Comput. Math. Applic. Vol. 15, No. 11, pp. 969–973, 1988
Printed in Great Britain.

0097-4943/88 $3.00 + 0.00

PASCAL FUNCTIONS FOR THE GENERATION OF RANDOM NUMBERS

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(*Received* 19 *November* 1987)

Communicated by E. Y. Rodin

Abstract—Random number generators are of great importance for many scientific and engineering applications. Although the PASCAL computer language is becoming increasingly important for such applications, in many computer systems no PASCAL functions are available for this purpose. This short note is concerned with the development of PASCAL functions for the generation of random numbers with uniform, exponential and normal distributions. The functions are based on well established mathematical methods and have been developed and tested on a VAX 11/785 minicomputer. However, the functions are written in standard PASCAL and can be used for other machines.

INTRODUCTION

The need for random number generators

Computer software functions for the generation of random numbers are of great importance for many scientific and engineering applications. Such applications include: system modeling, computer simulation, sampling, numerical analysis, testing the effectiveness of computer algorithms, decision making, etc. [1, 2]. Some computer languages have access to special libraries of statistical functions that include random number generators. An example of this is the FORTRAN ISML (International Mathematical and Statistical Libraries) [3].

The PASCAL computer language

The computer language PASCAL is becoming of increasing importance in many scientific and engineering applications, as it provides facilities for the definition of multi-level data structures, and allows the easy manipulation of such structures [4, 5]. However, many computer systems have no random number generators available to the language, such as the King Saud University IBM 370/3033 mainframe, the various VAX11 series computers of the University, and the different types of the IBM-PC and their compatibles [6, 7].

This paper

The objective of this short paper is to provide functions for the generation of random numbers written in the computer language PASCAL, so that they can be used by interested scientists and engineers. The basic three distributions usually used for random numbers are considered; they include: the *uniform* distribution, the *exponential* distribution, and the *normal* distribution [1, 2]. The developed PASCAL functions, are based on well established mathematical methods [1]. In addition, the various statistical distributions provided by the functions have been tested for some examples, with testing factors that include: mean value and standard deviation.

THE UNIFORM DISTRIBUTION

Method

The method used for the development of PASCAL functions for the generation of random numbers with uniform distribution is the *linear congruential method* [1, 2]. According to this method, the sequence of random numbers, X_n, is obtained using the following equation:

$$X_n + 1 = (a\,X_n + c) \bmod m, n \geqslant 0, \tag{1}$$

where

X_0—the starting value (the seed) $X_0 \geqslant 0$;
a—a multiplier constant $a \geqslant 0$;
c—an increment constant $c \geqslant 0$;
m—the modulus $m \geqslant X_0$, $m \geqslant a$, $m \geqslant c$.

This equation can provide a sequence of values in the range between (0) and $(m - 1)$. It is desirable for the sequence of random numbers to have a long repeatability cycle, which is usually called the period. To achieve this, the following recommendations are made for a computer of a word size of at least 4 binary digits [1]:

—m, should be chosen to have the largest prime number that can be held by a single computer word;
—a, should satisfy the following relation

$$a \bmod 8 = 3 \text{ or } 5; \tag{2}$$

—c, should be relatively prime to m.

In many applications, the range of the generated random numbers, with uniform distributions, is required to be between zero and one. This can be obtained using the following equation:

$$U_n = X_n/m, \tag{3}$$

where U_n is sequence of uniformly distributed random numbers in the range between zero and one.

PASCAL programming

A PASCAL function, *Uniform*, has been developed for the generation of uniformly distributed random numbers in the range from zero to one, using the above method. The computer used for this purpose is the VAX 11/785 of the College of Engineering, King Saud University. This computer has a word size of 32 bits. The factors m, a and c have been chosen in accordance with the above recommendations, to have the following values [1]:

$$m = 2^{31} - 1 = 2{,}147{,}483{,}647;$$
$$a = 314{,}159{,}269;$$
$$c = 453{,}806{,}245. \tag{4}$$

Figure 1 shows the developed PASCAL function. The function generates a single random value Uniform, in the range from 0 to 1, for a given *random value*, *Seed*, in the range from 0 to m. For the generation of a sequence of uniformly distributed random values, only one given value in the range from zero to m is needed, as such value would be the seed X_0 of the first generated value, and consequently of the whole repeatedly generated sequence.

Application and testing

The PASCAL function introduced above can be used for the generation of uniformly distributed random numbers of any required range. For example, a sequence of uniformly distributed integer

```
{*Function for generating Uniform Random Numbers *}
  Function Uniform (Var Seed : integer) : Real;
{*Uniform is a random number between zero and one*}
  const
        A = 314159269 ;
        C = 453806245 ;
        M = 2147483647;
  begin
       Seed     := A * Seed + C;
       Seed     := Seed MOD M  ;
       Uniform  := Seed/M
  end;
```

Fig. 1. A PASCAL function for the generation of uniformally distributed random numbers.

Table 1. Results obtained from testing the developed PASCAL random number generators

Test factors	Uniform distribution range: 0–100	Exponential distribution	Normal distribution
Total generated numbers	100,000	100,000	100,000
Given mean	5.00000E+01	2.50000E+01	2.50000E+01
Computed mean	4.99405E+01	2.49730E+01	2.49456E+01
Relative difference of mean	1.19078E−03	1.08010E−03	2.17795E−03
Expected standard deviation	2.91562E+01	2.50000E+01	1.00000E+00
Computer standard deviation	2.91340E+01	2.49623E+01	1.00987E+00
Relative difference of standard deviation	7.61271E−04	1.51217E−03	9.86946E−03

numbers, $U(\min, \max)$, in the range from min to max, can be generated using the above function within the following equation:

$$U(\min, \max) = \min + \mathrm{TRUNC}\,[(\max - \min + 1)\mathrm{Uniform}], \tag{5}$$

where TRUNC truncates the value in brackets.

The above example has been applied to the case where min = 0, and max = 100. For this case a function for computing $U(\min, \max)$ has been developed using equation (5). In addition, the function has been tested in accordance with the following assumptions:

—the test considers the generation of a sequence of 100,000 values;
—$X_0 = 577{,}215{,}665$;
—the test measures the frequency of each value in the range, the mean value and the standard deviation.

A PASCAL program using the function of Fig. 1 has been developed to implement the testing. The results produced are given in Table 1; it is shown that the relative difference between given and computed values is negligible.

EXPONENTIAL DISTRIBUTION

Method

This pattern of distribution can be developed from the uniform distribution using the *logarithm method* [1], if the term, *MeanX*, represents the mean of the exponential distribution required, the following equation gives the relationship between an exponentially distributed random number $Exponential_n$, and a uniformly distributed random number $Uniform_n$ of a range from zero to one [1]:

$$Exponential_n = -MeanX \ln(Uniform_n) \tag{6}$$

PASCAL programming

A PASCAL function, *Exponential*, has been developed for the generation of exponentially distributed random numbers with any specified mean. Figure 2 shows the developed function, which is based on equation (6), and the method used in Fig. 1 for the generation of $Uniform_n$.

```
{* Function for generating Random Numbers with    *}
{* Exponential Distribution                       *}
Function Exponential (MeanX:Real;Var Seed:integer):Real;
{* Exponential is a random number with exponential*}
{* distribution and with mean = MeanX             *}

  begin
      Exponential := -MeanX * Ln(Uniform (Seed))
  end;
```

Fig. 2. A PASCAL function for the generation of exponentially distributed random numbers.

Application and testing

To illustrate the use of the PASCAL function of Fig. 2, it has been run on the VAX 11/785 for an example, that has the same assumptions of the example given for the uniform distribution. The value of the mean is considered to be 25, and the example is tested using a PASCAL program developed for this purpose. The results obtained are given in Table 1.

THE NORMAL DISTRIBUTION

Method

The generation of random numbers with normal distribution can be obtained from the uniform distribution, using the *polar method* [1]. The method is based on the following algorithm:

Step 1.

$$V_{n1} = 2\ Uniform_{n1} - 1$$
$$V_{n2} = 2\ Uniform_{n2} - 1 \qquad (7)$$

Step 2.

$$S = V_{n1}^2 + V_{n2}^2,$$

if $S > = 1$ return to step 1

Step 3.

$$Y = V_{n2}\sqrt{[-2\ln(S)]/S}$$

Step 4.

$$Normal_n = MeanN + Stndrd\_Dev\ Y,$$

where:

V_{n1}, V_{n2} = variable of uniform distribution in the range of − to +1,
S = variable related to both V_{n1} and V_{n2};
Y = variable of normal distribution with mean zero and standard deviation one,
$Normal_n$ = variable of normal distribution with mean *MeanN* and standard deviation *Stndrd_Dev*.

PASCAL programming

A PASCAL function, *Normal*, has been developed for the generation of normally distributed random numbers with any specified mean, and standard deviation. Figure 3 shows the developed function, which is based on the mathematical algorithm (7), and the method used in Fig. 1 for the generation of $Uniform_n$.

```
{* Function for generating Random Numbers with    *}
{* Normal Distribution                            *}
Function Normal (MeanN,Stndrd_Dev:Real; Var Seed:integer):Real;
{* Normal is a random number with normal distribution     *}
{* and with mean = MeanN, standard deviation = Stndrd_Dev *}


  var  S, Y, V1, V2  :Real;
  begin
        repeat
             V1 := 2 * Uniform(Seed) - 1 ;
             V2 := 2 * Uniform(Seed) - 1 ;
             S  := V1 * V1 + V2 * V2     ;
        until (S < 1);
        Y        := V2 * SQRT(-2 * Ln(S)/S);
        Normal   := MeanN + Stndrd_Dev * Y
  end;
```

Fig. 3. A PASCAL function for the generation of normally distributed random numbers.

Application and testing

The function, *Normal*, has been run on the VAX 11/785 for an example that has the same assumptions of the above uniform and exponential distributions. The value of the mean is considered to be 25, and the value for the standard deviation is assumed to be one. The results obtained are given in Table 1, which shows that the function operates satisfactorily.

REMARKS

This short paper has introduced three PASCAL functions for the generation of random numbers with uniform, exponential, and normal distributions. Although the functions have been developed and tested on a VAX 11/785, they are written in standard PASCAL, and can be used for other machines. Results obtained from testing the functions show that they operate satisfactorily.

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