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Commentary on Robert Riley's article "A personal account of the discovery of hyperbolic structures on some knot complements"

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Abstract

We give some background and biographical commentary on the posthumous article [4] that appears in this journal issue by Robert Riley on his part of the early history of hyperbolic structures on some compact 3-manifolds. A complete list of Riley's publications appears at the end of this article.

MSC 2010: primary 01A60; 01A70; secondary 57M25; 57M50

Keywords: Robert Riley; Hyperbolic structures; Knot complements; Biography

1. Introduction

In the mid-1970s the study of the topology of 3-manifolds was revolutionized by the discovery that many 3-manifolds possess a hyperbolic structure. This discovery was made, in very different forms, independently and almost simultaneously, by Robert Riley and William Thurston, with Riley's results appearing in print first [1,2].

Riley's approach was algebraic while Thurston's was geometric. Riley's first results covered a small number of knot complements, while Thurston's covered large classes

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of 3-manifolds. Ultimately, a sweeping conjecture of Thurston [6] about the existence of geometric structures on all 3-manifolds (part of which implies the Poincaré Conjecture) was proven by Perelman. Riley's earliest results and conjectures are described in [5] as a motivating factor for Thurston's first result in this area.

Before Riley died in 2000, he wrote a short memoir, describing his recollection of the events leading to his meeting with Thurston in 1976. This was circulated among his colleagues and a few others, but has lain dormant since his death. A chance conversation between two people familiar with him from different decades of his life has revived interest in the memoir, and we have decided to publish it in this issue [4], along with this commentary which contains some biographical information and a full bibliography of Riley's publications. Riley's wording has not been altered since, to those who knew him, it reads as pure Riley. Our only modification to Riley's paper has been to add an abstract, MSC numbers, keywords, two footnotes and a reference to this paper.

2. Bob Riley's life and career

Robert F. Riley grew up on Long Island in New York State and studied mathematics at Cornell where he earned his bachelors degree in 1957. He enrolled in MIT for graduate work with an initial interest in number theory, but was unhappy with the modern algebraic geometry he was expected to learn there. Bob spent some time in industry where he became proficient in the use of computers. He regarded himself more as a 19th-century mathematician with the added advantage of being able to use modern computational tools. Much later, Bob proudly showed one of us a letter of rejection he had received from a reputable British journal, saying that they no longer publish 19th-century mathematics.

In 1966, Bob moved to Amsterdam. There he met Brian Griffiths, a topologist who was Professor of Pure Mathematics at the University of Southampton. Brian invited Bob to take a temporary post in Southampton, which he did in 1968. In Amsterdam he had become interested in knot theory and in Southampton he worked on the representations of knot groups in $PSL(2, \mathbb{C})$, which is the group of orientation preserving isometries of hyperbolic 3-space $\mathbb{H}^3$. After some time he realized that, at least for the figure-eight knot, he was getting a faithful representation, that the image was a discrete group and that the quotient of $\mathbb{H}^3$ by this group was the figure-eight knot complement. He had thus discovered a hyperbolic structure on this knot complement. He then showed that the same idea works for several other knots. Later, Thurston gave a necessary and sufficient condition for a knot complement to have a hyperbolic structure, and wrote that he was motivated by Bob's beautiful examples (P. 360 of [5]).

Bob discovered these examples with the help of a computer, making use of his previous industrial experience. Bob's work was one of the earliest examples of the extensive use of computers in a branch of mathematics traditionally dominated by the pure thought and abstraction method of mathematicians of the first half of the 20th century. Note that Bob was working when programs were submitted to the computer as decks of punched cards. It should be mentioned that Thurston, like Riley, was also an outstanding innovator in computational methods in pure mathematics.

At this time, Bob did not have a permanent academic job because he had left MIT before getting a Ph.D. David Singerman agreed to act as Bob's formal supervisor so that he could be a Ph.D. student at Southampton. Bob also obtained funding from the Science Research Council which financed him for four years. He obtained a Ph.D. for his thesis “Projective representations of link groups”, David Epstein from the University of Warwick being the external examiner.

Bob returned to the USA in 1980, initially joining Thurston at Boulder and then obtaining a permanent position in the Department of Mathematical Sciences at Binghamton University in 1982. He continued to work there until he died from complications following heart surgery in March, 2000.

3. Riley’s later mathematics

In Section 2 of his memoir, Bob wrote the following:

“In December 1991 I used Maple to extend the theorem to algebraic varieties of *nab*–reps and add some new material. In 1993 I told Tomotada Ohtsuki about this, giving no detail, and he promptly found a better proof and more new material. I hope to proceed to a joint paper soon”.

Bob exchanged emails with Ohtsuki, of Kyoto University, in the mid-1990s, resulting in a short joint manuscript on homomorphisms between two-bridge knot groups. This was still incomplete when Bob died, but it eventually evolved into a joint paper [7] with a third author, Makoto Sakuma, of Osaka University, who had also corresponded with Bob (though neither Ohtsuki nor Sakuma had the opportunity to meet him).

Bob’s research sometimes involved quite deep number-theoretic considerations. These brought him into contact with the number theorist Kunrui Yu, now Emeritus Professor at Hong Kong University of Science and Technology. In particular, in [3] Bob uses the Gel’fond–Baker theory of linear forms in the logarithms of algebraic numbers to demonstrate the expected growth of the first homology groups of k-sheeted branched covers of S^3 branched over a tame knot. This paper includes a three-page appendix by Yu.

Yu wrote to us as follows about a visit to Bob in Binghamton in 1990/1. “On the second day Bob took me with his very old green Toyota for a tour. We visited the factory of Corningware and the Campus of Cornell University. The tour was very interesting. I found that Bob was a very kind and nice gentleman, and he had very good sense of humor”.

3.1. Reminiscences

We remember Bob as an artful eccentric who practiced his art of bone dry humor, well aware of the effects he had on his audience. His accumulated oddities are too numerous to list and too difficult to explain. He was fiercely independent and carved out a life and career that were entirely of his own making. He ignored the fashionable, and stuck doggedly to his own ideas of what was important. His level of entertainment and his fellowship were hard to match and he is sorely missed.

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