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Study of magnetic and heat transfer on the peristaltic transport of a fractional second grade fluid in a vertical tube

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ABSTRACT

This study is concerned with the peristaltic flow of the fractional second grade fluid confined in a cylindrical tube. The effects of magnetic field in the presence of heat transfer are taken into account. Mathematical modeling is based upon continuity, momentum and energy equations. This analysis is carried out under the constraints of long wavelength ($0 << \lambda \to \infty$) and low Reynolds number ($Re \to 0$). Closed form solutions for velocity, temperature field and pressure gradient are obtained. Numerical integration is used to analyze the novel features of pressure rise and friction force. Effects of pertinent parameters such as Hartmann number M, heat source/sink parameter β, Grashof number Gr, material constant λ_1, pressure rise ΔP and friction force F alone with Reynolds number Re and Prandtl number Pr are discussed through graphs. It is found that an increase in constant of fractional second grade fluid results in the decrease of velocity profile for the case of fractional second grade fluid whereas the velocity remains unchanged for the case of second grade fluid. It is also observed that the absolute value of heat transfer coefficient increases with an increase in β and aspect ratio φ. It is due to the fact that the nature of heat transfer is oscillatory which is accordance with the physical expectation due to oscillatory nature of the tube wall. It is perceived that with an increase in Hartmann number, the velocity decreases. A suitable comparison has been made with the prior results in the literature as a limiting case of the considered problem, for instance, fractional second grade fluid model reduces to second grade models for $\alpha_1 = 1$ and classical Naiver Stokes fluid model can be deduced from this as a special case by taking $\overline{\lambda}_1 = 0$.

1. Introduction

It is well known that peristaltic flow is generated by means of moving contraction on the tube and channel walls. The mechanism of peristalsis is used in the body for pumping physiological fluids from one place to another. Due to indispensable role of peristaltic flows, it has been extensively studied in both mechanical and physiological situations under different conditions. Recently, several studies are being made on the peristaltic motion of Newtonian and non-Newtonian fluids. Moreover, the study of hydrodynamics has gained very much attention within the more general context of magnetohydrodynamics (MHD) in the last few years. The study of the motion of Newtonian and non-Newtonian fluids in the presence as well as in the absence of magnetic field has found several applications in different areas, including the biological fluids and the flow of nuclear fuel slurries, liquid metals, alloys, plasma, mercury amalgams and blood etc. To study the MHD effect on peristaltic flow of biological fluids is very important in connection with certain problems of the movement of conductive physiological fluids, for example the blood and the blood pump machines. Such analysis is of great value in medical research. Recently few investigations have been carried out to understand the interaction between heat transfer and peristaltic flow of non-Newtonian fluid. In fact heat transfer analysis is important because of its industrial and biological applications like sanitary fluid transport, blood pump in heart lungs machine and transport of corrosive fluid with the machinery part. Some relevant studies on the topic can be seen from the list of references [1–12]. Besides the concept of heat transfer analysis is also very useful in accessing the blood flow rate through thermal

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clearance rate and initial thermal conditions. It can be used to obtain information about the properties of tissues, for instance, the blood flow can be evaluated using a dilution technique. In this process, heat is either injected or generated locally and the thermal clearance is monitored. Specifically the bioheat transfer plays an energetic role in destroying uninvited tissues, hyperthermia, cryosurgery and laser therapy [13].

Furthermore, the study of the fractional calculus which is closely associated with the description of complex dynamics has achieved a great success, in particular it is quite flexible in describing the viscoelastic behavior of polymer solution. In general fractional model of the viscoelastic fluid is derived from well-known ordinary model by replacing, the ordinary time derivatives, to fractional order time derivatives and this plays an important role to study the valuable tools of viscoelastic properties. In many different situations fractional calculus has been used to handle various rheological problems [14–16] and several references therein. Among several models proposed for physiological fluids, fractional second grade fluid model is significant because this model reduces to second grade models for $\alpha_1 = 1$. Further, classical Naiver Stokes fluid model can be deduced from this as a special case by taking $\bar{\lambda}_1 = 0$.

In view of above studies, one can clearly observe that no analysis on the interaction of peristalsis flow of fractional second grade fluid in the presence of magnetic field and heat transfer has been accorded in available literature. In order to fill this gap, the current attempt is three folds. Firstly the relevant equations for the fluid under consideration are first modeled and then resulting mathematical problem is solved under low Reynolds number and long wavelength. The expressions for velocity, temperature and the pressure rise are obtained. Finally the impact of interesting and important features of emerging parameters are plotted and discussed in detail.

2. Problem formulation

Let us consider a fractional second grade fluid through a vertical tube. In axisymmetric cylindrical polar coordinate system (x, r), the $x-$ coordinate is along the axes of the tube and r is the radial coordinate. The geometry of the tube wall is mathematically given by

$$\bar{h} = a + \bar{\varphi}\cos^2\pi(x - t). \tag{1}$$

The constitutive relation for viscoelastic fluid with fractional second grade model is defined by the following relation

$$\bar{S} = \mu\left(1 + \bar{\lambda}_1^{\alpha_1}\frac{\partial^{\alpha_1}}{\partial \bar{t}^{\alpha_1}}\right)\dot{\gamma}, \tag{2}$$

where $\bar{S}$, $\bar{t}$, $\bar{\lambda}_1$, μ, $\dot{\gamma}$ and $\alpha_1 (0 \le \alpha_1 \le 1)$ are shear stress, time, material constant, viscosity, rate of shear strain and fractional time derivative parameter respectively.

The equations governing the flow of viscoelastic fluid with fractional second grade model for axisymmetric flows are

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\bar{v}) = 0, \tag{3}$$

$$\left.\begin{aligned}\rho\left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{r}}\right) = \frac{\partial \bar{p}}{\partial \bar{x}} + \mu\left(1 + \bar{\lambda}_1^{\alpha_1}\frac{\partial^{a_1}}{\partial \bar{t}^{a_1}}\right)\left[\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial \bar{u}}{\partial \bar{r}}\right) + \frac{\partial^2 \bar{u}}{\partial \bar{x}^2}\right] \\ -\sigma B_0^2 \bar{u} + \rho g \alpha (T - T_0)\end{aligned}\right\}, \tag{4}$$

$$\rho\left(\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{r}}\right) = \frac{\partial \bar{p}}{\partial \bar{r}} + \mu\left(1 + \bar{\lambda}_1^{a_1}\frac{\partial^{\alpha_1}}{\partial \bar{t}^{\alpha_1}}\right)\left[\frac{\partial}{\partial \bar{r}}\left(\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\bar{v})\right) + \frac{\partial^2 \bar{v}}{\partial \bar{x}^2}\right], \tag{5}$$

$$C_p\rho\left(\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{r}}\right) = k\left(\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + v\frac{\partial^2 \bar{T}}{\partial \bar{x}^2}\right) + Q_0. \tag{6}$$

We introduce the following non-dimensional quantities

$$\left.\begin{aligned} x &= \frac{\bar{x}}{\lambda}, r = \frac{\bar{r}}{a}, u = \frac{\bar{u}}{c}, v = \frac{\bar{v}}{c\delta}, \lambda_1^{\alpha_1} = \frac{c\bar{\lambda}_1^{\alpha_1}}{\lambda}, \delta = \frac{a}{\lambda} \\ p &= \frac{a^2\bar{p}}{\lambda\mu c}, Re = \frac{\rho c a \delta}{\mu}, M = \sqrt{\frac{\sigma}{\mu}}B_0 a, \varphi = \frac{\bar{\varphi}}{a}, t = \frac{c\bar{t}}{\lambda} \\ \Pr &= \frac{C_p\mu}{k}, Gr = \frac{\rho g a^3 \alpha \bar{T}_0}{\mu}, \theta = \frac{\bar{T} - \bar{T}_0}{T_0}, \beta = \frac{\alpha^2 Q_0}{k T_0}, h = \frac{\bar{h}}{a}, \end{aligned}\right\}, \tag{7}$$

where Re is the Reynolds number, δ is the dimensionless wave number, σ is the electrical conductivity of the fluid, B_0 is the magnetic field strength, T is the temperature, T_0 is the temperature of tube, α is the coefficient of expansion, Q_0 is the constant heat addition/absorption, β is the heat source/sink parameter, Gr is the Grashof number, Pr is Prandtl number and c is speed of sinusoidal wave train along the wall of tube. A uniform magnetic field is applied in the transverse direction to the flow. The magnetic Reynolds number is taken small so that the induced magnetic field can be neglected.

Eqs. (3)–(6) after invoking long wavelength approximation yield

$$\frac{\partial p}{\partial x} = \left(1 + \lambda_1^{\alpha_1}\frac{\partial^{\alpha_1}}{\partial t^{\alpha_1}}\right)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - M^2 u + Gr\theta, \tag{8}$$

$$\frac{\partial p}{\partial r} = 0, \tag{9}$$

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} + \beta = 0. \tag{10}$$

The non-dimensional boundary conditions are

$$\frac{\partial u}{\partial r} = 0 \quad \text{at} \quad r = 0, \quad u = 0 \quad \text{at} \quad r = h, \tag{11}$$

$$\frac{\partial \theta}{\partial r} = 0 \quad at \quad r = 0, \quad \theta = 0 \quad at \quad r = h. \tag{12}$$

3. Solutions of the problem

Solving the resulting Eq. (10) and then applying the corresponding boundary conditions given in Eq. (12), we get the solution for θ in the form as follows

$$\theta = \frac{\beta}{4}\left(h^2 - r^2\right). \tag{13}$$

Using Eq. (13) into the Eq. (8) and solving subject to boundary condition given in Eq. (11), we obtain the exact solution for the velocity field of the following form

$$u = \frac{1}{M^2}\frac{\partial p}{\partial x}\left[\frac{I_0(sr)}{I_0(sh)} - 1\right] + \frac{Gr\beta}{4M^2}\left[h^2 - r^2 + \frac{4}{M^2}\left(1 + \lambda_1^{a_1}\frac{\partial^{\alpha_1}}{\partial t^{\alpha_1}}\right) \times \left(\frac{I_0(sr)}{I_0(sh)} - 1\right)\right], \tag{14}$$

where

$$s(t) = \frac{M^2}{1+\lambda_1^{\alpha_1}\frac{\partial^{\alpha_1}}{\partial t^{\alpha_1}}} = s. \quad (\text{say}) \tag{15}$$

The volume flow rate is given as

$$\overline{Q} = \int_0^h 2rudr. \tag{16}$$

Solving Eq. (16), we get

$$\overline{Q} = \frac{h}{sM^2}\frac{\partial p}{\partial x}\left[\frac{2I_1(sh) - shI_0(sh)}{I_0(sh)}\right] + \frac{Gr\beta}{8M^2}\left[h^4 + \frac{8h}{sM^2}\left(1+\lambda_1^{\alpha_1}\frac{\partial^{\alpha_1}}{\partial t^{\alpha_1}}\right)\left(\frac{2I_1(sh) - shI_0(sh)}{I_0(sh)}\right)\right] \tag{17}$$

The transformation from fixed frame to wave frame is given by

$$X = x - t, \quad R = r, \quad U = u - 1, \quad V = v, \quad q = Q - h^2. \tag{18}$$

The wall of the tube in the wave frame is given as

$$h = 1 - \varphi\cos^2(\pi X). \tag{19}$$

The time averaged flow rate Q is calculated as

$$Q = q + 1 - \varphi + \frac{3\varphi^2}{8} = \overline{Q} - h^2 + 1 - \varphi + \frac{3\varphi^2}{8}. \tag{20}$$

Making use of Eq. (18) into Eqs. (13), (14) and (17) we get

$$\frac{\partial p}{\partial X} = \frac{sN^2 I_0(sh)}{h[2I_1(sh) - shI_0(sh)]}\left[Q + h^2 - 1 + \varphi - \frac{3\varphi^2}{8} - \frac{Gr\beta}{8M^2}\left\{h^4 + \frac{8h}{sN^2}\left(1+\lambda_1^{\alpha_1}\frac{\partial^{\alpha_1}}{\partial t^{\alpha_1}}\right)\left(\frac{2I_1(sh) - shI_0(sh)}{I_0(sh)}\right)\right\}\right], \tag{21}$$

$$U = \frac{Gr\beta}{4M^2}\left[h^2 - R^2 + \frac{4}{M^2}\left(1+\lambda_1^{\alpha_1}\frac{\partial^{\alpha_1}}{\partial t^{\alpha_1}}\right)\left(\frac{I_0(sR)}{I_0(sh)} - 1\right)\right] + \frac{1}{M^2}\frac{\partial p}{\partial X}\left[\frac{I_0(sR)}{I_0(sh)} - 1\right] - 1. \tag{22}$$

$$\theta = \frac{\beta}{4}\left(h^2 - R^2\right). \tag{23}$$

The dimensionless pressure rise and friction force are respectively given by

$$\Delta P = \int_0^1 \frac{\partial p}{\partial X}dX, \tag{24}$$

and

$$F = \int_0^1 \left(-h^2\frac{\partial p}{\partial X}\right)dX. \tag{25}$$

Finally the heat transfer coefficient at the outer wall is obtained as

$$\xi = \left[\frac{\partial\theta}{\partial R}\frac{\partial R}{\partial X}\right]_{R=h} = -\frac{\varphi\beta h\pi}{2}\sin(2\pi X). \tag{26}$$

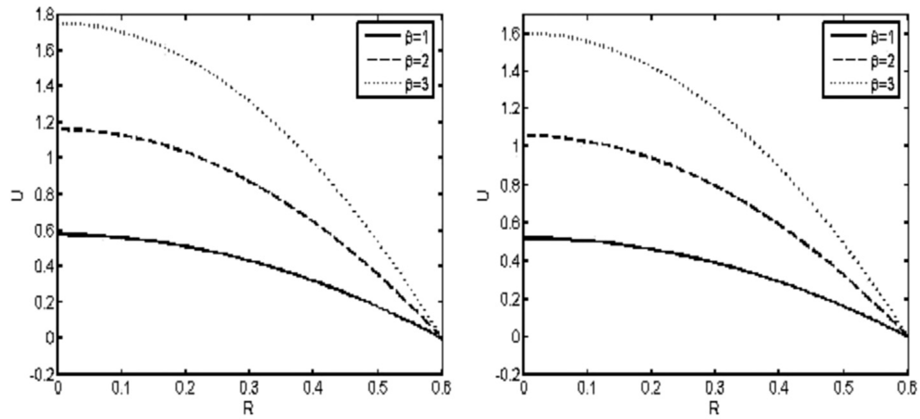
4. Results and discussion

The objective of this section is to present the effects of embedding parameters of interest on flow quantities such as velocity profile, coefficient of heat transfer, pressure rise and frictional force. Therefore, we have prepared Figs. 1–12.

Figs. 1–5 are plotted to see the variation of velocity profile for different physical parameters. Effects of heat source/sink parameter β and Grashof number Gr, are indicated in the Figs. 1 and 2. Since vertical tube exhibits elastic behavior that offers less resistance to flow which in turn speed up the velocity. Due to the fact these figures depict that the velocity for fractional second grade fluid is greater than the second grade fluid. Figs. 3 and 4 portray the effect of magnetic field M and aspect ratio φ on velocity profile. It is perceived that with an increase in M and φ the velocity decreases. The effect of λ_1 is shown in Fig. 5. It is found that an increase in constant of fractional second grade fluid results in the decrease of velocity profile for the case of fractional second grade fluid. But we noticed that velocity remains unchanged for the case of second grade fluid. The variation of heat transfer coefficient has been presented in Fig. 6. It is interesting to note that the absolute value of heat transfer coefficient increases with an increase in β and φ. It is noticed that the nature of heat transfer is oscillatory. This is accordance with the physical expectation due to oscillatory nature of the tube wall. Figs. 7–9 designate the effects of α_1, β, Gr, M, φ and λ_1 on pressure rise against the time average flow rate Q. Fig. 7 shows the effects of α_1 and β on the pressure rise versus Q. It can be seen that the pumping rate decreases with the increase of α_1 on ΔP with Q for pumping region. While in copumping the pumping rate increases as α_1 increase whereas pumping rate increases with increasing the values of β. The effects of Gr and M on the pressure rise are illustrated in Fig. 8. It is noted that the pumping rate increases with increasing the values of Gr, on the other hand in case of magnetic parameter, it reveals that pumping rate decreases by increasing the value of M in pumping region and such behavior is quite opposite in co-pumping region. Effects of φ and λ_1 on ΔP with Q are discussed in Fig. 9. It is observed that in pumping region the pumping increases with an increase of φ. However in both free pumping and co-pumping regions this situation is quite opposite to one another. The effect of λ_1 on ΔP with Q indicates that the pumping rate is increasing for time and material constant. Figs. 10–12 disclose the variation of α_1, β, Gr, M, φ and λ_1 on pressure rise against the time average flow rate Q. It is observed that the frictional force have opposite behavior when compared to pressure rise.

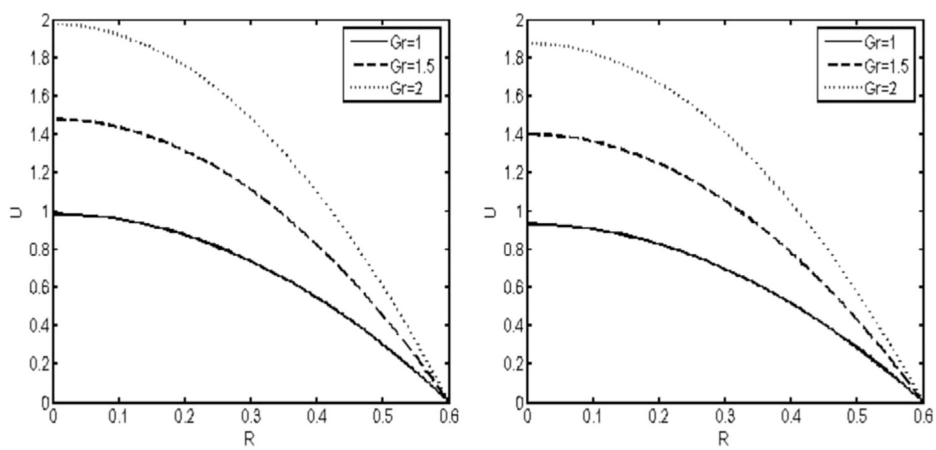
5. Concluding remarks

In this letter, we derived and analyzed a mathematical model subject to low Reynolds number and long wavelength approximations in order to study the peristaltic motion of fractional second grade fluid in a vertical tube. Analysis has been carried out in the presence of heat transfer and magnetic field. Exact expressions for velocity field, temperature, averaged flow rate and heat transfer coefficient are obtained. The results extracted are compatible with the physical expectations and are found to satisfy all the subjected conditions. A side by side comparative analysis is performed to compare our findings between second grade fluid and fractional second grade fluid. Moreover, fractional second grade fluid model reduces to second grade models for $\alpha_1 = 1$ and classical Naiver Stokes fluid model can be deduced from this as a special case by taking $\overline{\lambda}_1 = 0$. This provides a useful accuracy check about the correctness and validity of our results and provides a strong confidence into the presented mathematical descriptions.



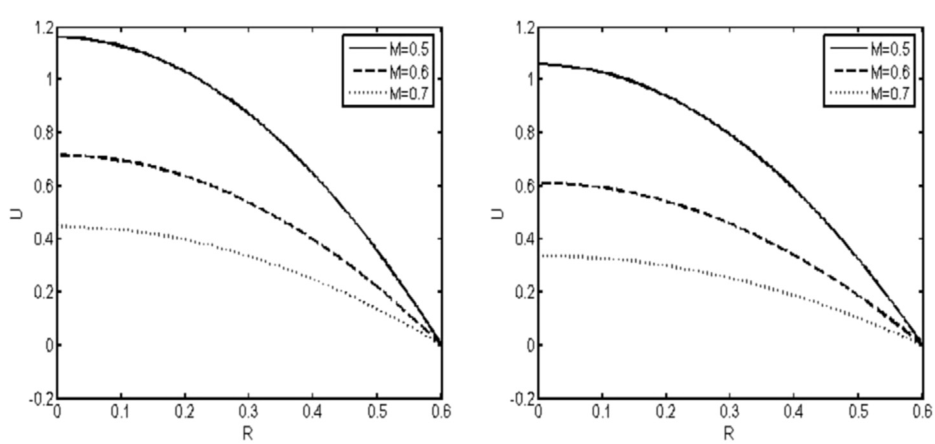
(a) Fractional second grade fluid ($\alpha_1 = 0.4$) (b) Second grade fluid ($\alpha_1 = 1$)

Fig. 1. Effects of β on velocity profile at φ=0.4, $Gr = 2$, $M = 0.5$, $\lambda_1 = 1$, $t = 0.2$, $\partial p/\partial X = 1$, $q = 1$ and $x = 1$.



(a) Fractional second grade fluid ($\alpha_1 = 0.4$) (b) Second grade fluid ($\alpha_1 = 1$)

Fig. 2. Effects of Gr on velocity profile at φ=0.4, β=2, $M = 0.4$, $\lambda_1 = 1$, $t = 0.2$, $\partial p/\partial X = 1$, $q = 1$ and $x = 1$.



(a) Fractional second grade fluid ($\alpha_1 = 0.4$) (b) Second grade fluid ($\alpha_1 = 1$)

Fig. 3. Effects of M on velocity profile at $\varphi = 0.4$, $Gr = 2$, $\beta = 2$, $\lambda_1 = 1$, $t = 0.2$, $\partial p/\partial X = 1$, $q = 1$ and $x = 1$.

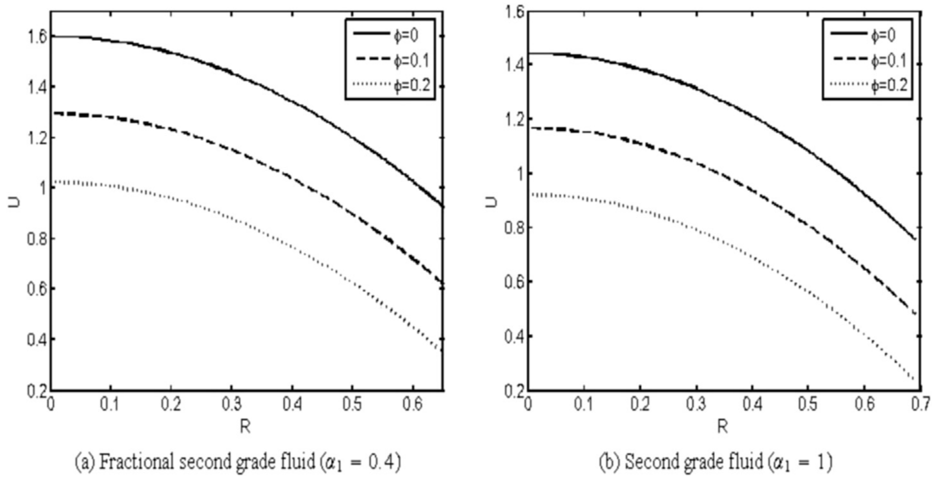


Fig. 4. Effects of φ on velocity profile at $Gr = 2$, $\beta = 1$, $M = 0.5$, $\lambda_1 = 1$, $t = 0.2$, $\partial p/\partial X = 1 = 1$, $q = 1$ and $x = 1$.

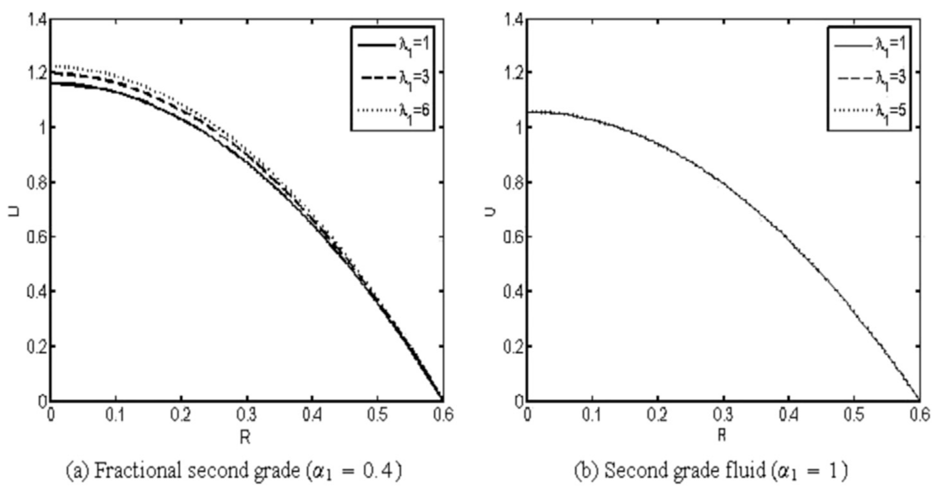


Fig. 5. Effects of λ_1 on velocity profile at $\varphi = 0.4$, $\beta = 2$, $M = 0.5$, $Gr = 2$, $t = 0.2$, $\partial p/\partial X = 1$, $q = 1$ and $x = 1$.

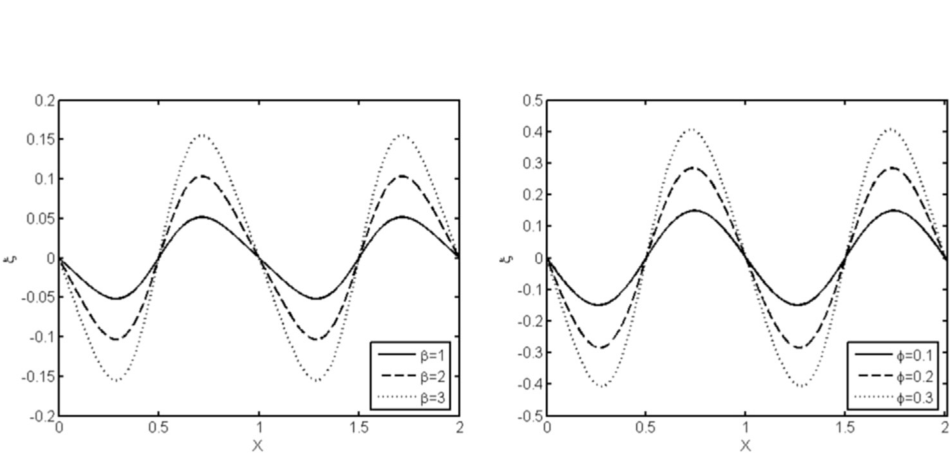


Fig. 6. Effects of coefficient of heat transfer for various values of β and φ.

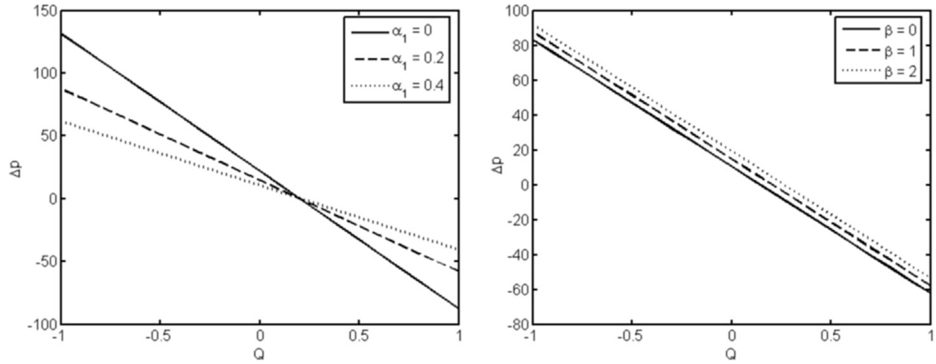


Fig. 7. Effects of α_1 and β on ΔP with Q at $\varphi = 0.4$, $Gr = 1$, $M = 1$, $\lambda_1 = 1$, and $t = 0.1$.

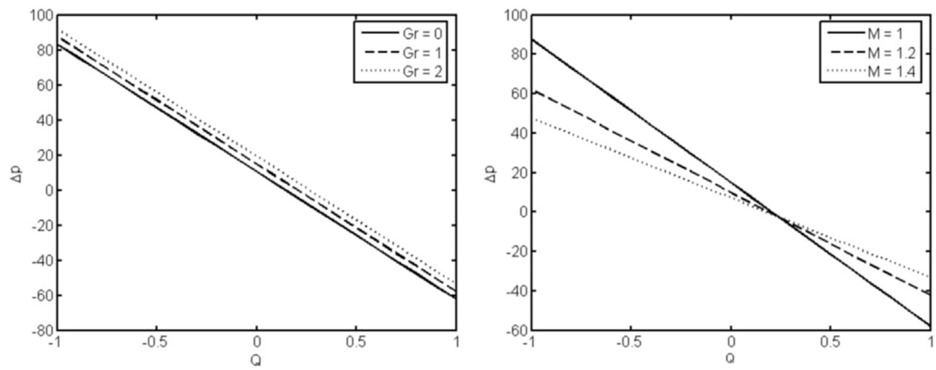


Fig. 8. Effects of G_r and M on ΔP with Q at $\varphi = 0.4$, $\alpha_1 = 0.2$, $\lambda_1 = 1$, $t = 0.2$ and $\beta = 1$.

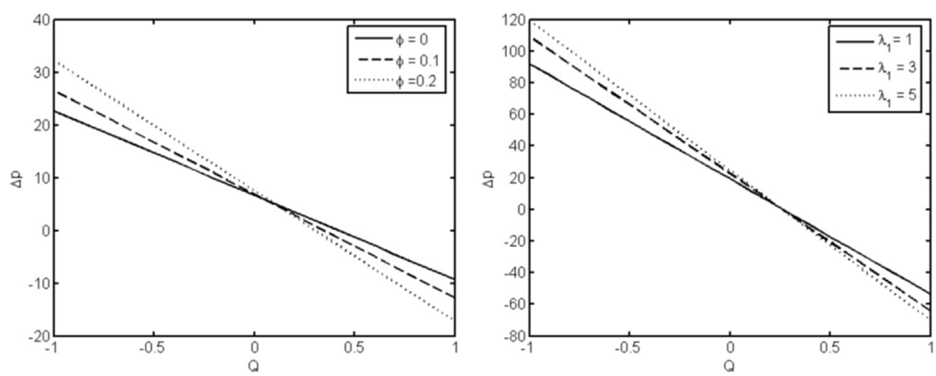


Fig. 9. Effects of φ and λ_1 on ΔP with Q at $\alpha_1 = 0.4$, $Gr = 2$, $M = 1$, $t = 0.2$ and $\beta = 1$.

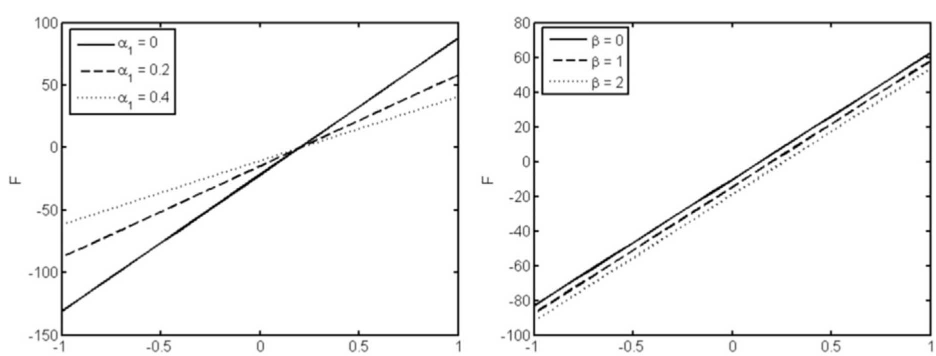


Fig. 10. Effects of α_1 and β on F with Q at $\varphi = 0.4$, $Gr = 1$, $M = 1$, $\lambda_1 = 2$, and $t = 0.1$

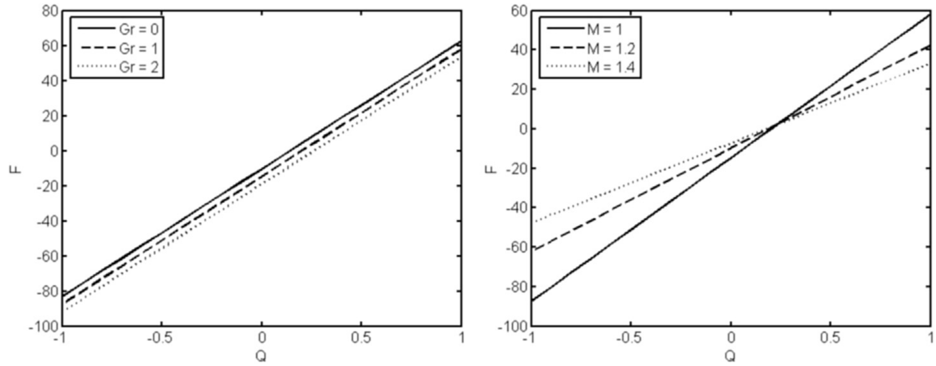


Fig. 11. Effects of G_r and M on F with Q at $\varphi = 0.4$, $\alpha_1 = 0.2$, $\lambda_1 = 1$, $t = 0.2$ and $\beta = 1$.

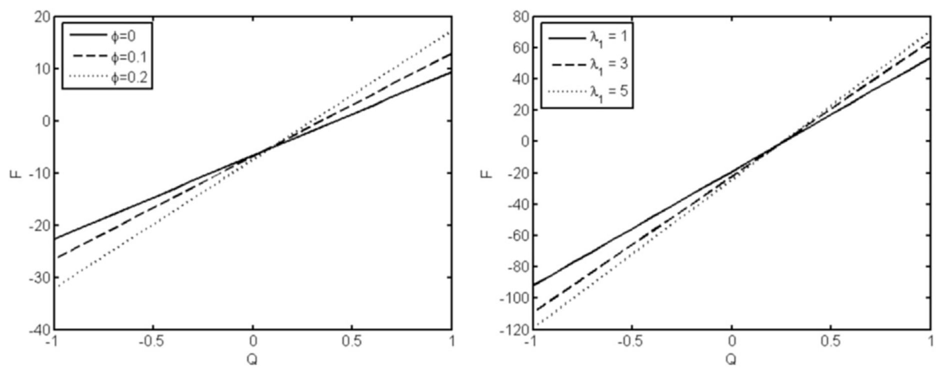


Fig. 12. Effects of φ and λ_1 on F with Q at $\alpha_1 = 0.2$, $Gr = 2$, $M = 1$, $t = 0.2$ and $\beta = 1$.

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