Recovering Depth Map from Enhanced Image Gradients

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Abstract

This paper presents a new and efficient approach for recovering surface height from its image gradients. In order to compensate noise effect in image intensities, Gaussian filter is used to smooth the intensity values. Image gradient is calculated from the smoothed image by taking the central difference of intensity values. A minimization problem is formulated in terms of gradient values. This minimization problem is solved in Fourier domain to avoid the iterative minimization. The experiments have been performed on real and synthetic images to demonstrate the accuracy of the approach using Gaussian filter. Several different measures have been used for error estimation.

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1. Introduction

Shape recovery of 3D object from one or more 2D images is a definitive discipline in computer vision. The retrieved shape can be expressed in terms of depth map, that can be observed as corresponding surface height above the xy plane. Shape recovery has a diverse range of applications in engineering field, medical science, designing and manufacturing of various products. Recovering 3D objects from 2D images is known as problem of surface reconstruction and it has been the core topic of interest for many researchers in the field of computer vision.

Surface reconstruction by evaluating depth map from measurements of spatial gradient is an important problem in the field of computer vision having usability in photometric stereo (PS), shape-from-shading (SfS), shape from contours and so on. The SfS problem deals with reconstruction of 3D object from one 2D image in two steps.1,2 In the first step, the surface gradients or surface normals are estimated for a discrete set of points on the object surface using different reflectance models. The second step is to find the surface height from the evaluated surface orientation, which results in height from gradient problem.

The height from gradient problem also comes into the picture from photometric stereo method.3,4,5 In photometric stereo method, 3D surface can be reconstructed from more than one images taken from the same viewpoint but with
different lightening conditions. Whereas in the shape from texture technique, three or more images are taken to
govern a dense gradient field, which in turn, integrated to obtain texture relief. In nutshell, it has been found that
entire surface reconstruction process can be branched into two independent steps: 1. Computation of the gradient. 2. Gradient integration, i.e. height from gradient.

Most of the algorithms related to this field produce discrete values of the gradient, so there arises a need of numerical
methods to evaluate the depths of the surface at each pixel of the given image. Traditional methods use integrability
constraint in height from gradient problem. But later on, many researchers have given new techniques by ignoring
the integrability enforcement. In\textsuperscript{6} Gaussian kernel approach is presented to reconstruct surface without enforcing
discrete integrability. However, it is always difficult to find the appropriate value of variance in this approach. Then
another kernel based method has been given in\textsuperscript{7}. They used box-spline function as a positive definite kernel due to its
compact support and parameter robustness. Ettl et al.\textsuperscript{8} proposed surface reconstruction from gradient data based upon
an approximation using radial basis functions. Agrawal et al.\textsuperscript{9} have proposed a general framework for the solution
of non integrable gradient field. They have used binary weighted Poisson’s equation in alpha surface method and
continuous weighted Poisson equation in M-estimators method. These methods iteratively revised the weights after
getting the initial result. In\textsuperscript{10}, discrete sine transform (DST) is used in order to solve Poisson’s equation numerically.

On the other hand, Harker and O’Leary\textsuperscript{11} proposed the global least square surface reconstruction without the use
of basis functions. This method did not incorporate regularization. In\textsuperscript{12,13} compressed sensing (CS) algorithm is used to
determine the horizontal and vertical gradients. Wei and Klette\textsuperscript{14} proposed a technique using Fourier transform
method to calculate depth map by taking different weight parameters. They have shown that their algorithm is capable
of reconstruct the surface accurately in case of noisy gradient data. In\textsuperscript{15}, an algorithm for reconstructing free-form
space curves in space has been proposed using a Non-Uniform Rational B-Spline (NURBS)-snake model.

In this paper, the image gradients are used as input data in the minimization problem to obtain depth map. These
image gradients are obtained from the intensity values of the image using central difference operator on smoothing
of image done by Gaussian filter. To solve the minimization problem, we assume that depth map can be written
as a linear combination of Fourier basis functions. The associated frequency parameters which minimize the error
functional in terms of image gradients are calculated.

Rest of the paper is organised as follows: Section 2 describes the proposed algorithm to find height from gradient. Section 3 explores the experimental results and finally the conclusions are given in Section 4.

2. Proposed approach for Height From Gradient

2.1. Formulation

In this section, a new algorithm to find depth map from image gradients is presented in which images are firstly
convolved by Gaussian filter in order to obtain improved gradient field. Let the intensity image be $f_1(x, y)$, Gaussian
filter be $h(n_1, n_2)$ and the operation of convolution on the image by the filter can be defined as:

$$f(x, y) = \sum_{n_1=-M/2}^{M/2} \sum_{n_2=-N/2}^{N/2} h(n_1, n_2) f_1(x - n_1, y - n_2)$$

where, $h(n_1, n_2) = \frac{h_g(n_1, n_2)}{\sum_{n_1} \sum_{n_2} h_g}$

$$h_g(n_1, n_2) = e^{-\frac{(n_1^2 + n_2^2)}{2\sigma^2}}$$

and $\sigma$ is the standard deviation of filter operated on an $M \times N$ size image.

When this preprocessing step of image is over, the gradients are evaluated using central difference operator in the
interior of the image grid and appropriate forward and backward difference operator on the boundaries of the image.
The formulae for calculating the image gradients are given as:

\[
p_{i,j} = \begin{cases} \frac{I_{i+1,j}-I_{i-1,j}}{2} & \text{if } (i, j) \in \text{image interior} \\ I_{i+1,j} - I_{i,j} & \text{if } (i, j) \in \text{left boundary} \\ I_{i,j} - I_{i-1,j} & \text{if } (i, j) \in \text{right boundary} \end{cases}
\]
\[
q_{i,j} = \begin{cases} \frac{I_{i,j+1}-I_{i,j-1}}{2} & \text{if } (i, j) \in \text{image interior} \\ I_{i,j+1} - I_{i,j} & \text{if } (i, j) \in \text{upper boundary} \\ I_{i,j} - I_{i,j-1} & \text{if } (i, j) \in \text{lower boundary} \end{cases}
\]

Now in the proposed algorithm, the only available input data is given by the gradient field \((p(x, y), q(x, y))\). The aim of the problem is to find a depth map \(z(x, y)\) such that

\[
z_x = \frac{\partial z}{\partial x} = p, \quad z_y = \frac{\partial z}{\partial y} = q
\]

The simplest error function on considered depth function \(z(x, y)\) with respect to given gradient \((p, q)\), can be written as

\[
E(z; p, q) = (z_x - p)^2 + (z_y - q)^2
\]

(1)

Afterwards, the issue of finding optimal \(z(x, y)\) can be expressed as a minimization of the following function in the image domain \(\Omega\).

\[
J(z) = \int_{\Omega} E(z; p, q) dxdy = \int_{\Omega} (z_x - p)^2 + (z_y - q)^2 dxdy
\]

(2)

The proposed method to evaluate height from gradients is given in algorithm 1.

**Algorithm 1:** Proposed algorithm for height from gradient

Step 1: Load the image and convert it into gray level image.
Step 2: Create a Gaussian low pass filter of size 7*7 with standard deviation 1.5.
Step 3: Process the input image by convolving it with filter prepared in step 2.
Step 4: Find the gradients of smoothed image using central difference operator.
Step 5: Input image gradients \(p(x, y), q(x, y)\).
Step 6: Calculate discrete Fourier transform of \(p(x, y)\) and \(q(x, y)\) : \(P(u, v)\) and \(Q(u, v)\).
Step 7: Compute the Fourier transform of \(z(x, y)\) as \(Z(u, v)\) using

\[
Z(u, v) = -i \frac{1}{2\pi} \left( \frac{u P(u,v)}{M^2} + \frac{v Q(u,v)}{N^2} \right)
\]

Step 8: Calculate the inverse Fourier transform of \(Z(u, v)\) as \(z(x, y)\) which is the required depth map.

2.2. Solution

Assume that the proposed depth map \(z\) can be written as a linear combination of basis function \(\phi(x, y; \omega)\) where \(\omega = (u, v)\) is a 2D parameter.

\[
z(x, y) = \sum_{\omega} A(\omega)\phi(x, y; \omega)
\]

(3)

After differentiating equation (3) with respect to \(x\) and \(y\), we get

\[
\frac{\partial z}{\partial x} = \sum_{\omega} A(\omega)\phi_x(x, y; \omega); \quad \frac{\partial z}{\partial y} = \sum_{\omega} A(\omega)\phi_y(x, y; \omega)
\]

(4)
Define $p(x,y)$ and $q(x,y)$ as

$$p = \sum_{\omega} A_1(\omega) \phi(x,y;\omega); \quad q = \sum_{\omega} A_2(\omega) \phi(x,y;\omega)$$

To evaluate the value of the coefficients $A(\omega)$ in equation (3), two new parameters $B_1$ and $B_2$ are defined as:

$$B_1(\omega) = \int \int |\phi_z(x,y;\omega)|^2 dx dy; \quad B_2(\omega) = \int \int |\phi_y(x,y;\omega)|^2 dx dy$$

The best value of the coefficients $A(\omega)$ in the equation (3) that minimizes the cost function given in equation (2) with square error (1) is

$$A(\omega) = \frac{B_1(\omega)A_1(\omega) + B_2(\omega)A_2(\omega)}{B_1(\omega) + B_2(\omega)}$$

In our proposed algorithm, discrete Fourier basis is used for its computational efficiency.

Here, $\phi(x,y;\omega) = \exp\left(2\pi i \left(\frac{ux}{M} + \frac{vy}{N}\right)\right)$

The derivatives of Fourier basis are calculated as

$$\phi_z = \left(2\pi i \frac{u}{M}\right) \phi(x,y;\omega); \quad \phi_y = \left(2\pi i \frac{v}{N}\right) \phi(x,y;\omega)$$

The value of $B_1$ and $B_2$ can be calculated using equation (6) as follows

$$B_1(\omega) = \left(\frac{2\pi u}{M}\right)^2; \quad B_2(\omega) = \left(\frac{2\pi v}{N}\right)^2$$

The expansion coefficients $A_1(w)$ and $A_2(w)$ can be calculated from the Discrete Fourier Transform (DFT) of $p$ and $q$, written as $\mathcal{F}(p)$ and $\mathcal{F}(q)$ using (5)

$$A_1(w) = -\frac{iN}{2\pi u} \mathcal{F}(p) M N; \quad A_2(w) = -\frac{iM}{2\pi v} \mathcal{F}(q) N$$

Putting the values of $A_1, A_2, B_1 and B_2$ from equation (10) and (11) in equation (7) to get $A(\omega)$ and then substituting the value of $A(\omega)$ in equation (3) and by taking the DFT of equation (3), we get

$$\mathcal{F}(z) = -i \left(\frac{2\pi u}{M} \mathcal{F}(p) + \frac{2\pi v}{N} \mathcal{F}(q)\right) = \frac{-i}{2\pi} \left(\frac{\mathcal{F}(p)}{\mathcal{F}(q)} + \frac{\mathcal{F}(q)}{\mathcal{F}(p)}\right)^2$$

Now, the final output $z$ with respect to input $p$ and $q$ can be written as

$$z = \mathcal{F}^{-1}\left(-i \frac{2\pi}{\mathcal{F}(p) + \mathcal{F}(q)}\right)$$

Here $\mathcal{F}(-\omega)$ and $\mathcal{F}^{-1}(-\omega)$ are DFT and inverse DFT operations respectively.

Thus depth map $z(x,y)$ is evaluated at discrete points using equation (13), which in turn, used in reconstructing surface. The brief overview of the whole process is presented in the block diagram given by Fig. 1

3. Results and Discussions

In order to check the performance of the algorithm for evaluating height from image gradients, judgement can be made by two types of measurements: (a) Subjective quality measurement (b) Objective quality measurement.

Subjective quality measurement contains Mean Opinion Score (MOS) which is based upon the perceptions of the individual. It is a person specific and time consuming process. The grading scales of MOS are given on the following 5-point-scale as: pleasant or excellent quality - 5, good - 4, acceptable - 3, poor quality - 2, unacceptable - 1.

Where as the objective quality measurements are time saving than subjective quality measurements. Various formulae for objective quality measurements viz. Mean Square Error (MSE), Mean Absolute Error (MAE), Structural content (SC), Normalized Absolute Error (NAE), Normalized Cross-Correlation (NCC) have been depicted in Table 1. Let $h(x,y)$ denotes the original height of the surface at pixel $(x,y)$ and $\hat{h}(x,y)$ denotes the height obtained from image gradients. Here $M$ and $N$ denotes the number of rows and columns respectively of an image.
3.1. Synthetic data

Several experiments have been performed to recover height from gradient using real as well as synthetic images. Fig. 2 represents the results of our proposed algorithm for two synthetic images, viz. Mozart and vase. It can be found from these results that the surfaces obtained using Gaussian filter are better and more similar to ground truths as compared to surfaces obtained by evaluating gradient directly. In our problem using Gaussian filter, the value of $\sigma$ has been taken as 1.5 and the size of the Gaussian filter has been taken as $7 \times 7$, tuned by hit-and-trial approach for the optimal performance. Error analysis for depth map of these synthetic images have been reported in table 2 for the proposed algorithm.

Fig. 3 describes the use of Gaussian filter in case of noisy images. To illustrate the importance of our algorithm, first we add noise in the image and then perform algorithm with and without Gaussian smoothing. Fig. 3 (c) is much better than Fig. 3 (b), this is because, filter gives better results in presence of noise. This betterment can be judged in terms of smoothness and reconstruction quality of the reconstructed surfaces.

3.2. Real images

Fig. 4 shows the performance of our algorithm on real image. The central portion of this real image is of white colour. The upper white portion of 3D reconstructed surface in circular highlighter in Fig. 4 (b) and Fig. 4 (c) has
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Fig. 2. Depth map for synthetic images of mozart (top) and vase (bottom): (a) intensity image; (b) ground truth depth; (c) depth using simple image gradients; (d) depth using improved image gradients by Gaussian filter

Table 2. Results for Assessment Parameters

<table>
<thead>
<tr>
<th>Image</th>
<th>Assessment Parameter</th>
<th>without using filter</th>
<th>using filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mozart</td>
<td>MSE</td>
<td>$10.320 \times 10^3$</td>
<td>$9.8618 \times 10^4$</td>
</tr>
<tr>
<td></td>
<td>MAE</td>
<td>93.2286</td>
<td>90.3340</td>
</tr>
<tr>
<td></td>
<td>NCC</td>
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<td>1.8176</td>
</tr>
<tr>
<td></td>
<td>SC</td>
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<td>0.1338</td>
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<tr>
<td></td>
<td>NAE</td>
<td>2.9032</td>
<td>2.8131</td>
</tr>
<tr>
<td>Vase</td>
<td>MSE</td>
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<td>531.8925</td>
</tr>
<tr>
<td></td>
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<td>20.0612</td>
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</tr>
<tr>
<td></td>
<td>SC</td>
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</tr>
<tr>
<td></td>
<td>NAE</td>
<td>316.9560</td>
<td>312.4377</td>
</tr>
</tbody>
</table>

Fig. 3. Depth map for synthetic image of vase by the addition of noise: (a) noisy image; (b) depth using without Gaussian smoothing; (c) depth using improved image gradients by Gaussian filter;

been compared and white colour spreads more in Fig. 4 (c) due to Gaussian filter. It shows the significance of our algorithm using filter in case of real images also.

It can be seen from the presented results that the quality of 3D reconstruction of surfaces has been improved using Gaussian filter.

4. Conclusion

In this paper, an algorithm to recover depth map of intensity images has been presented by calculating the gradients of the image. Firstly the image has been smoothed by convolving it with Gaussian filter of size $7 \times 7$ and standard deviation 1.5. Afterwards the gradients have been evaluated using appropriate finite difference operators. Here gra-
Fig. 4. Depth map for real image: (a) real image; (b) depth using simple image gradients; (c) depth using improved image gradients by Gaussian filter.

diagnostic values are the only available input data acquired from intensity values of the 2D images. Then depth recovery has been done by minimization of the error function having image gradients as input data. The algorithm has been implemented using discrete Fourier transform as a numerical technique. It has been observed from the results that the proposed algorithm is able to recover more accurate reconstructed surfaces as compared to surfaces obtained by just evaluating gradient directly.

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References