the aforementioned sequences map functions \( N \) and \( M \) from \( B_T \) into \( B_T \).

(ii) \( j \to \infty \Rightarrow N^{(j)}, M^{(j)} \to 0 \).

The conditions detailed in Remark 3 can generally hold only for bounded time intervals or for particular initial/boundary conditions. In addition we point out that the question of existence and uniqueness of solutions, under suitable assumptions on the initial and boundary conditions, is possible in terms of the analysis of Eq. (23), but it is not considered in this paper, which deals only with mathematical methods to obtain solutions. Nevertheless, verification of conditions (i), and (ii) can be difficult for large time-space domains for general initial and boundary conditions. In other words, a large number of terms may be necessary to achieve accuracy. On the other hand, we can show that the decomposition method can provide a "continuous" approximation of the solution in all points of a bounded time-space domain discretized into finite elements.
Consider then the domain $D = [t_0, T] \oplus [x_0, X]$ discretized into finite elements:

$$D_{ij} = [t_i, t_{i+1}] \oplus [x_j, x_{j+1}]: \Delta t = (t_{j+1} - t_j), \Delta x = (x_{j+1} - x_j).$$

Figure 1 shows both the discretization and the actual realization of the continuous approximation. The arrows indicate the initial and boundary conditions of the approximation in each domain $D_{ij}$ denoted by a dot. For instance, the following sequence can be realized:

1. **Approximation in $D_{00}$ with initial conditions on $[t_0, t_1; x = x_0]$ and boundary conditions on $[x_0, x_1; t = t_0]$.**

2. **Approximation in $D_{10}$ with initial conditions on $[t_1, t_2; x = x_0]$ and boundary conditions on $[x_0, x_1; t = t_1]$.**

3. **Approximation in $D_{01}$ with initial conditions on $[t_0, t_1; x = x_1]$ and boundary conditions on $[x_1, x_2; t = t_0]$.**

Note that steps 2 and 3 can be realized only after the approximation of step 1:

$$D_{00} \rightarrow (D_{01}, D_{10}) \rightarrow (D_{02}, D_{11}, D_{20}) \rightarrow \cdots \rightarrow (D_{0j}, ..., D_{jk}).$$

Other equivalent sequences can also be proposed.

5. **Discussion and Application**

This paper considers the semidiscrete Boltzmann equation, a nonlinear mathematical model in the kinetic theory of the gases, and provides an
analysis of some mathematical problems related to the application of such a model to the study of a large class of initial and initial/boundary value problems.

The analysis consists essentially of the following three steps:

—First, the partial differential integral equation is transformed, by the inverse operator method [5–7], into an operator equation which includes initial and boundary conditions and replaces the original initial/boundary value problem.

—Second, we seek the actual solution for quite general initial/boundary conditions by means of the decomposition method [5].

—Third, the decomposed solution is utilized as an algorithm for the continuous approximation of the solution in bounded domains discretized into finite elements.

The methods and analysis can be regarded as very general. In fact, they can be applied to several evolution equations with initial and boundary conditions on open domains. The extension to closed domains can be regarded as a technical problem.

In particular, the third step of the analysis appears useful for the solution of a large class of problems and provides a continuous approximation instead of the usual localized approximation in an analogous fashion of the method, proposed in Ref. [8], for ordinary differential equations. Various experiments indicate the usefulness of these techniques although the problem of the error bounds has not been considered in this paper, which essentially deals with methods.

However, the proof proposed in [9] for expansion of the solution for ordinary differential equations using the decomposition method may be technically extended to partial differential and operator equations also. Such a proof should provide an estimate of the error bounds to be related to the number of terms utilized in the decomposition and to the size of the discretized elements.

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REFERENCES


