ELSEVIER

Available online at www.sciencedirect.com

SCIENCE DIRECT•

Computers and Mathematics with Applications 50 (2005) 425–432

An International Journal
computers & mathematics with applications

www.elsevier.com/locate/camwa

Properties of Optimal Survivable Paths in a Graph

Z. R. BOGDANOWICZ
Armament Research, Development and Engineering Center (ARDEC), Picatinny, NJ 07806, U.S.A.

R. RAMAMURTHY
CIENA Corporation, 5965 Silver Creek Valley Road
San Jose, CA 95138, U.S.A.

(*Received January 2004; revised and accepted April 2005*)

Abstract—We introduce the concept of a survivable path in an undirected graph G. A survivable path between a pair of vertices in G is a pair of edge-disjoint paths consisting of a working path and a redundant protection path. Protection paths share edges in such a manner as to provide guaranteed recovery upon the failure of any single edge. Survivable paths play an important role in the design of survivable communication networks. We demonstrate several results on the properties of the optimal set of survivable paths. © 2005 Elsevier Ltd. All rights reserved.

Keywords—Undirected graph, Shortest paths, Survivability, Routing optimization, Diverse routing, Multicommodity flow, Optical network.

1. INTRODUCTION

In this paper, we consider simple undirected graphs (i.e., undirected graphs without parallel edges or loops) [1]. We introduce the concept of survivable paths that should have significant importance in the design of survivable communication networks. In fact, this topic has been inspired by work on real problems encountered in optical networks in the industry [2]. The problem of finding an optimal set of survivable paths in a graph is related to the multicommodity network flow problem that is known to be *NP-complete* [3,4]. More specifically, finding diverse paths with shared back-up is NP-complete [5], finding diverse paths with complex shared risk groups (SRGs) is NP-complete [5], and finding diverse paths with shared back-up and the shortest primary path is NP-complete [6]. The reason for this is the sharing capability of survivable paths that we describe and define later. Furthermore, there is not always an on-line sequence that achieves the optimal solution [7]. In this paper, we focus on the problem of finding properties of the given optimal set of survivable paths.

Communications networks can be modeled as an undirected simple graph $G = (V, E)$ with $V(G)$ vertices and $E(G)$ edges, where the vertices represent switching stations, and edges represent capacitated links. Circuits are routed through the communication networks along corresponding paths on the undirected graph model. Failures of switching stations or links on the communication network are modeled as failures of vertices and edges on the graph model. In a survivable

0898-1221/05/$ - see front matter Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX
doi:10.1016/j.camwa.2005.04.011

communication network, circuits can survive the failure of links or nodes. In this paper, we consider single-link failures only, since this is the dominant failure mode in a communication network. Upon a single-link failure, all circuits that traverse the failed link are considered to have failed.

An approach to providing survivable communications is to provide redundant paths for each circuit. Each circuit has a working path and a redundant protection path. The bandwidth on the working path is used under normal operating conditions, and the bandwidth on the protection path is idle. Further, the bandwidth on the backup paths may be shared among multiple circuits on a per-link basis, such that the single-link failure guarantee is met. That is, if two or more working paths don't use any common link in the network, then we can assign shared links for the corresponding protection paths. So, if two or more restoration paths would use the same links then the cost of these links per path would proportionally decrease. The working path and the protection path are edge-disjoint so that a single link failure will not affect them simultaneously. Upon a failure of a link, failed circuits are rerouted along their protection paths.

A survivable path, $S = (W, P)$, in graph G is an ordered pair of working and protection paths (W, P) between the pair of vertices, and further, W and P do not have any edges in common. Survivable paths may be defined between arbitrary pairs of vertices and multiple survivable paths may be defined between the same pair of vertices. Some related work has been done in the field of survivable paths [8,9].

The cost of a survivable path, C, is expressed as the sum of the number of used but not shared edges on the working or protection paths plus the cost of edges that are shared with other survivable paths. If two working paths $W_i = w_i^1, w_i^2, \ldots, w_i^a$ and $W_j = w_j^1, w_j^2, \ldots, w_j^b$ are diverse (i.e., no edge $w_i^k w_i^{k+1}$ in W_i equals an edge $w_j^l w_j^{l+1}$ in W_j), and their protection paths use the same edges, then both protection paths share the cost of common edges. Similarly, if k working paths are pair-wise diverse then the corresponding protection paths share the cost of common edges. That is, if i protection paths share edge j, then edge j contributes cost c_j, where $c_j = 1/i$, to the cost of each corresponding survivable path. Let $|E'(W_i)|$, $|E'(P_i)|$, represent the number of all unshared working and protection edges in the working and protection paths, respectively, of the i^{th} survivable path. Let $E''(P_i)$ represent all shared edges used by P_i, and let c_e be the cost of sharable edge e for P_i. Then, the cost of i^{th} survivable path is expressed by,

$$C_i = |E'(W_i)| + |E'(P_i)| + \sum_{e \in E''(P_i)} c_e .$$

We define an *optimal set of survivable paths* in G to be a set of survivable paths $\{S_1, S_2, \ldots, S_k\}$ that minimizes $\sum_{i=1}^{k} C_i$.

We now introduce a few additional definitions. A working path for a survivable path is called a *direct working path* if it follows on a shortest-hop path between the end-vertices Two-edge-connected graph G is *k-optimal* if for any combination of paths defined between end-vertices of distance not less than k in G, there exists an optimal set of survivable paths defined between the same end-vertices with all direct working paths. We also will call two survivable paths *distinct* if they are not defined between the same end-vertices.

In this paper, we show several properties of the optimal set of survivable paths. We characterize working and backup paths for optimal set of survivable paths defined for certain kinds of graphs. First, we show that if all survivable paths are defined on k pairs of adjacent vertices, then, an optimal set of survivable paths must include k distinct direct working paths. Then, we show that on a certain set of survivable paths defined on a graph, an optimal set of survivable paths includes all direct working paths, and all the protection edges lie on a spanning tree. If the graph is Hamiltonian, then on a certain set of survivable paths, an optimal set of survivable paths includes all direct working paths, and all the protection edges lie on the Hamiltonian cycle. Finally, we define two conditions for a graph to be two-optimal.

2. OPTIMAL SET OF SURVIVABLE PATHS IN ARBITRARY GRAPHS

We first investigate the properties of survivable paths defined on an arbitrary two-edge connected graph.

THEOREM 1 *Let $G = (V, E)$ be a two-edge connected graph with all m survivable paths defined between k ($m = k$) pairs of adjacent vertices. Then, there exists an optimal set of survivable paths that contains k distinct direct working paths.*

PROOF. Consider an optimal set of survivable paths in G. Let $\langle x_a, x_b\rangle$ denote a sequence of consecutive intermediate vertices of a path between vertices x_a and x_b, and assume that it contains at least one such vertex. Suppose that an optimal set of survivable paths contains paths $W = x_a \langle x_a, x_b\rangle x_b$, $P = x_a \langle x_a, x_b\rangle' x_b$, but doesn't contain a working or protection path of type $x_a x_b$, for some pair of vertices x_a, x_b, ($x_a \neq x_b$). Then, we can use the following transformation of paths that does not worsen the solution. We replace every protection path that shares an edge with P and whose corresponding working path uses edge (x_a, x_b) with a direct one-hop path. If there are m such paths then the cost of survivable paths will increase by at most m. Otherwise, the cost will remain at most m greater from the original cost. Note that these steps do not violate diversity of the individual paths. Now, we swap every previously replaced protection path except for P with its corresponding working path. Thus, m new protection paths will share edge (x_a, x_b) with each other and the cost will decrease to original cost plus one. Finally, we replace working path $W = x_a\langle x_a, x_b\rangle x_b$ with $W' = x_a x_b$ that will also be diverse with P. This last operation will decrease the cost by exactly one and will not violate diversity of individual paths. Thus, our transformation decreases the number of adjacent nodes that do not contain direct either working or protection path and does not increase the cost of a solution. Hence, we need now to consider only adjacent vertices x_a, x_b for which paths $W = x_a \langle x_a, x_b\rangle x_b$ and $P = x_a x_b$ exist, but path $W' = x_a x_b$ doesn't exist.

If no other protection path in G shares edge (x_a, x_b) with P, then by swapping W with P we obtain a feasible solution that is not worse than the original solution. Consider then P that shares (x_a, x_b) with at least one other protection path. In this case, we use the following transformation. We first swap W with P that produces a feasible solution containing working path $W' = x_a x_b$ and new protection path $P' = x_a \langle x_a, x_b\rangle' x_b$. Such a swap might worsen a solution by at most one unit of cost. That is, path P' in the worst case will cost exactly the same as original path W, and path W' will add one unit of cost to the solution. Now, we substitute every protection path of form $P_{ij} = x_i \dots x_a x_b \dots x_j$ that shared edge (x_a, x_b) with P, with protection path $P'_{ij} = x_i \dots x_a \langle x_a, x_b\rangle' x_b \dots x_j$. We observe that $x_a \langle x_a, x_b\rangle' x_b$ cannot violate diversity with W_{ij}, because otherwise P_{ij} couldn't be shared with P. Since we divert all such shared paths then the cost of solution will decrease by one unit of cost. So, our transformation doesn't worsen the cost of a solution. Hence, for all survivable paths with direct protection paths we can apply either direct swapping of W with P or the above transformation, producing new survivable paths that don't worsen a solution. Consequently, we are left with at least k distinct direct working paths contained in an optimal set of survivable paths. ∎

COROLLARY 2. *Let $G = (V, E)$ be a two-edge-connected graph with all survivable paths defined between adjacent vertices and at most one survivable path defined for any pair of adjacent vertices. Then, there exists an optimal set of survivable paths that contains all direct working paths.*

PROOF. Follows directly from Theorem 1 by letting $m = k$. ∎

Let $H_G = (V, E)$ be a graph derived from G as follows. Graph H_G consists of the same set of vertices as graph G, and in addition it consists of every edge (v_i, v_j), where v_i, v_j are the end-vertices of a survivable path in G. Define $S(G)$ to be a subgraph of G, consisting only of all edges and corresponding vertices of working paths in an optimal set of survivable paths of G. Let $S'(G)$ be a complement of $S(G)$ in respect to G, (i.e., $V(S'(G)) = V(S(G))$, each edge e in

$E(S'(G))$ is such that e is not in $E(S(G))$, and $e \in E(G)$). If $S'(G)$ is connected let $T'(G)$ be a spanning tree of $S'(G)$.

THEOREM 3. *Let $G = (V, E)$ be a two-edge-connected graph having all mutually diverse direct working paths in an optimal set of survivable paths. Let $S'(G)$ be a connected spanning subgraph of G corresponding to the mutually diverse direct working paths and let H_G be a connected graph. Then, there exists an optimal set of survivable paths consisting of all mutually diverse direct working paths and all protection paths lying on a single spanning tree $T'(G)$.*

PROOF. All mutually diverse direct working paths in G define $S(G)$. Since H_G is connected, therefore, there must be at least $|V| - 1$ protection edges in G. Since $S'(G)$ is connected spanning subgraph it contains a spanning tree $T'(G)$. Furthermore, if $T'(G)$ consists of all protection edges then, for every working path in $S(G)$, there corresponds a diverse protection path in $T'(G)$. Since $T'(G)$ consists of $|V| - 1$ edges, therefore, all protection edges lie on $T'(G)$ and the proof is complete. ∎

If G is Hamiltonian let $C(G)$ be a Hamiltonian cycle in G.

THEOREM 4. *Let $G = (V, E)$ be a Hamiltonian graph with at most one survivable path defined over any pair of adjacent vertices. Let $S'(G)$ be a disconnected graph, and let H_G be a connected graph. Then, there exists an optimal set of survivable paths consisting of all direct working paths and all protection paths lying on a single Hamiltonian cycle $C(G)$.*

PROOF. Since G is Hamiltonian, then it's two-edge-connected. By Theorem 1, consider all working paths as direct one-hop paths. So, all working paths in G are mutually diverse. All edges in $S(G)$ correspond to direct working paths. Since H_G is a connected then protection edges must contain a spanning tree. Because $S'(G)$ is not connected then at least one edge in $S(G)$ must be used by a protection path, and consequently there must be a cycle formed by protection edges. Hence, there must be at least $|V|$ protection edges. Since G is Hamiltonian, protection edges belonging to $C(G)$ are feasible protection edges. ∎

Based on Theorems 3 and 4 we have the following result.

COROLLARY 5. *Let $G = (V, E)$ be a Hamiltonian graph with at most one survivable path defined over adjacent pairs of vertices , and let H_G be a connected graph. Then, there exists an optimal set of survivable paths consisting of all mutually diverse direct working paths and all protection paths lying either on a single spanning tree $T'(G)$ or on a single Hamiltonian cycle $C(G)$.* ∎

We are now extending the results to the case of paths between vertices of distance ≤ 2.

THEOREM 6. *Let $G = (V, E)$ be a Hamiltonian graph having all mutually diverse optimal paths of distance ≤ 2 corresponding to survivable paths defined in G, and let H_G be a connected graph. Then, there exists an optimal set of survivable paths consisting of all mutually diverse direct working paths and all protection paths lying either on a single spanning tree of G or on a single Hamiltonian cycle $C(G)$*

PROOF. Since G is Hamiltonian, then it's two-edge-connected. Thus, for every working path of length ≤ 2, there exists a corresponding protection path. Consider all diverse working paths that correspond to the optimal paths. The cost of these working paths is the least possible. Furthermore, if there exist the corresponding protection paths all lying on a spanning tree of G than together their constitute an optimal set of survivable paths. Suppose now that they cannot lie on a single spanning tree of G. So, they must form at least one cycle. Let $C_i(G) = x_1, x_2, \dots, x_n$ be a Hamiltonian cycle in G. We show that for every type of working path W there corresponds a protection path P lying completely on C_i. For $W = x_i x_{i+1}$, there corresponds $P = x_{i+1}, x_{i+2}, \dots, x_i$. For $W = x_i x_j$ (where $j \neq i + 1$), there corresponds $P = x_i, x_{i+1}, \dots, x_i$. For $W = x_i x_{i+1} x_{i+2}$, there corresponds $P = x_{i+2}, x_{i+3}, \dots, x_i$ For $W = x_i x_{i+1} x_j$ (where $j \neq i + 2$) there corresponds $P = x_j, x_{j+1}, \dots, x_i$. Finally, for $W = x_i x_j x_k$ (where $j \neq i + 1$ and $k \neq j + 1$), there corresponds $P = x_k, x_{k+1}, \dots, x_i$. ∎

Finally, we introduce Lemma 7 pertaining to an arbitrary two-edge-connected graph that will be useful to prove Theorem 10 in the next section.

LEMMA 7. *Let $G = (V, E)$ be a two-connected graph with k survivable paths. If a set of $k+1$ survivable paths that contains an optimal set of k survivable paths defined on the same pairs of vertices as before is more expensive by 1 then this set is an optimal set of $k+1$ survivable paths.*

PROOF. By adding a survivable path to an optimal set of k survivable paths, we add at least one unit of cost corresponding to at least one additional working hop. Furthermore, if so constructed $k+1$ survivable paths of cost increased by 1 could be transformed in such a way that the cost of solution would decrease, then the cost of original k survivable paths could be decreased without need of an addition survivable path—a contradiction. ∎

3. OPTIMAL SET OT SURVIVABLE PATHS IN COMPLETE GRAPHS

In this section, we investigate the properties of survivable paths defined on complete graphs. Let $C_1, C_2, \dots, C_i$ be connected components of H_K. Let R_i be a spanning tree of C_i', if C_i' is connected, or a Hamiltonian cycle on vertices $V(C_i)$, if C_i' is disconnected..

THEOREM 8. *Let K_n be a complete graph of order $n > 2$, with k survivable paths defined between k distinct pairs of vertices Let $C_1, C_2, \dots, C_i$ be connected components of H_K. Then, an optimal set of survivable paths consists of all direct working paths and protection paths lying on $R_1, R_2, \dots, R_i$.*

PROOF. Since K_n is a complete graph of order $n > 2$ then, it satisfies the survivable paths. By Theorem 2 all working paths must be direct one-hop paths. Let $V_1, V_2, \dots, V_i$ be subsets of vertices of G corresponding to $C_1, C_2, \dots, C_i$. The protection paths must span vertices $V_1 \cup V_2 \cup \cdots \cup V_i$ that requires $T_1, T_2, \dots, T_i$ to be covered by protection paths. In addition, for every component C_j, whose complement is disconnected graph, there must correspond (by Theorem 4) at least one more cycle spanned by protection edges. Let there be m such components. Then there must be at least $(|V_1| - 1) + (|V_2| - 1) + \cdots + (|V_i| - 1) + m$ edges covered by protection paths which is satisfied by $R_1, R_2, \dots, R_i$. ∎

As the direct consequence of Theorem 7, we have the following result.

COROLLARY 9. *Let K be a complete graph of order > 2, with at most one survivable path defined for any pair of adjacent vertices, and let H_K be a graph consisting of components that are either triangles, or squares, or a combination of both. Then, there exists an optimal set of survivable paths in which the protection paths cover exactly the same set of edges as do the working paths.* ∎

It's easy to observe that the above condition (i.e., H_K being a graph consisting of either triangles or squares) is also necessary condition for this interesting property. The cost of solution for m survivable paths in this case equals $2m$.

Let's call a survivable path to be *incident to a vertex* if that vertex is one of the end-vertices of that path.

THEOREM 10. *Let K_n be a complete graph of order $n(n > 2)$ with at most two survivable paths defined for any pair of vertices, and at most $n-1$ survivable paths incident to any vertex. Let $S'(K)$ be a disconnected graph, and let H_K be a connected graph. Then, there exists an optimal set of survivable paths consisting of all direct working paths and all protection paths lying on a single Hamiltonian cycle $C(K)$.*

PROOF. Since K_n is a complete graph of order $n > 2$, then it satisfies the survivable paths. Consider first a subset of m survivable paths that does not contain two survivable paths defined for the same pair of vertices. Since $S'(K)$ is disconnected and H_K is connected, then by Theorem 4 there exists an optimal set of survivable paths consisting of all direct working paths, and all protection paths lying on a single Hamiltonian cycle $C(K)$. Furthermore, since there are at most $n-1$ survivable paths incident to any vertex in K, then there exists a Hamiltonian cycle $C'(K)$, in which every edge corresponds to a pair of vertices for which there is defined at most one survivable path. Thus, for m survivable paths under consideration there exists an optimal set of survivable paths in which all protection paths lie on a single Hamiltonian cycle $C'(K)$. Let $C'(K) = x_1, \dots, x_n, x_1$. Each edge in $C'(K)$ is contained in at least one protection path We now add the remaining survivable paths, one at a time, to the current optimal set of survivable paths. That is, we add direct working path of form $x_i x_j$ (where $j \neq i+1$ if $j < n$ and $j \neq 1$ otherwise) that adds one unit of cost to the current solution. In addition, if there existed protection path $x_i, x_{i+1}, \dots, x_j$, that corresponded to already established working path $x_i x_j$, then every edge on new protection path $x_i, x_{i-1}, \dots, x_j$ will be shared with other protection paths. So, protection path of a new survivable path will not add any additional cost to the current solution. Hence, every time we add a survivable path the cost of solution increases by exactly one. Then, by Lemma 7 adding survivable paths in the manner described above, one at a time, to the current optimal set of survivable paths results in the new optimal sets of survivable paths. This in turn implies that there exists an optimal set of all survivable paths consisting of all direct working paths and all protection paths lying on a single Hamiltonian cycle $C'(K)$. ∎

4. OPTIMAL GRAPHS

As we defined in the introduction, G is *two-optimal*, if for any combination of survivable paths defined between end-vertices of distance not less than two, there exists a corresponding optimal set of survivable paths defined between the same end-vertices with all direct working paths. We now give two conditions for a graph to be two-optimal.

THEOREM 11. *Let $G = (V_1, V_2, E)$ be a complete bipartite graph and let $|V_1|, |V_2| \geq 2$. Then G is two-optimal.*

PROOF. Since $|V_1|, |V_2| \geq 2$ then G is two-edge-connected and satisfies the survivable paths. Suppose that there exists an optimal set of survivable paths that contains some indirect working paths. An indirect working path W for non-adjacent vertices in G must be of length at least four. So, a survivable path with indirect working path must cost at least four. Hence, we can preserve optimality of survivable paths by substituting any such survivable path (W, P) defined for non-adjacent vertices a, b with a new survivable path (W', P') of form $(x_a x_i x_b, x_a x_j x_b)$ that would have to cost exactly four. Furthermore, W' would be a direct working path in G. ∎

THEOREM 12. *Let $G = (V, E)$ be a two-connected graph of order at least seven. Then, G is two-optimal only if it contains a square.*

PROOF. First, suppose that G is of girth at least seven. Let $x_1, x_2, x_3, x_4, x_5, x_6, x_7, \dots, x_k, x_1$ be a cycle of minimum length k ($k \geq 7$) in G. Then, it's easy to see that G is not two-optimal for two survivable paths defined for pairs x_1, x_3, and x_1, x_4. In particular, for two direct working paths the cost would be $2k$. But, for combination $(W', P') = (x_1, x_2, x_3, x_1, x_k, \dots, x_5, x_4, x_3)$, and $(W', P') = (x_1, x_k, \dots, x_6, x_5, x_4, x_1, x_2, x_3, x_4)$ the cost would be $2k-1$, since edge (x_2, x_3) could now be shared.

Now, suppose that G is of girth either five or six Let $C = x_1, x_2, x_3, x_4, x_5, \dots, x_1$ be a cycle of minimum length in G (i.e., a cycle of length either five or six). Since $|V| > 6$, then at least one vertex in C must be of degree at least three. Without loss of generality assume that x_1 is such a vertex. Because G is two-connected then there must be another cycle $x_1, y_1, y_2, \dots, x_i, \dots x_1, (x_i \neq x_1)$ in G, such that x_i is one of the vertices in C. Let C$'$ be a cycle of minimum length that is of this form. Vertices y_1, y_2 cannot be part of C because it would imply a cycle of length less then length of C. Denote by $|C|$, $|C'|$ the lengths of two cycles. Without loss of generality cycles C and C' can be represented as $C = x_1, x_2, x_3, x_4, x_5, \dots, x_1$ and $C' = x_1, y_1, y_2, \dots, x_3, x_2, x_1$. Consider two survivable paths defined for pairs of vertices y_1, x_2,

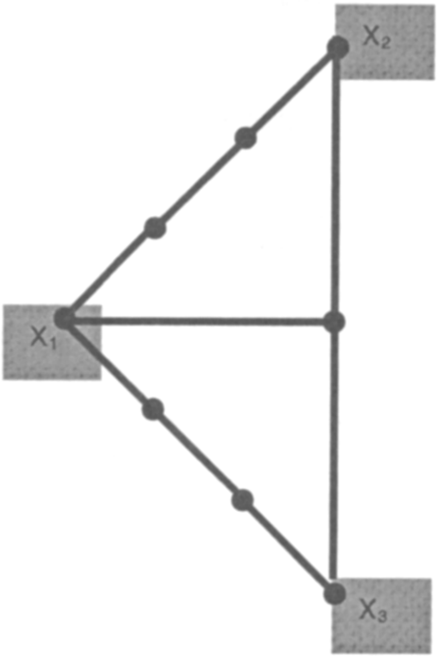

Figure 1 Graph without a direct working path for an optimal set of two survivable paths defined between X_1, X_2 and X_1, X_3.

and x_1, x_3 in G. In this case, two survivable paths $(W, P), (W', P')$ would have to cost at least $|C| + |C'|$ if the working paths were direct This would happen because the working paths would have to be y_1, x_1, x_2 and x_1, x_2, x_3 (otherwise a cycle of length four would exist) not allowing sharing any edges by protection paths. Survivable path $(W', P') = (x_1, x_2, x_3, x_1, x_6, x_5, x_4, x_3)$ satisfies the minimum cost in such a case. Then, by swapping W' with P' we would reduce cost by at least one since edge (x_2, x_3) could now be shared. So, G is not two-optimal for two survivable paths defined for pairs y_1, x_2 and x_1, x_3.

Finally, suppose that G is of girth three but doesn't contain a square. Then, G must contain an edge shared by cycles of lengths three and at least five. Let $C = x_1, x_2, x_3, x_4, x_5, \ldots, x_k, x_1$, be a cycle of shortest length k ($k \geq 5$) that shares edge (x_2, x_3) with a cycle C' of length three. Without loss of generality assume that $C' = x_2, x_3 y, x_2$. If y could be a part of C, then it would imply that C is not a shortest length cycle—a contradiction. So, y is not a part of C. Consider two survivable paths defined between x_1, x_3, and between x_2, x_4 If the working paths are direct then they are x_1, x_2, x_3, and $x_2, x_3 y, x_4$, and the corresponding protection paths cannot share the cost of other edges. Then each protection path would have to be of length at least $k - 2$ and they would cost $2k - 4$. So, the total cost of these two survivable paths would be at least $2k$. But, for working paths x_1, x_2, y, x_3, and x_2, x_3, x_4, there would correspond protection paths $x_1, x_k, x_{k-1}, \ldots, x_3$, and $x_2, x_1, x_k, x_{k-1}, \ldots, x_4$ for a total cost of $k + 4$. This would mean that for $k \geq 5$ an optimal set of two survivable paths defined between these vertices wouldn't contain direct working paths, i.e., $2k \geq k+4$. So, G would not be two-optimal. Since by Theorem 11 there exist bipartite graphs of order at least seven that are two-connected and two-optimal, then G is two-optimal only if it contains a cycle of length four. ∎

Figure 1 illustrates that there are graphs, and survivable paths defined on those graphs, in which any optimal set of survivable paths does not include even a single direct working path.

Finally, we emphasize that the results of Theorem 1 and Corollary 2 have practical implications on the real networks analysis. These two results allow the identification of the shortest primary paths in network design scenarios. Hence, the extensions to Theorem 1 would be of great interest to real network design issues, and would be worthwhile to pursue. In particular, relaxing the constraint on the adjacency of survivable path definition in Theorem 1 would have a significant impact in that regard.

REFERENCES

1. C. Berge, *Graphs and Hypergraphs*, North-Holland, Reading, (1979)
2. R Ramamurthy, Z Bogdanowicz, *et al.*, Capacity performance of dynamic provisioning in optical networks, *IEEE Journal of Lightwave Technology* **19**, 40–48, (2001).
3. M Gondran and M. Minoux, *Graphs and Algorithms*, John Wiley and Sons, Reading, (1984).
4. R M Karp, On the computational complexity of combinatorial problems, *Networks* **5**, 45–68, (1975)
5. G Ellinas, E Bouillet, *et al.*, Routing and restoration architectures in mesh optical networks, *Optical Networks Magazine* **4** (1), (Jan /Feb. 2003).
6. D. Xu, Y Chen and C Qiao, *Optical Fiber Communication (OFC 2004)*, Los Angeles, CA, February 2004.
7. E Bouillet, P Mishra, J. Labourdette, *et al.*, *Networks and Optical Communication (NOC)*, Darmstadt, Germany, June 2002.
8. J Doucette and W D Grover, Third International Workshop on the Design of Reliable Communication Networks, (DRCM 2001), Budapest, Hungary, (Oct. 2001).
9. Z. Bogdanowicz and S. Datta, Analysis of backup route reoptimization algorithms for optical shared mesh networks, *Mathl. Comput. Modelling* **40** (9/10), 1047–1055, (2004).