Convective cooling/heating induced thermal stresses in a fluid saturated porous medium undergoing local thermal non-equilibrium

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Abstract

Local thermal non-equilibrium (LTNE) may have profound effects on the pore pressure and thermal stresses in fluid saturated porous media under transient thermal loads. This work investigates the temperature, pore pressure, and thermal stress distributions in a porous medium subjected to convective cooling/heating on its boundary. The LTNE thermo-poroelasticity equations are solved by means of Laplace transform for two fundamental problems in petroleum engineering and nuclear waste storage applications, i.e., an infinite porous medium containing a cylindrical hole or a spherical cavity subjected to symmetrical thermo-mechanical loads on the cavity boundary. Numerical examples are presented to examine the effects of LTNE under convective cooling/heating conditions on the temperature, pore pressure and thermal stresses around the cavities. The results show that the LTNE effects become more pronounced when the convective heat transfer boundary conditions are employed. For the cylindrical hole problem of a sandstone formation, the thermally induced pore pressure and the magnitude of thermal stresses are significantly higher than the corresponding values in the classical poroelasticity, which is particularly true under convective cooling with moderate Biot numbers. For the spherical cavity problem of a clay medium, the LTNE effect may become significant depending on the boundary conditions employed in the classical theory.

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1. Introduction

Local thermal non-equilibrium (LTNE) may have profound effects on the pore pressure and thermal stresses in fluid saturated porous media. This work investigates the temperature, pore pressure, and thermal stress distributions in a porous medium subjected to convective cooling/heating on its boundary. The LTNE thermo-poroelasticity equations are solved by means of Laplace transform for two fundamental problems in petroleum engineering and nuclear waste storage applications, i.e., an infinite porous medium containing a cylindrical hole or a spherical cavity subjected to symmetrical thermo-mechanical loads on the cavity boundary. Numerical examples are presented to examine the effects of LTNE under convective cooling/heating conditions on the temperature, pore pressure and thermal stresses around the cavities. The results show that the LTNE effects become more pronounced when the convective heat transfer boundary conditions are employed. For the cylindrical hole problem of a sandstone formation, the thermally induced pore pressure and the magnitude of thermal stresses are significantly higher than the corresponding values in the classical poroelasticity, which is particularly true under convective cooling with moderate Biot numbers. For the spherical cavity problem of a clay medium, the LTNE effect may become significant depending on the boundary conditions employed in the classical theory.
medium (He and Jin, 2011). We note that the continuum mixtures theory of porous media generally assumes LTNE, i.e., the solid and fluid have different temperatures (Schrefler, 2002).

In the existing studies of LTNE thermo-poroelasticity problems, the same boundary temperature was specified for both solid and fluid phases. The boundary conditions for the solid and fluid, however, are generally different due to the difference in heat transfer coefficient at the boundary between the two phases. It is expected that the LTNE effects will become more pronounced when the difference in the boundary temperature between the solid and fluid is taken into account. In this paper, we employ a convective boundary condition for the solid (or fluid) with a finite heat transfer coefficient and a prescribed temperature boundary condition for the fluid (or solid) to investigate the LTNE effects on the temperature, pore pressure and thermal stresses. The thermo-poroelastic solutions for an infinite medium with a cylindrical or a spherical cavity are obtained using the LTNE thermo-poroelasticity equations presented by He and Jin (2010). The solutions of the problems may find applications in petroleum engineering and in evaluating the safety of underground nuclear waste storage. The remainder of the paper is organized as follows. Section 2 reviews the basic equations of the LTNE poroelastic theory. In Section 3, the temperature, pore pressure and thermal stress fields around a cylindrical hole in an infinite medium are derived and the numerical results are presented to illustrate the LTNE effects under convective cooling boundary conditions. Section 4 obtains the solutions of temperature, pore pressure and thermal stresses around a spherical cavity subjected to convective heating. Finally, the conclusions are provided in Section 5.

2. Basic equations of local thermal non-equilibrium thermo-poroelasticity

This section summarizes the basic equations of thermo-poroelasticity for fluid saturated porous media undergoing local thermal non-equilibrium (LTNE). Under the LTNE conditions, the solid and fluid phases have different temperature variations which are governed by the following heat conduction equations (see Sections 2.2.2 and 6.5 in Nield and Bejan, 2006)

\[
(1 - \phi_0)\rho_C c_\theta \frac{\partial \theta}{\partial t} = (1 - \phi_0)k_C \nabla^2 \theta + \dot{h}_{\text{int}}(\theta_j - \theta_l)
\]  
\[
\phi_0 \rho_f c_\theta \frac{\partial \theta}{\partial t} = \phi_0 k_f \nabla^2 \theta - \dot{h}_{\text{int}}(\theta_j - \theta_l)
\]

(1a) (1b)

where \(\theta_l\) and \(\theta_j\) are the solid and fluid temperature variations, respectively, \(t\) time, \(\phi_0\) the reference porosity, \(k_C\) and \(k_f\) the thermal conductivities of the solid and fluid, respectively, \(\rho_C\) and \(\rho_f\) the densities for the solid and fluid, respectively, \(c_\theta\) and \(c_f\) the specific heat for the solid and fluid, respectively, \(\dot{h}_{\text{int}}\) the specific heat for the solid–fluid interface, \(\nabla^2\) the Laplacian operator. Heat transfer by convection is neglected in this study.

The LTNE constitutive equations may be formulated using the weighted average of the solid and fluid temperatures as follows (He and Jin, 2010)

\[
\sigma_{ij} = 2G0_{ij} + \frac{2Gv}{1 - 2v} \eta_{s} \delta_{ij} - \frac{3}{2} \mu_0 \phi_0 \delta_{ij} - K \chi' \theta_{\text{ave}} \delta_{ij}
\]  
\[
\zeta = \frac{\alpha}{3K} (\theta_{sk} + 2\phi_0 \chi' (\theta_f - \theta_s)) \theta_{\text{ave}}
\]

(2) (3)

where \(\theta_{\text{ave}}\) is the weighted average temperature for the porous medium

\[
\theta_{\text{ave}} = (1 - \phi_0)\theta_l + \phi_0 \theta_j
\]  
\[
\sigma_{ij} \text{ denotes stresses, } \epsilon_{ij} \text{ strains, } p \text{ the pore pressure, } \zeta \text{ the fluid content variation, } \delta_{ij} \text{ the Kronecker delta, } G \text{ the shear modulus, } v \text{ (drained) Poisson’s ratio, } K \text{ the (drained) bulk modulus, } \chi_1 \text{ the Biot–Williams coefficient, } B \text{ Skempton’s coefficient, } \chi' \text{ the volumetric thermal expansion coefficient for the solid (drained porous medium), } \chi' \text{ the second volumetric thermal expansion coefficient for the solid (solid matrix), and } \chi' \text{ the volumetric thermal expansion coefficient for the fluid. The constitutive equations (2) and (3) are the same as those of McTigue (1986) except that the temperature is a weighted average one which bridges the LTNE heat transfer theory and the classical thermo-poroelasticity. Eqs. (2)–(4) also reduce to those of McTigue (1986) when local thermal equilibrium prevails (\(\theta_{\text{ave}} = \theta_l\)).}

We propose to use the weighted average temperature to formulate the thermoporoelasticity Eqs. (2) and (3). This kind of quantities is standard in establishing effective constitutive relations and effective properties in micromechanics of composite materials (Christensen, 1979). Use of \(\theta_{\text{ave}}\) is natural if the fluid saturated porous medium is treated as a composite. Eqs. (2) and (3) are only an exploratory model aiming to modify the existing models using an improved temperature quantity.

Using the equilibrium equations, strain–displacement relations, the constitutive equations, and Darcy’s law, the governing equations for the displacements and pore pressure can be obtained as follows (He and Jin, 2010, 2011)

\[
G\nabla^2 \epsilon_{ij} + \frac{G}{1 - 2v} \sigma_{ii} - \frac{3v}{2(1 - 2v)} \frac{\partial p}{\partial x_i} - K \chi' \theta_{\text{ave}} = 0
\]  
\[
\alpha \frac{\partial p}{\partial t} + \alpha \frac{\partial \epsilon_{ij}}{\partial x_j} - \frac{k}{\mu} \frac{\partial \theta_{\text{ave}}}{\partial t} - K \chi' \phi_0 (\theta_f - \theta_s)
\]

(5) (6)

where \(\epsilon_{ij}\) are displacements, \(k\) is the intrinsic permeability and \(\mu\) is the fluid viscosity.

Eqs. (5) and (6) are coupled partial differential equations for the displacements and pore pressure. The pore pressure and displacements become uncoupled when the displacement field is irrotational in an infinite medium, i.e., \(u_j = u_j\). The governing equation of the pore pressure can be obtained as (He and Jin, 2010)

\[
\frac{\partial p}{\partial t} = c \nabla^2 p + c \frac{\partial \theta_{\text{ave}}}{\partial t}
\]

(7)

where

\[
c = \frac{k}{\mu S}, \quad c' = \frac{1}{S} \left[ (2(1 - 2v)) \chi' + \phi_0 (\chi' - \chi') \right],
\]

\[
S = \frac{\alpha}{3K} \left( 1 - \frac{4\phi_0 B}{3} \right), \quad \eta = \frac{1}{(2(1 - v))}
\]

(8) (9)

The equilibrium equation in terms of displacements/strains now becomes

\[
\epsilon_{sk} = \left( 1 + 2G0 \right) \frac{c}{\chi' + \alpha} \frac{\partial \theta_{\text{ave}}}{\partial x_k}
\]

(10)

where \(\chi' = 2Gv(1 - 2v)\) is the Lamé constant.

In the next two sections, we employ the above LTNE heat conduction and thermo-poroelasticity equations to investigate the effects of LTNE on the temperature, pore pressure and thermal stresses around a cylindrical hole or a spherical cavity in an infinite medium subject to convective cooling/heating.

3. A cylindrical hole in an infinite porous medium

We consider a cylindrical hole in an infinite porous medium as shown in Fig. 1, where \(r\) is the radial coordinate and \(a\) is the radius of the hole. The boundary of the hole is subjected to a sudden pressure applied by a fluid whose temperature is different from that of the medium. This is a fundamental problem for drilling applications in petroleum engineering. We first solve the LTNE heat conduction Eqs. (1) for the solid and fluid temperatures as the usual
uncoupled approach is adopted in this work. The pore pressure and stresses are subsequently obtained by solving the thermo-poro-elasticity equations in Sections 3.2 and 3.3.

3.1. Temperature field

In the existing investigations of LTNE thermo-poroelasticity problems, the same boundary temperature was assumed for both the solid and fluid phases. A more realistic assumption for the present borehole problem is to use a convective condition for the solid and a prescribed temperature condition for the fluid phase. The initial and boundary conditions for the temperatures can thus be formulated as follows:

$$\theta_s(r, t)|_{t=0} = \theta_{10}(r, t), \quad \theta_f(r, t)|_{t=0} = \theta_{10}, \quad a \leq r < \infty$$

$$- k_s \frac{\partial \theta_s(r, t)}{\partial r} \bigg|_{r=a} = h_{ts}(\theta_s - \theta_1)|_{r=a}, \quad \theta_s(r, t)|_{r=a} = \theta_s, \quad t > 0$$

$$\theta_s(r, t)|_{r=\infty} = 0, \quad \theta_f(r, t)|_{r=\infty} = 0, \quad t > 0$$

(11)

where $\theta_s$ is a constant (temperature of the drilling fluid) and $h_{ts}$ is the heat transfer coefficient. Introducing the following parameters

$$\kappa_s = \frac{k_s}{\rho_s c_s}, \quad h = \frac{h_{ts}}{(1 - \phi_s) \rho_s c_s}, \quad b_1 = \frac{\phi_0 \rho_s c_s}{(1 - \phi_0) \rho_s c_s}, \quad b_2 = \frac{\phi_0 k_{fs}}{(1 - \phi_0) k_{fs}}, \quad b_a = \frac{1}{Bi} - k_{ts}$$

(13)

the governing equations of the temperatures for the solid and fluid phases can be rewritten as

$$\frac{\partial \theta_s}{\partial t} - \kappa_s \nabla^2 \theta_s - h(\theta_f - \theta_s) = 0$$

$$\frac{\partial \theta_f}{\partial t} - b_1 \kappa_s \nabla^2 \theta_f + h(\theta_f - \theta_s) = 0$$

(15a)

(15b)

where the Laplacian operator for the cylindrical symmetric problem is

$$\nabla^2 \theta = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r}$$

Clearly, $\kappa_s$ is the thermal diffusivity of the solid and $b_a$ is the inverse of the Biot number $Bi$ associated with the solid. Applying the Laplace transform to Eqs. (15) and (12) and using the initial conditions (11), we have the transformed equations and boundary conditions as follows:

$$s \tilde{\theta}_s - \kappa_s \nabla^2 \tilde{\theta}_s - h(\tilde{\theta}_f - \tilde{\theta}_s) = 0$$

$$b_1 s \tilde{\theta}_f - b_2 \kappa_s \nabla^2 \tilde{\theta}_f + h(\tilde{\theta}_f - \tilde{\theta}_s) = 0$$

$$- k_s \frac{\partial \tilde{\theta}_s(r, s)}{\partial r} \bigg|_{r=a} = h_{ts}(\tilde{\theta}_s(s) - \tilde{\theta}_s)|_{r=a}, \quad \tilde{\theta}_s(r, s)|_{r=a} = \theta_s, \quad t > 0$$

$$\tilde{\theta}_s(r, s)|_{r=\infty} = 0, \quad \tilde{\theta}_f(r, s)|_{r=\infty} = 0, \quad t > 0$$

(16a)

(16b)

(17)

where $s$ is the Laplace transform variable and an overbar of a quantity denotes its Laplace transform, i.e.,

$$\tilde{\theta}_s(r) = \int_0^\infty \theta_s(r, t) e^{-st} dt$$

(18)

The transformed temperatures of the solid and fluid can be obtained by solving the equations above as follows:

$$\tilde{\theta}_s(r, s) = c_1 k_0(r \sqrt{s}) + c_2 k_0(r \sqrt{\frac{h}{h}})$$

(19a)

$$\tilde{\theta}_f(r, s) = c_1 \left(1 + \frac{s - \kappa_s \lambda_1}{h}\right) k_0(r \sqrt{s}) + c_2 \left(1 + \frac{s - \kappa_s \lambda_2}{h}\right) k_0(r \sqrt{\frac{h}{h}})$$

(19b)

where $k_0(\cdot)$ is the second kind modified Bessel function of order zero, $\lambda_1$ and $\lambda_2$ are given by

$$\kappa_s \lambda_{1,2} = A_1 s + A_2 \pm \sqrt{A_3 s^2 + A_4 s + A_5}$$

(20)

in which

$$A_1 = \frac{1}{2} \left(1 + \frac{b_1}{b_2}\right), \quad A_2 = \frac{h}{2} \left(1 + \frac{1}{b_2}\right), \quad A_3 = \frac{1}{4} \left(\frac{b_1}{b_2} - 1\right)^2$$

(21)

$$A_4 = \frac{h}{2} \left(1 - \frac{1}{b_2}\right) \left(\frac{b_1}{b_2} - 1\right)$$

(22)

The constants $c_1$ and $c_2$ in Eqs. (19a) and (19b) are determined from the transformed boundary conditions Eq. (17) as follows:

$$c_1 = \frac{\theta_{10}}{s} \left(\frac{1}{c_1(k_0(\alpha_1))} \left(s - \kappa_s \lambda_2 - ab_a h \sqrt{\frac{h}{h}} K_1(\alpha_1) \sqrt{\frac{h}{h}}\right)\right)$$

$$c_2 = \frac{\theta_{10}}{s} \left(\frac{1}{c_2(k_0(\alpha_2))} \left(s - \kappa_s \lambda_1 - ab_a h \sqrt{\frac{h}{h}} K_1(\alpha_1) \sqrt{\frac{h}{h}}\right)\right)$$

(23)

where $K_1(\cdot)$ is the second kind modified Bessel function of order one and $c_3$ is given by

$$c_3 = \kappa_s (\lambda_1 - \lambda_2) + ab_a h \sqrt{\frac{h}{h}} K_1(\alpha_1) \sqrt{\frac{h}{h}} (h + s - \kappa_s \lambda_1)$$

(24)

Substituting Eq. (22) into Eqs. (19a) and (19b) gives the following transformed temperatures of solid and fluid

$$\tilde{\theta}_s(r, s) = \frac{\theta_{10}}{s} \left(s - \kappa_s \lambda_2 - ab_a h \sqrt{\frac{h}{h}} K_1(\alpha_1) \sqrt{\frac{h}{h}}\right) \frac{k_0(r \sqrt{s})}{k_0(\alpha_1)}$$

$$- \frac{\theta_{10}}{s} \left(s - \kappa_s \lambda_1 - ab_a h \sqrt{\frac{h}{h}} K_1(\alpha_1) \sqrt{\frac{h}{h}}\right) \frac{k_0(r \sqrt{s})}{k_0(\alpha_1)}$$

$$\tilde{\theta}_f(r, s) = \frac{\theta_{10}}{s} \left(s - \kappa_s \lambda_2 - ab_a h \sqrt{\frac{h}{h}} K_1(\alpha_1) \sqrt{\frac{h}{h}}\right) \frac{k_0(r \sqrt{s})}{k_0(\alpha_1)}$$

(24a)

(24b)

In the classical local thermal equilibrium heat transfer theory, the solid and fluid temperatures are the same, i.e., $\theta_s = \theta_f$. The corresponding initial and boundary conditions of the heat conduction problem are

$$\theta(r, t)|_{t=0} = \theta_0, \quad a \leq r < \infty$$

$$- k_m \frac{\partial \theta(r, t)}{\partial r} \bigg|_{r=a} = h_m (\theta_0 - \theta)|_{r=a}, \quad \theta(r, t)|_{r=\infty} = 0, \quad t > 0$$

(25)

(26)
where $k_{tm}$ is the effective thermal conductivity and $h_{tm}$ is the heat transfer coefficient for the porous medium. The transformed classical temperature can be obtained as follows:

$$
\bar{u}(r, s) = \frac{\theta_0}{s(K_0(\alpha \sqrt{s}) + ab_3 \sqrt{2}K_1(\alpha \sqrt{s}))K_0(r/\alpha \sqrt{s})}
$$

(27)

where $\kappa$ is the thermal diffusivity of the porous medium and $b_3$ is a constant given by

$$
b_3 = \frac{k_{tm}}{\alpha h_{tm}}
$$

(28)

We note that $b_3$ is the inverse of the Biot number associated with the heat transfer between the porous medium and the fluid in the borehole (drilling fluid for petroleum engineering applications).

3.2. Pore pressure

The pore pressure is decoupled from the displacement field and can be obtained by solving Eq. (7) as the displacement in the axisymmetric borehole problem is irrational. The initial and boundary conditions for pore pressure diffusion are

$$
p(r, t)|_{t=0} = 0, \quad a \leq r < \infty
$$

(29)

$$
p(r, t)|_{r=a} = p_a H(t), \quad t > 0
$$

(30)

$$
p(r, t)|_{r=\infty} = 0, \quad t > 0
$$

(31)

where $p_a$ is the fluid pressure applied on the boundary.

The Laplace transform of the basic equation (7) can be written as

$$
s\bar{p} = s\bar{T} + s\bar{\theta}_{ave}
$$

(32)

The transformed pore pressure can be obtained by solving the equation above and using the boundary conditions Eqs. (30) and the transformed temperature fields Eq. (24) as follows:

$$
p(r, s) = \left\{\frac{p_a}{s} + \frac{c'm_1(s)}{(c_0^2 - s)} + \frac{c'm_2(s)}{(c_0^2 - s)}\right\}\frac{K_0(r/\sqrt{s})}{K_0(\alpha \sqrt{s})} - \frac{c'm_1(s)}{(c_0 - s)}\frac{K_0(r/\alpha \sqrt{s})}{K_0(\alpha \sqrt{s})} - \frac{c'm_2(s)}{(c_0 - s)}\frac{K_0(r/\alpha \sqrt{s})}{K_0(\alpha \sqrt{s})}
$$

(33)

where

$$
m_1(s) = \frac{\theta_0}{c_3} \left\{s - K_{12} - ab_3 h \sqrt{2} K_1(\alpha \sqrt{s})\right\}
$$

$$
m_2(s) = \frac{\theta_0}{c_3} \left\{s - K_{12} - ab_3 h \sqrt{2} K_1(\alpha \sqrt{s})\right\}
$$

For the classical local thermal equilibrium case, the transformed pore pressure is given by

$$
p^{(\infty)}(r, s) = \left\{\frac{p_a}{s} + \frac{c'\theta_{0} k}{(c' - \kappa)(1 + ab_1 \sqrt{s}/k_{tm} \sqrt{K_0(r/\sqrt{s})})}\right\}
$$

$$
\times \frac{1}{2}\frac{K_0(r/\sqrt{s})}{K_0(\alpha \sqrt{s})} - \frac{c'\theta_{0} k}{(c' - \kappa)(1 + ab_1 \sqrt{s}/k_{tm} \sqrt{K_0(r/\sqrt{s})})}\frac{1}{2}\frac{K_0(r/\alpha \sqrt{s})}{K_0(\alpha \sqrt{s})}
$$

(34)

3.3. Thermal stresses

The basic equation governing the displacement for the axisymmetric problem can be obtained from Eq. (10) and the strain–displacement relations as follows:

$$(s + 2G)\left(\frac{\partial \bar{u}}{\partial s} + \frac{u}{s}\right) = 2p + K_2' \bar{\theta}_{ave}
$$

(35)

where $u$ is the radial displacement. The boundary conditions for the stress field are

$$
\sigma_r(r, t)|_{r=a} = -p_a
$$

$$
\sigma_r(r, t)|_{r=\infty} = 0
$$

Once the temperature and pore pressure are obtained in the time domain, the displacement and stresses can be calculated from the following expressions

$$
u(r, t) = \frac{a^2}{2G} \frac{\alpha}{r} p_a + \frac{1}{2G} \frac{1}{r^2} \left\{\alpha \int_{r}^{\infty} \bar{x}p(x, t) dx + Kx' \int_{a}^{\infty} \bar{x} \bar{\theta}_{ave}(x, t) dx\right\}
$$

(36a)

$$
\sigma_r(r, t) = \frac{a^2}{r^2} p_a - \frac{2G}{r^2} \frac{1}{2 + 2Gr^2} \left\{\alpha \int_{a}^{\infty} \bar{x} \bar{\theta}_{ave}(x, t) dx\right\}
$$

(36b)

$$
\sigma_o(r, t) = -\frac{2G}{r^2} \left\{\bar{x} \bar{p}(r, t) + \bar{K}_x' \bar{\theta}_{ave}(r, t)\right\} + 2G \frac{\bar{h}(r, t)}{r}
$$

(36c)

where $\bar{p}(r, t)$ and $\bar{\theta}_{ave}(r, t)$ can be obtained by numerical inversion of the Laplace transformed temperature and pore pressure given in Eqs. (24) and (32), respectively. This study is mainly concerned with thermally induced pore pressure and stresses which can be obtained from Eqs. (32) and (36) by taking $p_a = 0$.

3.4. Numerical results

Eqs. (24) and (32) give the temperature and pore pressure fields in the Laplace transformed domain. The temperatures and pore pressure in the time domain are obtained using a numerical inversion technique proposed by Durbin (1974) as the analytical inversions are not feasible. Denote by $F(s)$ the Laplace transform of $f(t)$. The inverse Laplace transform may be obtained using the following Durbin formula

$$
f(t) = \frac{a^2}{2G} \left\{\Re\{F(s)\} + \sum_{n=1}^{N} \left\{2 \Re\{F(s + n\pi i)\} \cos n\pi \frac{t}{T} - \Im\{F(s + n\pi i)\} \sin n\pi \frac{t}{T} \right\}\right\}
$$

(37)

where $i = \sqrt{-1}$. In general, accurate and convergent results can be obtained when the parameter $T$ is selected such that $T > t/2$ and $5 \leq T' \leq 10$.

In the numerical examples, a material representative of sandstone rock (material B) is considered. Table 1 lists the thermal properties of the solid and fluid phases and Table 2 lists the poroelastic parameters for the fluid saturated porous medium in the numerical study. Following McGee and Tigue (1986), the two volumetric thermal expansion coefficients for the solid are assumed the same, i.e., $\alpha_s = \alpha_f$. The thermal conductivity, mass density and specific heat of the porous medium are determined from the micromechanical consideration and a rule of mixtures as described in He and Jin (2010). The radius of the cylindrical hole is taken as $a = 0.1$ m. In all calculations, we assume that $b_3 = b_5$ in Eqs. (14) and (28), i.e., the Biot number associated with the porous medium in the classical thermo-poroelasticity is assumed to be the same as that associated with the solid. Moreover, the nondimensional solid–fluid interface heat transfer coefficient is taken as $h = 1.0$, where $h$ is given by

$$
h = \frac{a^2 h_{tot}}{(1 - \phi_f) b_3 k_{tm}}
$$

(38)

in which

$$
b_3 = \frac{1}{(1 - \phi_f)^2 + \phi_f (1 - \phi_f) \left\{2 + \frac{p_a}{K_2'} \right\} + \phi_f^2 \frac{a^2}{2G} \left\{1 + \frac{\phi_f (k_2 - k_{tm})}{K_2' (k_2 - k_{tm}) (1 - \phi_f) / 3}\right\}
$$

(39)

3.4.1. Temperature fields

Fig. 2a shows the weighted average temperature (normalized by $\theta_0$) of the LTNE theory along the radial direction at a nondimen-
the fluid temperature is higher than the solid temperature in a smaller region near the hole boundary. Finally, both the weighted average temperature of LTNE and the classical one at the hole boundary become closer to the normalized, prescribed temperature of 1.0. We note that when the Biot number approaches infinity (corresponding to an infinite heat transfer coefficient at the boundary), all temperatures at the boundary approach the prescribed temperature \( \theta_{\text{p}} \).

### 3.4.2. Thermally induced pore pressure

Fig. 3a shows the normalized, thermally induced LTNE and classical pore pressures (normalized by \( c' \theta_{\text{h}} / a^2 \)) along the radial direction at a nondimensional time of \( t = \kappa t / a^2 = 0.1 \) for a Biot number of \( Bi = 1.0 \). The thermally induced pore pressure first increases with increasing \( r/a \) reaches a peak value, and then decreases with further increases in the distance from the hole boundary. The LTNE peak pore pressure is significantly higher than that of the classical theory. For example, the peak LTNE pore pressure is about 15.0 which is approximately 2.0 times the classical peak pressure. Fig. 3b shows the normalized, thermally induced LTNE and classical pore pressures along the radial direction at the same nondimensional time but the Biot number is increased to \( Bi = 10.0 \). The pore pressures exhibit similar behavior to that for \( Bi = 1.0 \) observed in Fig. 3a. The normalized peak LTNE pressure, however, increases to about 36 which is more than twice the peak pressure for \( Bi = 1.0 \). Again, the peak LTNE pore pressure is significantly higher than the classical one.

### 3.4.3. Thermal stresses

Fig. 4a shows the normalized, thermally induced LTNE and classical radial stresses (normalized by \( c' h_0 / a^2 \)) along the radial direction at a nondimensional time of \( t = \kappa t / a^2 = 0.1 \) for a Biot number of \( Bi = 1.0 \). The magnitude of the normalized radial stress initially increases with increasing \( r/a \) reaches a peak value, and then decreases with further increasing distance from the hole. The magnitude of the LTNE peak radial stress is dramatically higher than that of the classical solution. For example, the magnitude of the peak LTNE stress is about 3.4 which is approximately 2.0 times the classical peak value. Fig. 4b shows the thermally induced LTNE and classical radial stresses along the radial direction at the same nondimensional time but for a larger Biot number of \( Bi = 10.0 \). The radial stresses exhibit similar behavior to that observed in Fig. 4a. The difference between the peak LTNE stress and the classical one becomes smaller compared with the result for \( Bi = 1.0 \).

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**Table 1**

Thermal parameters for the solid and fluid phases.

<table>
<thead>
<tr>
<th></th>
<th>Solid (material A)</th>
<th>Solid (material B)</th>
<th>Fluid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity ([W/(mK)])</td>
<td>(k_s = 3.3)</td>
<td>(k_s = 2.4)</td>
<td>(k_f = 0.6)</td>
</tr>
<tr>
<td>Density ([kg/m^3])</td>
<td>(\rho_s = 2600)</td>
<td>(\rho_s = 2600)</td>
<td>(\rho_f = 1000)</td>
</tr>
<tr>
<td>Specific heat ([J/(kgK)])</td>
<td>(c_s = 937)</td>
<td>(c_s = 920)</td>
<td>(c_f = 4200)</td>
</tr>
<tr>
<td>Coefficient of thermal expansion (volumetric) ([1/C])</td>
<td>(\alpha_s = 3.0 \times 10^{-5})</td>
<td>(\alpha_s = 3.3 \times 10^{-5})</td>
<td>(\alpha_f = 3.0 \times 10^{-4})</td>
</tr>
</tbody>
</table>

**Table 2**

Poroelastic parameters for the fluid-saturated porous media.

<table>
<thead>
<tr>
<th>Material A</th>
<th>Material B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(clay)</td>
<td>(sandstone)</td>
</tr>
<tr>
<td>Reference porosity</td>
<td>(\phi_0 = 0.375)</td>
</tr>
<tr>
<td>Shear modulus ([MPa])</td>
<td>(G = 1.2)</td>
</tr>
<tr>
<td>Poisson’s ratio (drained) ([\cdot])</td>
<td>(\nu = 0.2)</td>
</tr>
<tr>
<td>Bulk modulus (drained) ([MPa])</td>
<td>(K = 1.6)</td>
</tr>
<tr>
<td>Biot-Willis coefficient ([\cdot])</td>
<td>(\alpha = 1.0)</td>
</tr>
<tr>
<td>Shermpton’s coefficient ([\cdot])</td>
<td>(B = 1.0)</td>
</tr>
<tr>
<td>Intrinsic permeability ([m^2])</td>
<td>(k = 5.0 \times 10^{-17})</td>
</tr>
<tr>
<td>Diffusivity coefficient ([m^3/s])</td>
<td>(c = 1.6 \times 10^{-17})</td>
</tr>
<tr>
<td>Pore pressure–temperature coupling coefficient ([Pa/C])</td>
<td>(c' = 3.72 \times 10^2)</td>
</tr>
</tbody>
</table>

---

**Fig. 2a.** Normalized temperature distributions along the radial direction around a cylindrical hole \((Bi = 1.0)\).

**Fig. 2b.** Normalized temperature distributions along the radial direction around a cylindrical hole \((Bi = 10.0)\).
However, the peak LTNE radial is still substantially higher than the classical one.

Fig. 5a shows the normalized, thermally induced LTNE and classical tangential stresses (normalized by \( c^0c \times 10^{-2} \)) along the radial direction at a nondimensional time of \( t = 0.1 \) for a Biot number of \( Bi = 1.0 \). The magnitude of the tangential stress monotonically decreases with increasing distance from the hole when \( r/a < 1.5 \). The tangential stresses are compressive near the hole boundary and become tensile when \( r/a > 1.5 \). The maximum magnitude of the LTNE stress is about 21 which significantly exceeds the corresponding classical stress of about 11.5. Fig. 5b shows the thermally induced LTNE and classical tangential stresses along the radial direction at the same nondimensional time but for a larger Biot number of \( Bi = 10.0 \). Now the difference between the peak LTNE stress and the classical one becomes smaller compared with the result for \( Bi = 1.0 \).

The numerical results above show that the LTNE effects can substantially increase the thermally induced pore pressure and stresses, particularly under boundary conditions with moderate Biot numbers. When a porous medium is subjected to rapid heating or cooling, the solid and fluid initially undergo different temperature variations due to the difference in thermal diffusivity between the solid and fluid, and the finite heat transfer coefficient between the two phases at the microscopic level. The differential temperature variations are further exacerbated by the difference in the heat transfer coefficient at the boundary between the solid and fluid. As a result, the effective temperature field at the macroscopic level in the porous medium deviates from that in the classical local thermal equilibrium theory in which the temperatures of the solid and fluid are assumed to be the same. The differences in the temperature and its (temporal and spatial) gradients induce additional pore pressure and thermal stresses which become significant when the Sparrow number is not very large. In the present numerical study, the equivalent Sparrow number in terms of \( \bar{h} \) has an order of \( \bar{h}b^3 \) which is 0.5409 for the rock (material B) indicating significant LTNE effects. We note that the effective stresses are approximately the same as the corresponding stresses because the pore pressure is three orders of magnitude smaller than the stresses.

4. A spherical cavity in an infinite porous medium

Next we consider a spherical cavity in an infinite porous medium as shown in Fig. 6, where \( r \) is the radial coordinate and \( a \) is the radius of the cavity. The boundary of the cavity is assumed to be impermeable and fixed. This is a fundamental problem for nuclear waste disposal applications in which metal canisters containing nuclear waste are placed in a deep geologic vault (Zhou et al., 1998b).

Fig. 3a. Normalized thermally induced pore pressure distributions along the radial direction around a cylindrical hole (\( Bi = 1.0 \)).

Fig. 3b. Normalized thermally induced pore pressure distributions along the radial direction around a cylindrical hole (\( Bi = 10.0 \)).

Fig. 4a. Normalized thermally induced radial stresses along the radial direction around a cylindrical hole (\( Bi = 1.0 \)).

Fig. 4b. Normalized thermally induced radial stresses along the radial direction around a cylindrical hole (\( Bi = 10.0 \)).

4.1. Temperature field

He and Jin (2011) considered this problem using the same boundary temperature for both solid and fluid phases, which is
consistent with the study of classical thermo-poroelasticity (Zhou et al., 1998b). For nuclear waste storage applications, however, a convective condition for the fluid and a prescribed temperature condition for the solid phase may be more appropriate if we assume the canisters and the surrounding geological material are in perfect contact. The initial and boundary conditions for the temperatures thus can be formulated as follows:

\begin{align}
\theta_i(r, t)|_{t=0} &= \theta_i(r, t)|_{t=0} = 0, \quad a \leq r < \infty \quad (40) \\
\theta_i(r, t)|_{r=0} &= \theta_i(r, t)|_{r=0} = 0, \quad t > 0 \quad (41)
\end{align}

where \( \theta_i \) is a constant (temperature of the nuclear waste source) and \( h_{ij} \) is the heat transfer coefficient. Here the temperature decay of the nuclear waste is not considered because the LTNE effects are most significant at relatively short times.

By using the Laplace transform, we can obtain the following transformed temperatures for the solid and fluid

\begin{align}
\tilde{\theta}_i(r, s) &= \frac{sC_2}{C_5} e^{-\sqrt{s}h} + \frac{C_2}{C_5} \frac{r}{h} e^{-\sqrt{s}h} \quad (42a) \\
\tilde{\theta}_j(r, s) &= C_4 \left( 1 + \frac{s - \kappa \lambda_1}{h} \right) \frac{1}{r} e^{-\sqrt{s}h} \quad (42b)
\end{align}

where \( \lambda_1 \) and \( \lambda_2 \) are given in Eqs. (20) and (21), and \( c_4 \) and \( c_5 \) are given by

\begin{align}
c_4 &= \frac{\theta_0}{s} \frac{a^2 e^{\sqrt{s}/2}}{C_9} \left[ s - \kappa \lambda_2 + b_q(1 + a \sqrt{s}/2)(h + s - \kappa \lambda_2) \right] \\
c_5 &= -\frac{\theta_0}{s} \frac{a^2 e^{\sqrt{s}/2}}{C_9} \left[ s - \kappa \lambda_1 + b_q(1 + a \sqrt{s}/2)(h + s - \kappa \lambda_1) \right]
\end{align}

in which \( b_q \) and \( C_9 \) are given by

\begin{align}
b_q &= \frac{k_f}{\rho_f C_p} \quad (44) \\
c_s &= \kappa (\lambda_1 - \lambda_2) - b_q(1 + a \sqrt{s}/2)(h + s - \kappa \lambda_1) + b_q(1 + a \sqrt{s}/2)(h + s - \kappa \lambda_2) \quad (45)
\end{align}

Substituting Eq. (43) into Eqs. (42a, b) gives the following transformed temperatures of solid and fluid

\begin{align}
\tilde{\theta}_i(r, s) &= \frac{s C_2}{C_5} \left[ s - \kappa \lambda_2 + b_q(1 + a \sqrt{s}/2)(h + s - \kappa \lambda_2) \right] \frac{a}{r} e^{-\sqrt{s}h} \\
&\quad - \frac{\theta_0}{s} \frac{a}{C_9} \left[ s - \kappa \lambda_1 + b_q(1 + a \sqrt{s}/2)(h + s - \kappa \lambda_1) \right] \frac{a}{r} e^{-\sqrt{s}h} \quad (46a) \\
\tilde{\theta}_j(r, s) &= \frac{s C_2}{C_5} \left[ s - \kappa \lambda_2 + b_q(1 + a \sqrt{s}/2)(h + s - \kappa \lambda_2) \right] \left( 1 + \frac{s - \kappa \lambda_2}{h} \right) \frac{a}{r} e^{-\sqrt{s}h} \\
&\quad - \frac{\theta_0}{s} \frac{a}{C_9} \left[ s - \kappa \lambda_1 + b_q(1 + a \sqrt{s}/2)(h + s - \kappa \lambda_1) \right] \left( 1 + \frac{s - \kappa \lambda_1}{h} \right) \frac{a}{r} e^{-\sqrt{s}h} \quad (46b)
\end{align}

For the classical local thermal equilibrium case (\( \theta_i = \theta_j = 0 \)), either a prescribed temperature or a convective boundary condition (for consideration of heat transfer between the pore fluid and solid canister) may be used. The transformed classical temperature under the convective boundary condition (CBC) can be obtained as follows:

\begin{align}
\tilde{\theta}(r, s) &= \frac{s a}{s + b_q(1 + a \sqrt{s}/2)} \frac{a}{r} e^{-\sqrt{s}/\kappa} \quad (47a)
\end{align}

where \( \kappa \) is the thermal diffusivity of the porous medium and \( b_q \) is a constant given in Eq. (28) and is the inverse of the Biot number associated with the convective heat transfer between the solid canister and the surrounding fluid saturated porous medium. The classical temperature under the prescribed temperature boundary condition (PTBC) is given by

\begin{align}
\theta(r, t) &= \theta_0 \frac{a}{r} \text{erfc} \left( \frac{r - a}{2 \sqrt{\kappa t}} \right) \quad (47b)
\end{align}

where \( \text{erfc}(\cdot) \) is the complementary error function.

### 4.2. Pore pressure

For nuclear waste storage applications, the wall of the cavity can be assumed to be impermeable, i.e., no fluid flow occurs between the waste storage vault and the surrounding medium (Zhou et al., 1998b).
et al., 1998b). The initial and boundary conditions for pore pressure diffusion are thus given by

\[ p(r, t)|_{t=0} = 0, \quad a \leq r < \infty \]  

\[ \frac{\partial p(r, t)}{\partial t} |_{t=a} = 0, \quad t > 0 \]  

\[ p(r, t)|_{r \to \infty} = 0, \quad t > 0 \]  

Using the Laplace transform, the transformed pore pressure can be obtained by solving Eq. (31) and using the boundary conditions Eqs. (48) and (49), and the transformed temperature fields Eq. (46) as follows:

\[ \hat{p}(s, r) = \left\{ \begin{array}{ll} c m_2(s) & \left( 1 + a \sqrt{\frac{s}{2}} \right) \left( 1 + a \sqrt{\frac{s}{c}} \right) \frac{a}{r} e^{-\left( a \sqrt{s/2} \right)} \\
& \left( c \frac{a}{r} e^{-\left( a \sqrt{s/2} \right)} \right) \\
& \left( c - k \right) \left[ 1 + b_s \left( 1 + a \sqrt{s/c} \right) \right] \end{array} \right. \]  

\[ \hat{m}_2(s) = \frac{a}{r} e^{-\left( a \sqrt{s/2} \right)} \left( c - k \right) \left[ 1 + b_s \left( 1 + a \sqrt{s/c} \right) \right] \]  

\[ \hat{m}_4(s) = \frac{a}{r} e^{-\left( a \sqrt{s/2} \right)} \left( c - k \right) \left[ 1 + b_s \left( 1 + a \sqrt{s/c} \right) \right] \]  

(50)

where

\[ m_2(s) = \frac{a}{r} e^{-\left( a \sqrt{s/2} \right)} \left( c - k \right) \left[ 1 + b_s \left( 1 + a \sqrt{s/c} \right) \right] \]  

\[ m_4(s) = \frac{a}{r} e^{-\left( a \sqrt{s/2} \right)} \left( c - k \right) \left[ 1 + b_s \left( 1 + a \sqrt{s/c} \right) \right] \]  

(51a)

For the classical local thermal equilibrium case, the transformed pore pressure under the convective boundary condition can be obtained as follows:

\[ \hat{p}^{\text{lin}}(s, r) = \left\{ \begin{array}{ll} c \theta_0 k & \left( 1 + a \sqrt{s/k} \right) \left( 1 + a \sqrt{s/c} \right) \frac{a}{r} e^{-\left( a \sqrt{s/2} \right)} \\
& \left( c - k \right) \left[ 1 + b_s \left( 1 + a \sqrt{s/c} \right) \right] \end{array} \right. \]  

(51b)

The classical transformed pore pressure under the prescribed temperature boundary condition is given by

\[ \hat{p}^{\text{lin}}(s, r) = \frac{c \theta_0 k}{c - k} \left( 1 + a \sqrt{s/k} \right) \frac{a}{r} e^{-\left( a \sqrt{s/2} \right)} - \frac{a}{r} e^{-\left( a \sqrt{s/2} \right)} \]  

(51c)

4.4. Numerical results

In the numerical examples, a material representative of clay (material A) is considered. This material was investigated by Zhou et al. (1998a,b) in nuclear waste storage applications. The thermal properties of the solid and fluid phases are listed in Table 1 (from Zhou et al., 1998b) and the poroelastic parameters for the fluid saturated porous medium are listed in Table 2. In all calculations, we assume that \( b_s = b_t \) in Eqs. (44) and (28), i.e., the Biot number associated with the porous medium in the classical thermo-poroelasticity is assumed to be the same as that associated with the pore fluid in the LTNE theory. Moreover, the nondimensional solid-fluid interface heat transfer coefficient is also taken as \( \tilde{h} = 1.0 \) as for the cylindrical hole problem. Hence, the Sparrow number is now 0.3553 for the clay also indicating significant LTNE effects. Only the results for a nondimensional time \( f = \kappa t/\alpha^2 = 0.1 \) (the actual time \( t = 51,952 \) s) are presented as the responses are similar at other short times at which the LTNE effects are significant. Finally, the radius of the cavity is taken as \( a = 0.5 \) m.

Figs. 7a and 7b show the normalized average temperature (normalized by \( \theta_0 \)) of the LTNE theory along the radial direction. The Biot number Bi is taken as 1.0 and 10.0, respectively. The corresponding normalized temperatures for the solid and fluid, as well as those in the classical heat transfer theory under both prescribed temperature (PTBC) and convective boundary conditions (CBC) are also included. The normalized solid temperature is always higher than the fluid temperature as the solid temperature at the hole boundary immediately attains the prescribed boundary temperature (perfect contact between the canister and clay) but the pore fluid temperature increases gradually due to convection. Moreover, the thermal diffusivity of the solid is much larger than that of the fluid, which also contributes to higher solid temperature in the medium. The weighted temperature of LTNE is always higher than that of the classical theory under the convective boundary condition at a given radial position although the difference between the weighted average temperature of LTNE and the classical one becomes smaller for the larger Biot number shown in Fig. 7b. Compared with the classical temperature under prescribed temperature boundary conditions, the LTNE temperature is lower near the cavity boundary (because the LTNE temperature is lower at the boundary by convective heat transfer) but becomes higher at radial distances of \( r/a > 1.5 \) (because the thermal diffusivity of the solid is higher than that of the porous medium and therefore thermal diffusion in the solid is faster). The temperature responses for the spherical cavity problem differ from those for the cylindrical hole problem in Section 3 wherein the convective boundary conditions apply to the solid phase of the porous medium.

Figs. 8a and 8b show the normalized, thermally induced LTNE pore pressure (normalized by \( c \theta_0 \times 10^{-2} \)) along the radial direction. The classical pore pressures under both convective and prescribed temperature boundary conditions are also given. The
thermally induced pore pressure decreases monotonically with increasing distance from the cavity boundary. For a Biot number of $\text{Bi} = 1.0$, the maximum normalized LTNE pore pressure of 42 now occurs at the cavity boundary and is about 3.5 times the value of the classical theory using the convective boundary condition as shown in Fig. 8a. The peak LTNE pore pressure, however, is smaller than the classical one under the prescribed temperature boundary conditions. We note that in the prescribed temperature boundary condition the heat transfer coefficient at the boundary is assumed to be infinity, which leads to significantly higher pore pressure and stresses. For a Biot number of $\text{Bi} = 10.0$ shown in Fig. 8b, the differences between the maximum LTNE and classical pore pressures become smaller but still remain significantly large. We note that the pore pressure response for the clay is different from that for the sandstone in Section 3. This is because there is a four orders of magnitude difference in pore pressure diffusivity coefficient $c$ between the two materials as shown in Table 2.

Figs. 9a and 9b show the normalized, thermally induced LTNE and classical radial stresses (normalized by $c'\theta_{a} x 10^{-2}$) along the radial direction. The magnitude of the normalized radial stress initially increases with increasing $r/a$, reaches a peak value, and then decreases with further increasing distance from the hole. The magnitude of the LTNE peak radial stress is dramatically higher than that of the classical solution under the convective boundary conditions but is smaller than the classical one under the prescribed temperature boundary conditions. For example, the magnitude of the peak LTNE stress is about 13.6 which is approximately 3.8 times the classical peak value for a Biot number of $\text{Bi} = 1.0$ (Fig. 9a). It is noted that although the nondimensional pore pressure for the clay is higher than that for the sandstone in Section 3, the pore pressure for the clay is actually much lower because the value of $c'$ used for normalization for the clay is three orders of magnitude smaller than that of the sandstone as shown in Table 2.

Figs. 10a and 10b show the normalized, thermally induced LTNE and classical tangential stresses (normalized by $c'\theta_{a} x 10^{-2}$) along the radial direction. Near the cavity boundary, the magnitude of the tangential stress monotonically decreases with increasing distance from the hole. The tangential stresses are compressive near the cavity boundary and become tensile at larger distances from the cavity. The magnitude of the compressive stress, however, is much larger than the tensile one. For $\text{Bi} = 1.0$ (Fig. 10a), the maximum magnitude of the LTNE stress is about 38 which significantly exceeds the corresponding classical stress of about 11 under the convective boundary conditions.

Figs. 11a and 11b show the normalized, thermally induced effective LTNE and classical radial and tangential stresses (normalized by $c'\theta_{a} x 10^{-2}$) along the radial direction, respectively, for a Biot number of $\text{Bi} = 1.0$. The effective stresses are defined as
\[ \sigma'_r = \sigma_r + 2\rho \]
\[ \sigma'_\psi = \sigma_\psi + 2\rho \]  

It can be seen from Fig. 11a that the effective radial stresses are tensile near the cavity boundary with the peaks occurring at the hole boundary. The peak LTNE effective stress is higher than the classical.
one under the same convective boundary condition but smaller than the classical stress under the prescribed temperature boundary condition. Fig. 11b shows that the effective tangential stresses are all tensile with the peaks occurring near the hole boundary. Again, the LTNE peak is higher than the classical one if the same convective boundary condition is used.

5. Concluding remarks

In this paper, we employ the LTNE thermo-poroelasticity equations (He and Jin, 2010) to investigate the temperatures, pore pressure and thermal stresses in a fluid saturated porous medium subjected to convective cooling/heating boundary conditions. The temperatures, pore pressure and thermal stresses are obtained using the Laplace transform method for a cylindrical as well as a spherical cavity in an infinite porous solid. Numerical results show that the LTNE effects become more pronounced when the convective boundary conditions are employed. The peak thermally induced pore pressure and the magnitude of thermal stresses may reach several times the corresponding peak values in the classical poroelasticity. This is particularly true under convective boundary conditions with moderate Biot numbers and can not be observed when the prescribed temperature boundary conditions are employed.

In general, the LTNE theories should be used to evaluate thermally induced pore pressure and stresses in a fluid-saturated porous medium subjected to convective cooling/heating boundary conditions when the Sparrow number is not very large. The Sparrow number proposed by Minkowycz et al. (1999) is given by

$$Sp = \frac{\phi_0 h L^2 \Delta A_p}{k_e \Delta V_p}$$

where $h$ is the solid/fluid interface heat transfer coefficient, $L$ is a characteristic length parameter, $k_e$ is the effective thermal conductivity of the porous medium, $\phi_0$ is the porosity, and $\Delta V_p$ and $\Delta A_p$ are the mean volume and surface area of the pores, respectively. Consider a porous medium with spherical pores of radius 10 $\mu$m, a porosity of 0.2, and an effective thermal conductivity of 3 W/(m$^2$K). The Sparrow number now becomes

$$Sp = 2hL^2 \times 10^4$$

which is large in most geomechanical applications. For problems involving fracture initiation at a micro-crack tip, $Sp$ may become small and LTNE effects could become quite significant (for example, $Sp$ is on the order of 2 for a crack length of $L = 1000 \mu$m and an interface heat transfer coefficient of $h = 100$ W/(m$^2$K)). While this work does not consider crack initiation, the solutions obtained are needed in calculating the stress intensity factors at the crack tips.

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References


