INFORMATION AND CONTROL 4, 95–96 (1961)

Errata

Corrections for: "Symbolic Analyses of a Decomposition of Information Processing Machines," by J. Hartmanis, Volume **3**, Number 2, pp. 154–178, June 1960.

In the last paragraph on p. 166 it is stated: "Furthermore, it can be seen that π and π^1 are permutable." This is not correct, the two partitions do not have to permute. Note that no use has been made of this incorrect statement in this paper. The mistake was pointed out by M. B. Freeman, University of California and J. Mezei, Massachusetts Institute of Technology.

On p. 170, in the middle paragraph, it is described how to compute a partition with the substitution property. It should be stated explicitly that at each stage of this computation we must add all the identifications which are induced by the transitive law: "if S_i is identified with S_j and S_j with S_k, then S_i and S_k must be identified."

For a related application of these ideas and examples, see "On the State Assignment Problem for Sequential Machines I," by J. Hartmanis, to be published in the *IRE Trans. on Electronic Computers* (June, 1961).

Corrections for, and additional remarks to, "Generalization of the Sampling Theorem," by D. A. Linden and N. M. Abramson, Volume **3**, Number 1, pp. 26–31, March 1960.

The coefficients of Eq. (1) and (3) are incorrect. Equation (1) should read

$$\hat{\chi}(t) = \sum_{k=-\infty}^{\infty} \left[\xi(k\tau) + (t - k\tau)\xi^{(1)}(k\tau) + \frac{(t-k\tau)^2}{2!}\xi^{(2)}(k\tau) + \cdots + \frac{(t-k\tau)^R}{R!}\xi^{(R)}(k\tau) \right] \left[\operatorname{sinc} \frac{t - k\tau}{\tau} \right]^{(R+1)} \tag{1'}$$

The $\xi^{(j)}(k\tau)$ are linear combinations of the $\chi^{(j)}(k\tau)$:

$$\xi^{(j)}(k\tau) = \sum_{i=0}^{j} \binom{j}{i} \left(\frac{\pi}{\tau}\right)^{j-i} \Gamma_{R+1}^{(j-i)} \chi^{(i)}(k\tau) \tag{2'}$$

where

$$\Gamma_{\alpha}^{(\beta)} = \frac{d^{\beta}}{dt^{\beta}} (t/\sin t)^{\alpha}]_{t=0} \tag{3'}$$

The $\Gamma_{\alpha}^{(\beta)}$ may be expressed in terms of generalized Bernoulli numbers (Nörlund, 1922). From Nörlund, we obtain

$$\Gamma_{\alpha}^{(0)} = 1; \qquad \Gamma_{\alpha}^{(2)} = \alpha/3; \qquad \Gamma_{\alpha}^{(4)} = \alpha(5\alpha + 2)/15;$$

$$\Gamma_{\alpha}^{(6)} = \alpha(35\alpha^2 + 42\alpha + 16)/63; \qquad \Gamma_{\alpha}^{(\beta)} = 0 \quad \text{for odd } \beta.$$

Equation (2′) may be obtained by multiplying both sides of (1′) by $\{\mathrm{sinc}\,[(t - k\tau)/\tau]\}^{-(R+1)}$ and equating their jth derivatives at $t = k\tau$. The proof contained in Appendix A of the original paper is unaffected by the above correction.

Rearrangement of terms in (1′) yields an alternate form which emphasizes the $\chi^{(j)}(k\tau)$ rather than their linear combinations. In addition this alternate form relates the R-derivative sampling theorem to the Taylor Series. For simplicity we assume that all terms corresponding to $k \neq 0$ vanish so that we may examine the expansion about a single point. We also introduce the "truncated-Taylor" operator $T_j[\cdot]$, defined by

$$T_j[f(t)] = f(0) + tf'(0) + \frac{t^2}{2} f''(0) + \cdots + \frac{t^j}{j!} f^{(j)}(0) \tag{4'}$$

where the indicated derivatives exist. Then (1′) may be written as

$$\hat{\chi}(t) = \left(\mathrm{sinc}\,\frac{t}{\tau}\right)^{R+1} \sum_{j=0}^{R} \frac{t^j}{j!} \chi^{(j)}(0) T_{R-j}\left[\left(\mathrm{sinc}\,\frac{t}{\tau}\right)^{-R-1}\right] \tag{5'}$$

As R increases, (5′) takes the form of a Taylor series whose terms are multiplied by the weighting functions

$$\left(\mathrm{sinc}\,\frac{t}{\tau}\right)^{R+1} T_{R-j}\left[\left(\mathrm{sinc}\,\frac{t}{\tau}\right)^{-R-1}\right]$$

References

Linden, D. A. and Abramson, N. M. (1960). A generalization of the sampling theorem. *Information and Control* **3**, 26–31.

Nörlund, N. E. (1922). Mémoire sur les Polynômes de Bernoulli. *Acta Math.* **43**, 121–196.