Numerical study of mode waves in a deviated borehole penetrating a transversely isotropic formation

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Abstract

A 2.5 dimensional method is developed to investigate the mode waves in a deviated borehole penetrating a transversely isotropic formation. The phase velocity dispersion curves of the fast and slow flexural mode waves excited by a dipole source are computed accurately at different deviation angles for both hard and soft formation. The sensitivity of flexural waves to all the five elastic constants are calculated. Numerical results show that for a soft formation, the fast flexural mode wave is dominated by $C_{66}$ at high deviation angles and low frequencies, the slow flexural mode wave is dominated by $C_{44}$ at the same condition. An inversion procedure is presented to prove the sensitivity analysis.

Keywords: Deviated borehole; transverse isotropy; acoustic mode waves; dispersion curve; sensitivity analysis

1. Introduction

Acoustic wave propagation in boreholes is the theoretical basis for acoustic well logging. In a real situation, many of the sedimentary rocks with horizontal layers can be characterized as transversely isotropic (TI) when the wavelength of the elastic waves is much larger than the thickness of layers. For such a TI formation, the medium presents isotropy in the horizontal plane while the wave propagation along the vertical axis is different from that in the horizontal plane. When the borehole axis is parallel to the symmetrical axis of the TI formation, analytic methods such as the real axis integration (RAI) and the branch-cut integration can be used to investigate the borehole acoustic field. When the borehole axis is deviated, analytic solutions of the wave field can no longer be obtained [1]. A finite difference time domain (FDTD) method is widely used to study this problem by computing the waveforms [2]. Dispersion curves can be obtained from the results in time domain. The perturbation method is also used when the formation shows weak anisotropy [3].

In this paper, we present a 2.5-dimensional frequency wave-number method to investigate the mode wave characteristics in a deviated borehole. The mode distributions and their dispersion characteristics are observed and analyzed intuitively and accurately. The sensitivities of the mode waves to the elastic constants are obtained based on calculating the dispersion curves accurately. A simple inversion procedure is provided to support our results.
2. Method and results

2.1. Method validation

Consider a fluid-filled cylindrical borehole embedded in an infinite TI formation. The vertical $z'$ axis is the symmetrical axis of the formation while the borehole is deviated with deviation angle $\alpha$, see Fig. 1(a). Because the formation exhibits isotropy in the horizontal plane, without loss of generality, we assume that $y$ axis and $y'$ axis coincide so that we can obtain the Cartesian coordinate $O - xyz$ from $O - x'y'z'$ when it rotates around $y'$ axis with angle $\alpha$. The acoustic source is located at the origin. The receivers are located at the borehole axis. For a dipole source, there exists an orientation angle $\theta$, see Fig. 1(b).

When the borehole and surrounding formation are assumed invariant in the axial $z$ direction. Using the separation of variables technique, the wave propagation in the $z$ direction may be described by $\exp(ikzz)$ where $k_z$ is the wavenumber in the $z$ direction. We have:

$$\phi(x, y, z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(x, y, \omega, k_z)e^{i(k_zz-\omega t)}d\omega dk_z,$$  

where $\phi$ is the displacement potential which fulfills the wave equation. Thus, we can compute $\phi(x, y, \omega, k_z)$ for different $\omega$ and $k_z$ instead of computing $\phi(x, y, z, t)$ directly, i.e. the 2.5-dimensional method. From the $\omega - k_z$ domain results, mode distribution can be seen clearly and dispersion can be obtained. The computation area is shown in Fig. 1(b).

We use the PDE module of a commercial FEM software COMSOL to solve the equations. An artificial convolutional perfectly matched layer (PML) is realized to absorb the incident waves [4], [5]. The Bond Transform $C = MC_0M^T$ is used to obtain the elastic constants in the new coordinate [6], where $C_0$ is the elastic constants matrix expressed in the coordinate of TI formation and $M$ is a matrix related with deviation angle $\alpha$.

Firstly, we use a non-deviated borehole model to validate our method as the analytical solution exists for such a model. The acoustic velocity in the borehole fluid is $v_f = 1500m/s$ and the density is $\rho_f = 1000kg/m^3$. The borehole radius is $0.1m$. A dipole source is located at the origin. The parameters of TI formation are listed in Table 1.

<table>
<thead>
<tr>
<th>Formation</th>
<th>$C_{11}(GPa)$</th>
<th>$C_{33}(GPa)$</th>
<th>$C_{13}(GPa)$</th>
<th>$C_{44}(GPa)$</th>
<th>$C_{66}(GPa)$</th>
<th>$\rho_s(kg/m^3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cotton Valley shale</td>
<td>74.7</td>
<td>58.8</td>
<td>25.3</td>
<td>22.0</td>
<td>30.0</td>
<td>2640</td>
</tr>
<tr>
<td>Austin Chalk</td>
<td>22.0</td>
<td>14.0</td>
<td>12.0</td>
<td>2.4</td>
<td>3.1</td>
<td>2200</td>
</tr>
</tbody>
</table>

Fig. 2(a) shows the phase velocity dispersion curves of flexural modes for both fast and slow formations. The solid lines are the analytic results while the points are the results obtained using our method. The comparison indicates that our results are in good agreement with those of the analytic method. These results give us a confidence that the mode
2.2. Results

Here we present the flexural wave dispersion curves for different deviation angles. In Fig. 2(b) and 2(c), the slow flexural wave ($\theta = 0^\circ$, solid lines) and the fast flexural wave ($\theta = 90^\circ$, dashed lines) for $\alpha = 0^\circ, 30^\circ, 60^\circ, 90^\circ$ are presented. At low frequencies, the velocities of fast and slow flexural waves are near the SH or quasi-SV wave speeds of TI formation. When the borehole is deviated, the flexural wave splits, the velocity of fast flexural wave increases monotonically with the increase of the deviation angle while that of slow flexural wave does not.

In the theoretical modeling of guided-wave propagation in a borehole, many parameters can influence the wave propagation characteristics [7]. Sensitivity analysis is commonly used to analyze the importance of each parameter in affecting the wave propagating. It is simply defined as the normalized partial derivative of the wave phase velocity with respect to a model parameter $p$:

$$Sensitivity = \frac{p}{v_{\text{phase}}(\omega)} \frac{\partial v_{\text{phase}}(\omega)}{\partial p}$$

As the dispersion curves can be obtained accurately using our method, the sensitivity analysis is achievable. We have found that the fast flexural wave is more sensitive to $C_{66}$ at a high deviation angle while the slow flexural wave distributions of the wave field and the phase velocity dispersion curves of mode waves obtained using our method are accurate for our numerical analyses.
is more sensitive to $C_{44}$ at the same condition, as shown in Fig. 3. Now we present an inversion procedure to support the sensitivity analysis. The surrounding formation is Austin Chalk. A dipole source ($\theta = 90^\circ$) is used to excite a fast flexural wave at 2kHz when $\alpha = 90^\circ$. The real value of phase velocity $v = 1176.0 m/s$. All the parameters are assumed known except $C_{66}$. Firstly, give $C_{66}$ an initial value and conduct a forward simulation to obtain the phase velocity $v_1$ (the subscript stands for the iterate number). Secondly, as shown in Fig. 3(a), the sensitivity to $C_{66}$ is always positive, the phase velocity should increase monotonically when $C_{66}$ increases. So if $v_1 < v$, increase the guess value of $C_{66}$, otherwise decrease it. Repeat the above two steps until the error is less than the tolerance.

Let $C_{44}$ be the initial value of $C_{66}$ and conduct the computations. The result is shown in Fig. 4, marked with circles. Only several iterate times are needed. However, in a real situation, known parameters may deviate from the real value for some reasons such as the measurement error. The inversion procedure should be stable and still converge to the real value. Here we assume that $C_{33}$ has a 20% off set from the real value of Austin Chalk formation, then we conduct the inversion procedure again, see Fig. 4, marked with crosses. Very little difference is presented which means the sensitivity analysis is correct. Similar results can be obtained to support Fig. 3(b).

3. Conclusion

A 2.5-dimensional frequency wave-number method is used to investigate the mode wave characteristics in a deviated borehole penetrating a TI formation. Flexural wave dispersion curves at various deviation angles for both fast and slow formation are presented. The sensitivity analysis is conducted. At high deviation angles and slow frequencies, the fast flexural wave is dominated by $C_{66}$ while the slow flexural wave is dominated by $C_{44}$. A simple inversion procedure supports the results.

Acknowledgements

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References