Near threshold enhancement of $p\bar{p}$ system and $p\bar{p}$ elastic scattering

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ABSTRACT

The observed enhancement of $p\bar{p}$-production near the threshold in radiative decays of J/ψ and e^+e^--annihilations can be explained with final state interactions among the produced $N\bar{N}$ system, where the enhancement is essentially determined by $N\bar{N}$ elastic scattering amplitudes. We propose to use an effective theory for interactions in an $N\bar{N}$ system near its threshold. The effective theory is similar to the well-known one for interactions in an NN system but with distinctions. It is interesting to note that in the effective theory some corrections to scattering amplitudes at tree-level can systematically be summed into a simple form. These corrections are from rescattering processes. With these corrected amplitudes we are able to describe the enhancement near the threshold in radiative decays of J/ψ and e^+e^--annihilations, and the $p\bar{p}$ elastic scattering near the threshold.

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It has been observed the enhancement of the $p\bar{p}$-production near the threshold in various experiments. The enhancement has been observed near the threshold of the $p\bar{p}$ system in the decay $J/\psi \to \gamma p\bar{p}$ by BES [1] and in the decay of $B^+ \to K^+ p\bar{p}$ and $\bar{B}^0 \to D^0 p\bar{p}$ by Belle [2]. The enhancement also has appeared in the process $e^+e^- \to p\bar{p}$ measured by BaBar [3]. The enhancement of other baryonic system has been also seen [4,5]. In this work we focus on the enhancement of $p\bar{p}$ system.

Because BES was the first to publish the result about the enhancement with rather high statistical accuracy and precise information about the spectrum, many explanations for the observed enhancement at BES exist. A class of explanations is that the enhancement is interpreted as the existence of a baryonium bound state [6] or a glueball below the threshold [7]. Another class of explanations is to take the effect of final state interactions into account. There are different ways to take final state interactions into account. One can use a complex S-wave $p\bar{p}$ scattering length [8] or use a K-matrix formalism to include one pion exchange [11]. A more realistic way is by using potential models of $N\bar{N}$ interactions [9,10]. The observed enhancement in $e^+e^- \to p\bar{p}$ has also motivated theoretical studies [10–16]. It is interesting to note that by taking final state interactions into account through potential models of $N\bar{N}$ interactions, the enhancement in $J/\psi \to \gamma p\bar{p}$ and $e^+e^- \to p\bar{p}$ can be explained simultaneously [10,15]. However, these models are in general complicated and contain several

or more parameters which need to be fixed. It should be noted that with final state interactions the enhancement is predicted with the energy dependence of $p\bar{p}$-elastic scattering amplitudes, where one employs Watson–Migdal approximation [17].

A rather simple approach with final state interactions for explaining the enhancement has been give in [18]. The $p\bar{p}$-elastic scattering amplitude there is determined through the rescattering mechanism similar to Watson–Migdal approximation, where one re-sums the multi-pion exchange or multi-rescattering of an $N\bar{N}$ system. Because the coupling of πNN is well known, the amplitude is completely fixed in this approach. It has been shown in [18] that the observed enhancement in the $J/\psi \to \gamma p\bar{p}$ and $e^+e^- \to p\bar{p}$ can be well explained. But, with the fixed amplitude one cannot explain the $p\bar{p}$ elastic scattering near the threshold. In this work we make an attempt in the framework of effective field theories to give a unified explanation for the enhancement in the J/ψ decay and e^+e^- annihilation, and for the $p\bar{p}$ elastic scattering near the threshold.

An effective theory of $N\bar{N}$ interactions can be developed in analogy to the effective theory of NN interactions. The effective theory of NN interactions has been proposed in [19–21] and studied extensively in [20–22]. With the effective theory the experimental data of NN scattering of low partial waves near the threshold can be well described. In constructing such effective theories one makes a power expansion in the momentum near the threshold. The coefficients in the expansion characterize the properties of the NN- or $N\bar{N}$ system, like scattering length a_0, interaction range r_0, etc.

There are distinct differences between the effective theory of $N\bar{N}$ interactions and that of NN interactions. As for an effective

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field theory, one does not only need to construct the effective Lagrangian which gives tree-level amplitudes directly, but also one should be able to estimate relative importance of higher-order corrections and to systematically calculate these corrections. In other word, one needs a power counting for the estimation of loop corrections. If the interaction of a system is characterized by a momentum scale Λ and the quantities like $a_0, r_0, \ldots$ have the natural size, i.e., $a_0 \sim \Lambda^{-1}$, a simple power counting like that for the well-known chiral perturbation theory can be used. However, for NN systems it is not the case. It is well known from experiment that NN-systems have large scattering lengths. This fact makes the simple power counting invalid. An important idea has been suggested in [21], where one can use the power divergence subtraction scheme instead of the minimal subtraction scheme which is commonly used, implemented to the effective theory. With the power divergence subtraction scheme a power counting can be established. This makes the effective theory well defined for NN system with large scattering lengths. In the case of $N\bar{N}$ systems, the scattering lengths, according LEAR experiment [8] and model results [9], are around 1 fm or smaller. They are much smaller than those of NN systems. Therefore one can use the minimal subtraction scheme and hence the simple power counting for the effective theory of $N\bar{N}$ systems.

Another difference is that an $N\bar{N}$ system can be annihilated in to a multiple pion system and the pions can be real, while an NN system cannot be annihilated. The annihilation of an $N\bar{N}$ system into virtual- or real pions results in that the dispersive- and absorptive part of the $N\bar{N}$ scattering amplitudes are of the same importance. In order to incorporate this fact some coupling constants in the effective theory of an $N\bar{N}$ system are complex numbers. In the effective theory of an NN system the coupling constants are real. In this work we will first discuss the effective theory of an $N\bar{N}$ system and the NN elastic scattering. Then we study the rescattering mechanism for the enhancement in the decay $J/\psi \to \gamma p \bar{p}$ and the annihilation $e^+e^- \to p\bar{p}$. We will show that with our effective theory approach the enhancement in the decay $J/\psi \to \gamma p \bar{p}$ and the annihilation $e^+e^- \to p\bar{p}$ and the $p\bar{p}$ elastic scattering near the threshold can be well described.

We consider the $N\bar{N}$ scattering near the threshold:

$$N(\vec{p}, s_1) + \bar{N}(-\vec{p}, s_2) \to N(\vec{k}, s_1') + \bar{N}(-\vec{k}, s_2'),$$

$$|\vec{p}| = p = \beta\sqrt{m^2 + p^2}, \tag{1}$$

where $\vec{p}$ and $\vec{k}$ are three-momenta. The spins are denoted with s's. β is the velocity. Near the threshold, the momentum p or β approaches to zero. We are interested in the momentum region $p \leqslant m_\pi$. An effective Lagrangian can be obtained by an expansion in p or β. For this it is natural to use nonrelativistic fields to describe the nucleon N. The nonrelativistic fields are given as

$$\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}, \qquad \chi = \begin{pmatrix} \chi_p \\ \chi_n \end{pmatrix}. \tag{2}$$

The two-component field $\psi_p(\chi_p^\dagger)$ annihilates a proton (an antiproton) and the field $\psi_p^\dagger(\chi_p)$ creates a proton (an anti-proton). We denote the Pauli matrix acting in the $SU(2)$ isospin space as τ^i ($i = 1, 2, 3$). The π fields are given as $\pi = \tau^i \pi^i$ with $\pi^3 = \pi^0$. At leading order the interacting part can be written as:

$$\delta\mathcal{L} = \frac{c_0}{4}\psi^\dagger \chi \chi^\dagger \psi + \frac{c_1}{4}\psi^\dagger \tau^i \chi \chi^\dagger \tau^i \psi + \frac{d_0}{4}\psi^\dagger \sigma^i \chi \chi^\dagger \sigma^i \psi$$

$$+ \frac{d_1}{4}\psi^\dagger \tau^i \sigma^j \chi \chi^\dagger \sigma^j \tau^i \psi$$

$$+ \frac{g_A}{2F_\pi}\psi^\dagger \vec{\sigma} \cdot (\vec{\partial}\pi)\psi + \frac{g_A}{2F_\pi}\chi^\dagger \vec{\sigma} \cdot (\vec{\partial}\pi)\chi + \mathcal{O}(p^2), \tag{3}$$

with $g_A \approx 1.25$ and $F_\pi \approx 93$ MeV. c_I with $I = 0, 1$ is the coupling constant in the $^1S_0$ channel, while d_I with $I = 0, 1$ is the coupling constant in the $^3S_1$ channel. As discussed before, these coupling constants are in general complex because an $N\bar{N}$ can annihilate into π's. One should keep in mind that the complex coupling constants here do not mean the violation of time-reversal symmetry. The complex coupling constants can be understood as the following: One can imagine that the effective theory is obtained from a perturbative matching of a more fundamental theory. In the more fundamental theory with the time-reversal symmetry amplitudes at tree-level are real, but they receive imaginary parts beyond treelevel because absorptive parts are nonzero at one- or more loop level. The imaginary parts of the coupling constants are from these absorptive parts in the matching.

To clearly discuss the mentioned simple power counting for the above effective theory we first ignore the interactions of pion exchanges. Then in this case, the scattering amplitude at tree level is expanded in p, the leading orders are determined by the contact interactions given in Eq. (3) and are at $\mathcal{O}(p^0)$ if we take $c_{0,1}$ and $d_{0,1}$ as constants. The tree-level contributions at higher orders of p starting at $\mathcal{O}(p^2)$ are given by operators with derivatives in the effective theory. Therefore, at tree-level the amplitude is simply expanded in power of p and the power of p of each term is determined by the corresponding contact terms in the effective theory. However, this can be changed if we take loop-effects into account, i.e., the effects of scale-dependence of coupling constants. E.g., in the effective theory for an NN system of large scattering lengths the coupling constants corresponding to $c_{0,1}$ and $d_{0,1}$ should be taken as at order of p^{-1} after including loop effects with power subtraction scheme, where one reasonably takes the renormalization scale as $\mu \sim p$. In this way, a consistent power counting is established for an NN system with large scattering lengths [21].

As discuss before, it is expected that quantities characterizing interactions of an $N\bar{N}$ system have the nature size Λ. With this expectation the coefficients $c_{0,1}$ and $d_{0,1}$ in the effective theory scale like Λ^{-2} or $(m\Lambda)^{-1}$ with m as the nucleon mass. They are at order of $\mathcal{O}(p^0)$ with the minimal subtraction scheme in dimensional regularization as shown in [21]. The scale-dependence of these coupling constants are suppressed by certain power of p. The loop contributions formed only by the contact interactions can be then estimated as the following: Each loop contributes a factor $(mp/4\pi)$. Hence, the leading contribution is at p^0 and comes from $c_{0,1}$ and $d_{0,1}$ at tree-level. The next to-leading order is at p^1 and comes from $c_{0,1}$ and $d_{0,1}$ at one-loop level. The contribution at p^2 comes from $c_{0,1}$ and $d_{0,1}$ at two-loop level and from dimension-8 operators in $\delta\mathcal{L}$ at tree-level, etc. In the above we discuss the simple power counting without interactions with pions. Since we consider the momentum region of $p \leqslant m_\pi$, the interaction with π is taken at the order of $\mathcal{O}(p^0)$. In Eq. (3) the leading interactions are given explicitly. At higher order of p operators with derivatives and operators for emission of more than one pion will appear. We notice that the power counting of loop-contributions through exchange of pions is slightly different because some loop contributions can be more strongly suppressed than the estimated by the simple power counting.

With the interactions given in the above and also for our purpose, it is convenient to work with partial waves of the scattering. The scattering amplitude of Eq. (1), denoted as $\mathcal{T}_I(\vec{p}, \vec{k}, s_1, s_2, s_1', s_2')$ with the isospin $I = 0, 1$, can be decomposed with CG coefficients and harmonic oscillators into partial waves $\mathcal{T}_{[j\ell\ell's,I]}(E)$:

$$\mathcal{T}_I(\vec{p}, \vec{k}, s_1, s_2, s_1', s_2')$$

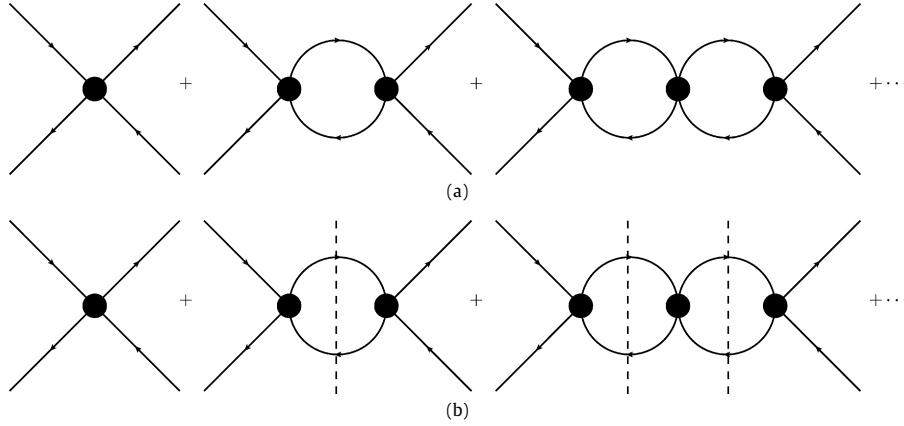$$= \sum Y^*_{\ell \ell_3}(\vec{p}/p) Y_{\ell' \ell_3'}(\vec{k}/k)(1/2, s_1; 1/2, s_2|s, s_3)$$

Fig. 1. The loop diagrams with the vertex of c_0. The black dots denote vertices of the contact term.

$$\times \left(1/2, s_1'; 1/2, s_2' | s', s_3'\right)(\ell, \ell_3, s, s_3 | j, j_3)$$
$$\times \left(\ell', \ell_3', s', s_3' | j', j_3'\right)\delta_{jj'}\delta_{j_3 j_3'}\delta_{ss'}\mathcal{T}_{[j\ell\ell's,I]}(E). \tag{4}$$

In the above the repeated indexes are summed and E is the total kinetic energy $E = p^2/m$ of the system. With the effective Lagrangian it is clear that the term with c_I will only contribute to $\mathcal{T}_{[0000,I]}(E)$ with $I = 0, 1$, respectively.

With the effective theory it is straightforward to work out the scattering amplitude and various partial waves at tree-level to compare with experimental results of $p\bar{p}$-scattering and enhancement near the threshold. In the comparison we perform a combined fit to fit the cross-section of $p\bar{p}$-scattering and the enhancement in $J/\psi \to \gamma\bar{p}$ and $e^+e^- \to p\bar{p}$, where the coupling constants $c_{0,1}$ and $d_{0,1}$ are taken as free parameters. However, as we will mention and show later, the theoretical results at tree-level can only fit the experimental data in a small region with $E \leqslant 20$ MeV. Although the χ^2/d.o.f. of the fit is close to 1, but the coupling constants can only be determined with the error from 40% to 100% or even more. In this work we will improve the situation by adding some corrections beyond tree-level. In the perturbation expansion of the effective theory some corrections appearing in n-loop level can be summed into a compact form. We will add these corrections to our tree-level results at tree-level and show that the improvement is significant. In the below we will study these corrections. This also illustrates some aspects of the effective theory.

We focus at moment on the contribution only from c_0 by ignoring the contribution from π-exchange. It is straightforward to obtain the contributions from tree-, one-loop, etc. as given from the diagrams in Fig. 1a. The tree-level amplitude is just $4\pi c_0$. The one-loop contribution is proportional to the loop integral regularized in d-dimension [21]:

$$I_0 = i\mu^{4-d}\int \frac{d^d q}{(2\pi)^d}\left[q^0 - \frac{|\vec{q}|^2}{2m} + i\varepsilon\right]^{-1}\left[E - q^0 - \frac{|\vec{q}|^2}{2m} + i\varepsilon\right]^{-1}$$
$$= -m(-mE - i\varepsilon)^{\frac{d-3}{2}}\Gamma\left(\frac{3-d}{2}\right)\frac{\mu^{4-d}}{(4\pi)^{(d-1)/2}}. \tag{5}$$

An interesting observation can be made for the above result. The integral is finite with $d = 4$. It has a pole at $d = 3$ corresponding to a power divergence of the integral. The power divergence subtraction scheme is to subtract from the above contribution the pole contribution at $d = 3$. The subtraction introduces the renormalization scale μ, hence a μ-dependence of the coupling constant. With this scheme one can show that for $N\bar{N}$-interactions with large scattering lengths a consistent power counting can be established [21] by setting $\mu \sim p \sim m_\pi$. As discussed before, for $N\bar{N}$-

interactions scattering lengths are much smaller than those from NN-interactions. Therefore we can employ the minimal subtraction scheme in which one subtracts the pole terms at $d = 4$. Because the above one-loop integral is finite at $d = 4$, no subtraction is needed. With $d = 4$ we have $4\pi I_0 = -imp = -im^2\beta$. Inspecting the contribution from n-loop, one will find that the contribution is proportional to I_0^n. In fact the sum of n-loop contributions forms a sum of a geometric series. The sum can easily be performed. We have then the exact the amplitude without π-exchange as:

$$\mathcal{T}_{[0000,0]}(E)|_{c_0} = \frac{4\pi c_0}{1 - i\frac{m^2\beta}{4\pi}c_0}. \tag{6}$$

Expanding the above expression in c_0 one can identify that the term with c_0^{1+n} comes from the n-loop diagram. Through the expansion one also sees that each loop brings a suppression factor p or β as indicated by the discussed simple power counting. The tree-level amplitudes obtained from the effective Lagrangian in Eq. (3) are at order of $\mathcal{O}(p^0)$. The corrections to them start at order of $\mathcal{O}(p)$. By considering corrections from higher orders in p, the coupling constants in our effective theory will be generally depend on the renormalization scale μ. With the power counting the μ-dependence is suppressed at least by $\mathcal{O}(p)$.

Observing the above result one can realize that the loops in Fig. 1a calculated with the minimal subtraction scheme are the same in Fig. 1b calculated only by taking the absorptive part of the loop integral I_0, i.e., calculated by putting the $N\bar{N}$ in the loop on-shell indicated by the cuts in Fig. 1b. Now we consider to include the contributions from π-exchanges. The tree-level contribution can be represented by the first diagram of Fig. 2a where the bubble contains the vertex of c_0 and one-π-exchange as indicated in Fig. 2b. The contribution of Fig. 2b is straightforward to obtain:

$$\mathcal{T}_{[0000,0]}^{(0)}(E) = 4\pi\left[c_0 - \frac{3}{2}g_0^2(2 - yL_y)\right], \tag{7}$$

with the notations:

$$g_0^2 = \left(\frac{g_A}{2F_\pi}\right)^2, \qquad y = \frac{m_\pi^2}{2p^2}, \qquad L_y = \ln\left(1 + \frac{2}{y}\right). \tag{8}$$

It is easy to find that the sum of bubble diagrams with cuts in Fig. 2a can be performed. It is a geometric series. We have then the summed amplitude:

$$\mathcal{T}_{[0000,0]}(E) = \mathcal{T}_{[0000,0]}^{(0)}(E)\left[1 - i\frac{m^2\beta}{(4\pi)^2}\mathcal{T}_{[0000,0]}^{(0)}(E)\right]^{-1} + \cdots, \tag{9}$$

where $\cdots$ stand for other corrections, like the dispersive part of one-loop contributions formed with π-exchanges. In the work we
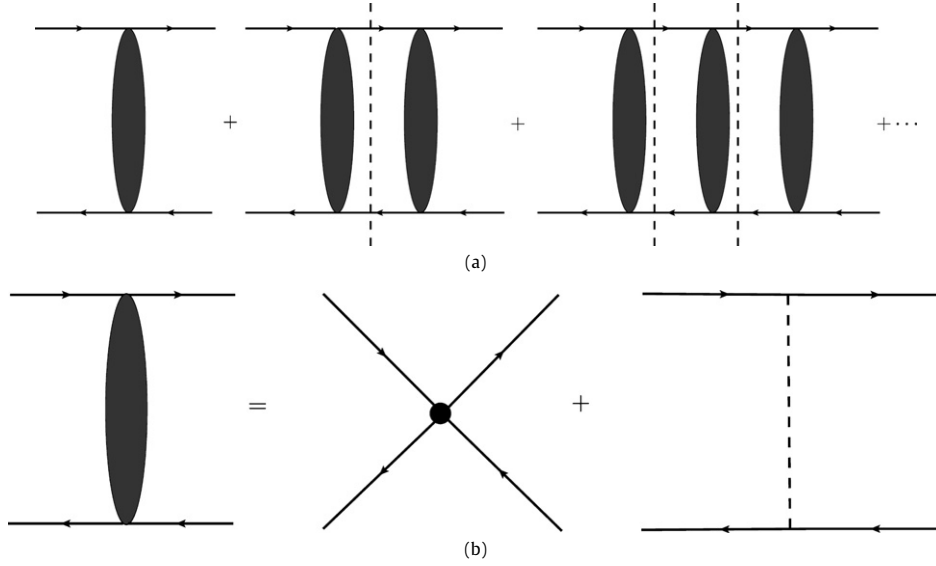
Fig. 2. (a) represents the sum of rescattering amplitudes, where the nucleon- and antinucleon lines going through the cuts are on-shell. The narrow and long bubble represents the tree-level amplitude with the contact interactions and pion-exchange as indicated by (b).

will neglect these corrections for our predictions. These corrections can be systematically studied. We will come back to the dispersive part in a future work, where a complete one-loop calculation will be performed. Physically the interpretation of Fig. 2a and the amplitude in Eq. (9) is the following: The $N\bar{N}$ undergoes a multiple scattering process $N\bar{N} \to N\bar{N} \to \cdots \to N\bar{N}$. Each scattering is due to the vertex with c_0 or exchange of one pion. Each pair of $N\bar{N}$ is on-shell in Fig. 2. We will call amplitudes for such a multiple scattering process as rescattering amplitudes. Similarly one can work out the tree-level and rescattering amplitude with $I = 1$:

$$\mathcal{T}_{[0000,1]}^{(0)}(E) = 4\pi \left[c_1 + \frac{1}{2}g_0^2(2 - yL_y) \right],$$

$$\mathcal{T}_{[0000,1]}(E) = \mathcal{T}_{[0000,1]}^{(0)}(E)\left[1 - i\frac{m^2\beta}{(4\pi)^2}\mathcal{T}_{[0000,1]}^{(0)}(E) \right]^{-1} + \cdots. \tag{10}$$

Now we turn to the amplitudes with $j = 1$ and $s = 1$. In this case it is little complicated because the πNN-interaction mixes amplitudes with difference ℓ. The difference caused by exchanging one π can only be ± 2. The summed or rescattering amplitude has to be expressed in a matrix form. We define the following matrix amplitude:

$$\mathcal{M}_{[I]}(E) = \begin{pmatrix} \mathcal{T}_{[1001,I]}(E), & \mathcal{T}_{[1201,I]}(E) \\ \mathcal{T}_{[1021,I]}(E), & \mathcal{T}_{[1221,I]}(E) \end{pmatrix}. \tag{11}$$

The tree-level results for $\mathcal{M}_{[0]}(E)$ reads:

$$\mathcal{M}_{[0]}^{(0)}(E) = \pi \begin{pmatrix} 4d_0 + 2g_0^2(2 - yL_y), & \sqrt{2}g_0^2(2 - 6y + (2y + 3y^2)L_y) \\ \sqrt{2}g_0^2(2 - 6y + (2y + 3y^2)L_y), & g_0^2[2 + 6y - (4y + 3y^2)L_y] \end{pmatrix}, \tag{12}$$

for $I = 1$ the matrix $\mathcal{M}_{[1]}^{(0)}(E)$ is obtained by replacing d_0 with d_1 and g_0^2 with $-g_0^2/3$. The summed or rescattering amplitude matrix from Fig. 2 can be found as:

$$\mathcal{M}_{[I]}(E) = \mathcal{M}_{[I]}^{(0)}(E)\left[I - i\frac{m^2\beta}{(4\pi)^2}\mathcal{M}_{[I]}^{(0)}(E) \right]^{-1}. \tag{13}$$
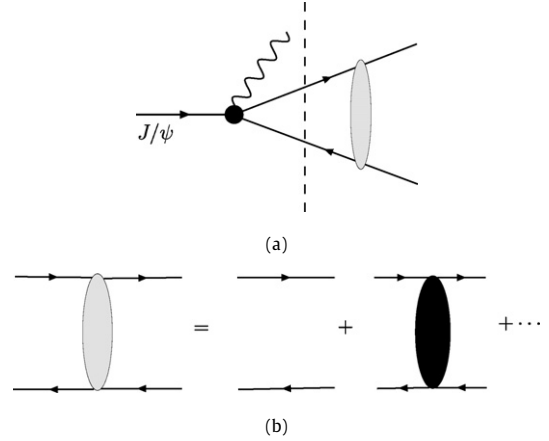

Fig. 3. The diagrammatic explanation of our approach. The dash line in (a) is the cut.

It should be noted that the contact terms in the Lagrangian are involved only in the above discussed amplitudes. Later we will use these amplitudes and those of other partial waves at tree-level to compare with experimental results of the cross section of $p\bar{p}$ scattering near the threshold.

Now we turn to the enhancement observed in $J/\psi \to \gamma p\bar{p}$ and $e^+e^- \to p\bar{p}$. In [18] we assumed that in these processes an $N\bar{N}$ system is produced first, then the $N\bar{N}$ system undergoes a scattering before observed. This can be explained with Fig. 3. As illustrated in Fig. 3, the production of a $p\bar{p}$ system can be thought that at first step an $n\bar{n}$ system or a $p\bar{p}$ system is produced and then the $N\bar{N}$ system through a rescattering is converted into the observed $p\bar{p}$ system. We assume that the production of an $N\bar{N}$ system is a short distance process, i.e., the energy scale characterizing the production is much large than m_π and the momentum near the threshold. Hence we can expand the vertex for the production in Fig. 3a in the momentum p or β. At the leading order we can approximate the black vertex for the production in Fig. 1a with constant form factors. We denote the form factors as $\mathcal{A}_{p\bar{p},n\bar{n}}$, respectively. At the leading order the $N\bar{N}$ – hence the final $p\bar{p}$ system can only be in the $^1S_0$ state. Then the decay amplitude can be written as:
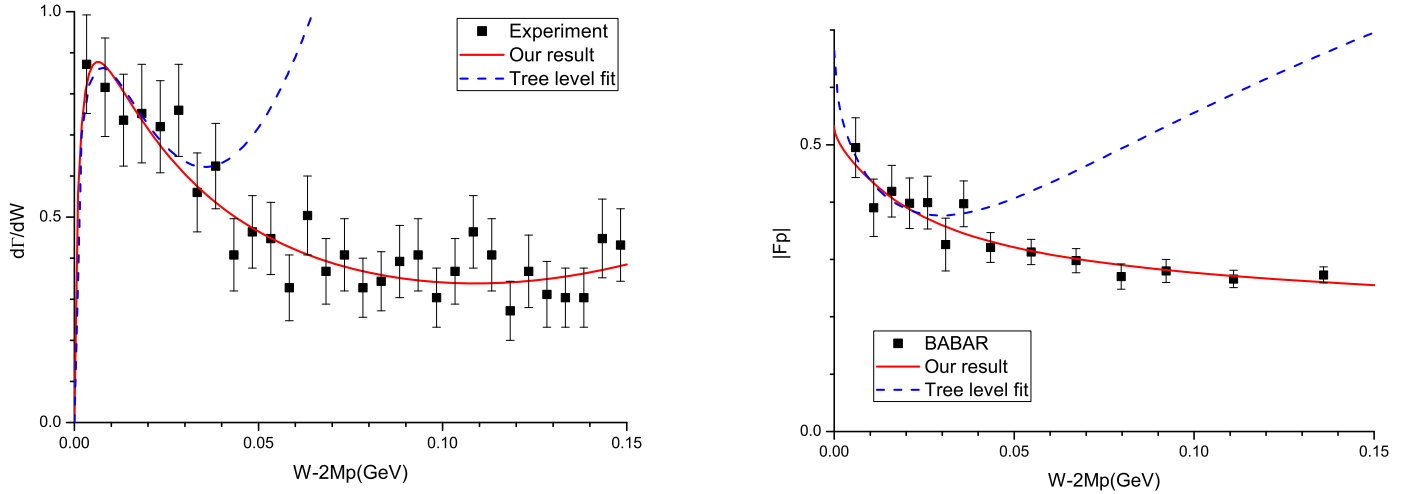
Fig. 4. The fitting results for the enhancement observed in $J/\psi \to \gamma p\bar{p}$ observed by BES (left plot) and in $e^+e^- \to p\bar{p}$ by BaBar (right plot). W is the total energy of the $p\bar{p}$ system.

$$\mathcal{T}_{J/\psi \to \gamma p\bar{p}}$$

$$\approx \varepsilon_{\mu\nu\sigma\rho} P^{\mu}_{J/\psi} k^{\nu}_{\gamma} \varepsilon^{\sigma}_{J/\psi} \varepsilon^{\rho}_{\gamma} \left\{ \frac{\mathcal{A}_{p\bar{p}} + \mathcal{A}_{n\bar{n}}}{2} \left[1 + i \frac{m^2 \beta}{(4\pi)^2} \mathcal{T}_{[0000,0]}(E) \right] \right.$$

$$\left. + \frac{\mathcal{A}_{p\bar{p}} - \mathcal{A}_{n\bar{n}}}{2} \left[1 + i \frac{m^2 \beta}{(4\pi)^2} \mathcal{T}_{[0000,1]}(E) \right] \right\} + \cdots. \quad (14)$$

The $\cdots$ stand for those partial wave amplitudes with $\ell \geqslant 1$ and the corrections proportional to β. With the approximation of the rescattering as discussed before, the $p\bar{p}$ amplitudes can be obtained from Eqs. (9), (10). We have then for the decay amplitude near the threshold as

$$\mathcal{T}_{J/\psi \to \gamma p\bar{p}} \approx \varepsilon_{\mu\nu\sigma\rho} P^{\mu}_{J/\psi} k^{\nu}_{\gamma} \varepsilon^{\sigma}_{J/\psi} \varepsilon^{\rho}_{\gamma} \left\{ \frac{\mathcal{A}_{p\bar{p}} + \mathcal{A}_{n\bar{n}}}{2} \right.$$

$$\times \left[1 - i \frac{m^2 \beta}{(4\pi)^2} \mathcal{T}^{(0)}_{[0000,0]}(E) \right]^{-1}$$

$$\left. + \frac{\mathcal{A}_{p\bar{p}} - \mathcal{A}_{n\bar{n}}}{2} \left[1 - i \frac{m^2 \beta}{(4\pi)^2} \mathcal{T}^{(0)}_{[0000,1]}(E) \right]^{-1} \right\}. \quad (15)$$

Setting $c_{0,1} = 0$ we recover our early results in [18] where in the corresponding amplitude the factor i should be replaced with $-i$ because the absorptive part has been identified with a wrong sign. The physical results in [18] will be not changed with this replacement. In our approach the intermediate state can only be $N\bar{N}$ systems according to our effective theory. It is possible to add contributions from mesons as intermediate states in some phenomenological models as shown in [23].

For the enhancement observed at BaBar the form factors of proton are involved. These form factors are defined as:

$$\langle p(p_1) \bar{p}(p_2) | J^{\mu} | 0 \rangle$$

$$= \bar{u}(p_1) \left[\gamma^{\mu} F_1(q^2) + i \frac{\sigma^{\mu\alpha}}{2m_p} q_{\alpha} F_2(q^2) \right] v(p_2). \quad (16)$$

It should be noted that near the threshold only the combination of the two form factors is involved. Near the threshold we have:

$$\langle p(p_1) \bar{p}(p_2) | \mathbf{J} | 0 \rangle$$

$$= -2m_p \left(F_1(q^2) + F_2(q^2) \right) \xi^{\dagger} \boldsymbol{\sigma} \eta + \mathcal{O}(\beta^2). \quad (17)$$

Similarly, we introduce the mechanism as in Fig. 3. We characterize the vertex of the $N\bar{N}$ production at the first step with constant

form factors, denoted as $\tilde{G}^{(p)}_M$ and $\tilde{G}^{(n)}_M$. Then the form factor with our approach is given as:

$$F_1(q^2) + F_2(q^2) = \frac{\tilde{G}^{(p)}_M + \tilde{G}^{(n)}_M}{2} \mathcal{A}_{11}(0)$$

$$+ \frac{\tilde{G}^{(p)}_M - \tilde{G}^{(n)}_M}{2} \mathcal{A}_{11}(1) \quad (18)$$

with $\mathcal{A}_{11}(I)$ as one matrix element of the matrix for the summed amplitudes as given in Eq. (13):

$$\mathcal{A}(I) = \left[I - i \frac{m^2 \beta}{(4\pi)^2} \mathcal{M}^{(0)}_{[I]}(E) \right]^{-1}. \quad (19)$$

We will use the above formula for $F_1(q^2) + F_2(q^2)$ to describe BaBar results.

We will use our theoretical results to perform a fit by combining the BES data, BaBar data in [3] and the data of the cross section of $p\bar{p}$ elastic scattering in [24]. For the BES data we use the measured results with BES3 experiment [25]. In the fit we are able to determine the coupling constants in the effective theory, i.e., $c_{0,1}$ and $d_{0,1}$. These constants are complex as discussed before. We make two fits with different theoretical results in our approach. One fit is done with tree-level results, another is performed with $N\bar{N}$ scattering amplitudes in which part of partial wave amplitudes includes summed high-order corrections as indicated in Eqs. (9)–(13) and in Eqs. (15), (18). The tree-level results for $p\bar{p}$ elastic scattering can be obtained from the effective theory in Eq. (3). In the fits we start to gradually add more data points to those nearest to the threshold until the $\chi^2/\text{d.o.f.}$ larger than 1. We do not take these data points below the $n\bar{n}$-threshold into account because we assume the isospin symmetry. Our fitting results are shown in Fig. 4 for BES and BaBar and in Fig. 5 for the cross section.

For the fit with tree-level results we are only able to describe the experimental data in the region with $E < \sim 20$ MeV. This can be seen from Figs. 4 and 5. Although the $\chi^2/\text{d.o.f.}$ of the fit for experimental data in this region is around 1, but the coupling constants are determined with errors from 40% to 100% or even more. With the results containing the summed or rescattering partial waves the regions of data which can be fitted become larger as shown in Figs. 4 and 5. For BES data and BaBar data the region is with $E \leqslant 150$ MeV. For the cross section of the $p\bar{p}$ elastic scattering the region is with $E \leqslant 70$ MeV. The results of the fit for the coupling constants in unit of GeV^{-2} are:
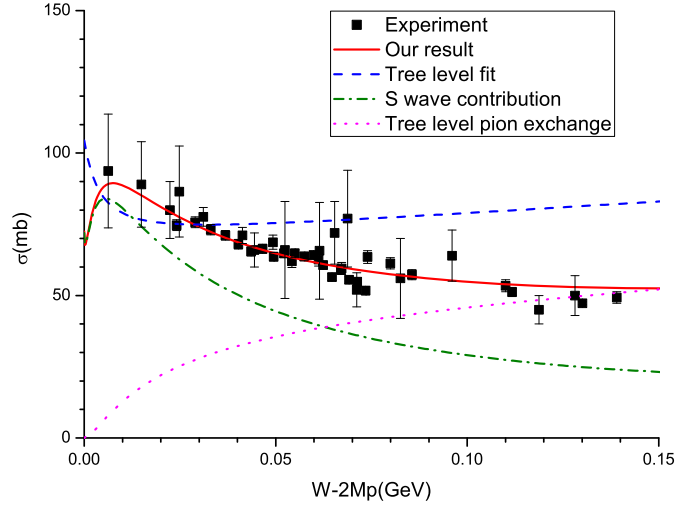
Fig. 5. The fitting result for the cross-section of the elastic $p\bar{p}$-scattering near the threshold.

$$c_0 = 123(15) - 22(11)i, \qquad c_1 = -177(55) - 57(27)i,$$
$$d_0 = 32(31) + 17(60)i, \qquad d_1 = 73(28) - 13(19)i. \tag{20}$$

The other constants appearing in $J/\psi \to \gamma p\bar{p}$ and $e^+e^- \to p\bar{p}$ and χ^2/d.o.f. from the fit are:

$$\mathcal{A}_{p\bar{p}} + \mathcal{A}_{n\bar{n}} = 0.23(37), \qquad \mathcal{A}_{p\bar{p}} - \mathcal{A}_{n\bar{n}} = 2.22(72),$$
$$\tilde{G}_M^{(p)} + \tilde{G}_M^{(n)} = 1.18(47), \qquad \tilde{G}_M^{(p)} - \tilde{G}_M^{(n)} = -0.11(20),$$
$$\chi^2/\text{d.o.f.} = 50/63. \tag{21}$$

In the above the coefficients $\mathcal{A}_{p\bar{p},n\bar{n}}$ are arbitrarily normalized. From the fitting results the coupling d_0 is not well determined. From Fig. 5 one can also see that without these contact terms in the effective theory the $p\bar{p}$ elastic scattering cannot be described with pion exchanges. This is also true if we include rescattering effects in S-waves. This is expected as discussed at beginning because without these contact terms in our effective theory the effects of an $N\bar{N}$ annihilation into virtual- and real pions are not included. These effects are important for $N\bar{N}$ scattering and should be not neglected. In our effective theory, the annihilation near threshold is described with these contact terms.

From Fig. 5 we can see that in our approach for the $p\bar{p}$ elastic scattering near the threshold the S-wave amplitudes are dominant. We can determine the scattering lengths of S-waves. In our notation the phase-shift and scattering length of a partial wave is given as:

$$\exp\big(2i\delta_{[j\ell\ell's,I]}(E)\big) - 1 = i\frac{mp}{8\pi^2}\mathcal{T}_{[j\ell\ell's,I]}(E),$$
$$p\cot\big(\delta_{[j\ell\ell's,I]}(E)\big) = -\frac{1}{a_{[j\ell\ell's,I]}} + \mathcal{O}(p^2). \tag{22}$$

From our fitting results we have the S-wave scattering lengths in unit of fm

$$a_{[0000,0]} = -1.80(19) + 0.32(17)i,$$
$$a_{[0000,1]} = 2.61(81) + 0.84(41)i,$$
$$a_{[1001,0]} = -0.48(46) - 0.25(89)i,$$
$$a_{[1001,1]} = -1.07(41) + 0.19(29)i. \tag{23}$$

These results are comparable with LEAR experiment [8] and model results [9]. The scattering lengths are determined by the coupling constants in Eqs. (3), (9). We notice here that our numerical results

of $c_{0,1}$ in Eq. (20) are roughly twice larger than the corresponding coupling constants in the effective theory of an NN system [21]. But the determined scattering lengths here are much smaller than the corresponding scattering lengths of the NN systems, because the relation between scattering lengths and coupling constants is different in the two different effective theories.

Before summarizing our study we briefly discuss our prediction relevant for the enhancement in $J/\psi \to \gamma p\bar{p}$. The measured spectrum can be fitted with the decay amplitude as an S-wave Breit–Wigner resonance form. The fitting done by BES gives the resonance mass around 1861 MeV and the width $\Gamma < 38$ MeV. The mass is near and below the threshold. We have observed that our formula in Eq. (15) can re-produce the shape of the S-wave Breit–Wigner resonance form above the threshold only by tuning the parameters $\mathcal{A}_{p\bar{p},n\bar{n}}$. If there is a resonance or structure below and near the threshold, one should also have roughly the same shape of the S-wave Breit–Wigner resonance below the threshold. But, our amplitude in Eq. (15) below the threshold through analytical continuation has a cut near the threshold because of the L_y-function in Eq. (8) appearing in the amplitude. Therefore, we cannot conclude from our result that there exists a resonance or structure below the threshold, although our result agrees with experimental data above the threshold.

To summarize: We have proposed an effective theory to study $N\bar{N}$ scattering near the threshold. Because scattering lengths of $N\bar{N}$ system are rather small, we have implemented the standard minimal subtraction scheme to the effective theory to establish a power counting. The power counting is used to determine the relative importance of higher-order corrections. We have found that certain higher-order corrections represented as multiple rescattering can be simply summed. Using these rescattering amplitudes and assuming that the enhancement in $J/\psi \to \gamma p\bar{p}$ and $e^+e^- \to p\bar{p}$ near the threshold of the $p\bar{p}$ system is due to final state interactions, we can simultaneously explain the experimental data of the enhancement and of the cross section of $p\bar{p}$ elastic scattering near the threshold. The S-wave scattering lengths are determined which are comparable with existing results. Given the fact of the successful description of experimental data, the proposed effective theory needs to be studied in more detail. In the future we will study higher-order corrections from loops formed through pion exchanges and the renormalization group of coupling constants in the effective theory. In this Letter we have ignored the one-loop corrections to dispersive parts of scattering amplitudes, especially, the one-loop dispersive parts formed through pion exchanges. It

has been found in NN-scattering that these parts can receive large corrections in [22]. It will be interesting to see if this also happens in $N\bar{N}$ scattering. This question can only be answered after our undergoing study of the complete one-loop correction has been done.

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