a nutshell through the reminiscences of his child. Next, it contains George's life story told by himself: a faithful, though sometimes sarcastic narrative, which covers periods that most of us know only from history books. Next comes the list of Lorentz' 130 papers and his six books, as well as the list of his 17 doctoral students. With excellent editorial insight Rudi included four unpublished essays of his father on mathematics in the first volume: an account on the pre-war University of Leningrad (lecturers and curricula), on the work of the mathematical mind, on proofs in mathematics, and on writing mathematical books. Then come the papers in their original form (in German or English). The work of Lorentz in the aforementioned main areas are summarized by experts in the different fields. The summary for summability is written by S. Baron and D. Leviatan, for (Birkhoff) interpolation by S. D. Riemenschneider, for real and functional analysis by C. Bennett, and for approximation theory by H. Berens. The summary papers reflect their original source: they are lively and elegant.

Volume II also contains an article by T. Erdélyi and P. Nevai on Lorentz' books. His books, namely *Bernstein Polynomials*, *Approximation*

volumes *Constructive Approximation* (with R. A. DeVore) and *Constructive Approximation*: *Advanced Problems* (with M. v. Golitschek and Y. Makovoz) are all related to approximation theory and they are among the most extensively used textbooks on approximation.

George once told me that he would write a problem article and then drop mathematics altogether. Seeing the immense work presented in these two volumes and the fact that two of his most important books were written *after* his retirement, one can hardly believe that. And we all hope that he will continue doing mathematics, with his elegance, enthusiasm and wisdom, well into the next millennium.

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I. Novikov and E. Semenov, *Haar Series and Linear Operators*, Mathematics and Its Applications **367**, Kluwer, Dordrecht, 1997, xv + 218 pp.

Alfred Haar introduced the orthogonal system that now bears his name in his 1909 Göttingen dissertation. Since then it has played a significant role in many contexts, some unexpected, for example in the Lévy–Ciesielski construction of Brownian motion. Providing the first example in wavelet theory, it is also a martingale difference sequence.

This book focuses mostly on the Haar system while at the same time showing how martingale theory and the study of the Haar system fruitfully interact. The concept of conditional expectation is defined in the first paragraph of Chapter 1. This chapter also contains introductory material on bases in Banach spaces, interpolation of operators, and rearrangement invariant (RI) spaces such as the Lebesgue, Orlicz, and Lorentz spaces. The study of operators on RI spaces is one of the central themes of the book. Chapters 2–4 contain inequalities showing how close the Fourier–Haar partial sums of a function approximate the function, the theorem that the Haar system is a monotone basis for every separable RI space, and a two-sided inequality comparing the size of a function and the size of the maximal function of the sequence of partial sums of its Fourier–Haar series, an inequality that holds for any RI space with an upper Boyd index strictly less than 1 (caution: the authors define these upper and lower indices as Boyd does; some of their references define them as the reciprocals of the original indices). Chapter 5 contains the Paley–Marcinkiewicz theorem that the Haar system is an unconditional basis of $L^p(0, 1)$, $1 < p < \infty$, and Burkholder's 1985 proof, shorter than his original proof, that the unconditional basis constant is $p^* - 1$, where p^* is the maximum of p and its conjugate index. Chapter 6 contains the square-function inequality for the Haar system not only for L^p, $1 < p < \infty$, but also for all RI spaces with a strictly positive lower Boyd index. Moreover, the Haar system is an unconditional basis in a separable RI space if and only if both Boyd indices are in the open unit interval. Chapter 7 contains information about the Fourier–Haar coefficients for various classes of functions. Chapter 8 contains the Johnson–Schechtman 1988 extension of the martingale square function inequality of Burkholder, Davis, and Gundy to RI spaces with a strictly positive lower Boyd index, which of course implies the analogous result, mentioned above, for the Haar system (see also, the slightly later papers of Antipa and Igor Novikov.) The main result of Chapter 9 is that the Haar basis of every separable RI space is precisely reproducible. Chapter 10 characterizes all monotone bases in some general classes of RI spaces. Chapter 11 is a study of operators arising from rearrangements of the Haar system. Chapters 12–14 focus on norm and pointwise estimates for Fourier–Haar multiplier operators, Chapter 15 focuses on subsequences of the Haar system, Chapter 16 on the equivalence problem for Haar and Franklin systems in RI spaces, and Chapter 17 on the Olevskii system.

The long list of over 350 references will be especially helpful to those readers who are not as familiar as they should be with the vast Eastern European literature on the subject. However, there is one cautionary remark: for a great many, perhaps for most, of the references numbers listed at the end of the book do not match those cited in the text itself. For example, at the end of the book, Haar's 1910 paper has the number [113], but is cited as [117] in the text. There are also many minor misprints but these cause less inconvenience.

Overall, this is a valuable book containing many deep results.

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G. A. Baker, Jr. and P. Graves-Morris, *Padé Approximants*, 2nd ed., Encyclopedia of Mathematics and Its Applications **59**, Cambridge Univ. Press, Cambridge, 1996, xiv + 746 pp.

This extensive work deals with rational approximants that have become known under the name *Padé Approximants*. They have been studied by Jacobi (1846), Frobenius (1881), and, systematically, by Padé (1892). Moreover many of the generalizations introduced later are incorporated. The main use in the 19th century of what is now called the *Padé table* was its application to analytic number theory (cf., the problem of transcendence of e as treated by Hermite (1873)).

The approximation problem is simple: given two nonnegative integers L and M and a (formal) power series

$$f(z) = \sum_{i=0}^{\infty} c_i z^i,$$

find two polynomials

$$A^{[L/M]}(z) = \sum_{i=0}^{L} a_i z^i, \qquad B^{[L/M]}(z) = \sum_{i=0}^{M} b_i z^i,$$

with $b_0 = 1$, such that

$$A^{[L/M]}(z)/B^{[L/M]}(z) = f(z) + O(z^{L+M+1}). \tag{1}$$

This is clearly a nonlinear problem and the existence and uniqueness of solutions (written as rational functions in their lowest terms) is governed by the nonvanishing of certain Hankel-determinants containing the coefficients of the power series f. The rational functions in (1) are placed in a two-dimensional table referred to as the *Padé table*. The determinants that play an important role (in existence and uniqueness problems, in explicit formulae, in recurrence relations and connections with continued