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Technifermion representations and precision electroweak constraints

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Abstract

We discuss the selection of fermion representations in technicolor models with a view toward minimizing technicolor contributions to the precision electroweak S parameter. We present and analyze models that involve one technifermion $\mathrm{SU}(2)_L$ doublet with standard-model singlet technifermion sectors that lead to walking behavior, which further reduces S. We also consider models that have technifermions in higher-dimensional representations and study embeddings in extended technicolor theories.

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1. Introduction

It is possible that electroweak symmetry breaking occurs as the result of the existence of a new, asymptotically free, vectorial gauge interaction, generically called technicolor (TC) [1], which becomes strongly coupled at a scale Λ_{TC} of several hundred GeV, producing a bilinear technifermion condensate with weak isospin $I = 1/2$ and weak hypercharge $Y = 1$. To communicate the electroweak symmetry breaking to the standard-model (technisinglet) fermions and to give them masses, one embeds technicolor in a larger, extended technicolor (ETC) theory [2] (reviews include [3]). Technicolor theories produce corrections to precisely measured electroweak quantities, in particular, to the W and Z propagators (called oblique corrections), and are stringently constrained by the requirement that these modifications not exceed experimental limits. Here we analyze technifermion representations from the viewpoint of minimizing these technicolor corrections, in particular, the S parameter. We construct models that can accomplish this, using (a) an $\mathrm{SU}(N_{\mathrm{TC}})$ gauge group with the minimal non-Abelian value, $N_{\mathrm{TC}} = 2$; (b) a minimal standard-model (SM)-nonsinglet sector consisting of technifermions that transform as a doublet under weak isospin $\mathrm{SU}(2)_L$ and also a doublet under $\mathrm{SU}(2)_{\mathrm{TC}}$; and (c) a SM-singlet technifermion sector that produces walking behavior, which further reduces S. We also consider models with technifermions in higher-dimensional representations of the TC group, and study embeddings of both types of technicolor models in extended technicolor theories.

2. Some basics

In a minimal technicolor model with gauge group G_{TC} the technifermions transform under $G_{\mathrm{TC}} \times \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ as

$$\begin{pmatrix} F^{1/2} \\ F^{-1/2} \end{pmatrix}_L : (\mathcal{R}_{\mathrm{TC}}, 1, 2)_{0,L},$$
$$F_R^{\pm 1/2}: (\mathcal{R}_{\mathrm{TC}}, 1, 1)_{\pm 1,R}, \tag{1}$$

where $\mathcal{R}_{\mathrm{TC}}$ denotes the representation of G_{TC}, and the superscripts (subscripts) denote electric charge (weak hypercharge Y and chirality), respectively. The Y values in Eq. (1) are determined by the requirement of no $\mathrm{SU}(2)_L^2\mathrm{U}(1)_Y$ or $\mathrm{U}(1)_Y^3$ gauge anomalies. Here we take $G_{\mathrm{TC}} = \mathrm{SU}(N_{\mathrm{TC}})$. Most studies have chosen for $\mathcal{R}_{\mathrm{TC}}$ the simplest nontrivial possibility, namely, the fundamental representation, but there have also been studies with higher-dimensional technifermion representations [5–7]. At certain points below it will be useful to compare predictions

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of TC theories based on Eq. (1) with those in which the technifermions transform as a SM family; some recent works on this latter type of model are Refs. [8–14].

Technicolor corrections to the W and Z propagators are summarized in terms of the S, T, and U parameters [15–17] (for reviews, see [4,18]). Of these, the S and T parameters provide the most important constraints on technicolor. We denote the technicolor contributions to S and T as $(\Delta S)^{(\mathrm{TC})}$ and $(\Delta T)^{(\mathrm{TC})}$. The T parameter measures corrections, from new physics (NP) beyond the standard model, to the custodial symmetry relation $\rho = 1$, where $\rho = m_W^2/(m_Z^2 \cos^2\theta_W)$ and $\Delta\rho^{(\mathrm{NP})} = \alpha_{\mathrm{em}}(m_Z)T$. Since the SM gauge interactions are small at the scale Λ_{TC}, technifermion condensates can naturally produce nearly degenerate dynamical masses for SM-nonsinglet technifermions with weak $I_3 = \pm 1/2$, preserving approximate custodial symmetry and yielding an acceptably small $|(\Delta T)^{(\mathrm{TC})}|$. One of the tasks that ETC theories take on is then how to explain the large t–b mass splitting while maintaining a small $|(\Delta T)^{(\mathrm{TC})}|$. One-family ETC models using relatively conjugate ETC representations for left- and right-handed $Q = -1/3$ quarks can account for this $m_t - m_b$ splitting without excessive contributions to T, but have problems with flavor-changing neutral currents (FCNC) [11,13]. (In contrast, for models with vectorial ETC representations, it was shown in Refs. [11,13] that FCNC constraints, in particular from K^0–$\bar{K}^0$ mixing, are not as serious as had been thought previously.)

The S parameter measures heavy-particle contributions to the Z self-energy via the term $4s_W^2 c_W^2 \alpha_{\mathrm{em}}^{-1}(m_Z)[\Pi_{ZZ}^{(\mathrm{NP})}(m_Z^2) - \Pi_{ZZ}^{(\mathrm{NP})}(0)]/m_Z^2$, where $s_W^2 = 1 - c_W^2 = \sin^2\theta_W$, evaluated at m_Z (see [4] for details). Here we shall focus on the minimization of $(\Delta S)^{(\mathrm{TC})}$, commenting on $(\Delta T)^{(\mathrm{TC})}$ briefly below. Global fits to data yield allowed regions in (S,T) depending on a reference value of the SM Higgs mass, $m_{H,\mathrm{ref.}}$. The comparison of these with a technicolor theory is complicated by the fact that technicolor has no fundamental Higgs field; sometimes one formally uses $m_{H,\mathrm{ref.}} \sim 1$ TeV for a rough estimate, since the SM with $m_H \sim 1$ TeV has strong longitudinal vector boson scattering, as does technicolor. However, this may involve some double-counting when one also includes contributions to S from technifermions, whose interactions and bound states (e.g., technivector mesons) are responsible for the strong $W_L^+ W_L^-$ and $Z_L Z_L$ scattering in a technicolor framework. The current allowed region in (S,T) [18] disfavors values of $S \gtrsim 0.2$ and $|T| \gtrsim 0.2$.

For fermions comprising an $\mathrm{SU}(2)_L$ doublet, plus right-handed $\mathrm{SU}(2)_L$ singlets, which have degenerate masses m_F satisfying $(2m_F/m_Z)^2 \gg 1$ and are weakly interacting, the well-known one-loop contribution to S is $N_D/(6\pi)$ (independent of Y). Since technifermions are strongly interacting on the scale m_Z used in the definition of S, it is of questionable validity to try to apply perturbation theory to calculate $(\Delta S)^{(\mathrm{TC})}$. Nevertheless, the estimate of $(\Delta S)^{(\mathrm{TC})}$ based on the perturbative one-loop contribution of the technifermions is often used as an approximate guide. Because the technifermions have dynamical masses Σ_{TC} that satisfy $(2\Sigma_{\mathrm{TC}}/m_Z)^2 \gg 1$ and which, moreover, are naturally approximately degenerate, it follows that a perturbative estimate is $(\Delta S)^{(\mathrm{TC})}_{\mathrm{pert.}} = N_D/(6\pi)$, where N_D denotes the total number of new technifermion $\mathrm{SU}(2)_L$ doublets. For the model of Eq. (1), commonly called the "one-doublet" TC model, this total number is $N_D = \dim(\mathcal{R}_{\mathrm{TC}})$, while for a one-family TC model, $N_D = (N_c + 1)\dim(\mathcal{R}_{\mathrm{TC}}) = 4\dim(\mathcal{R}_{\mathrm{TC}})$ (where $N_c = 3$ colors). Therefore, to minimize $(\Delta S)^{(\mathrm{TC})}$, one can reduce N_{TC} to its minimal non-Abelian value, $N_{\mathrm{TC}} = 2$ and $\mathcal{R}_{\mathrm{TC}}$ to its smallest nontrivial possibility, viz., the fundamental (fund.) representation. With these choices, the TC model of Eq. (1) yields

$$(\Delta S)^{(\mathrm{TC})}_{\mathrm{pert.}} = \frac{1}{3\pi}, \quad \text{for } N_{\mathrm{TC}} = 2,\ \mathcal{R}_{\mathrm{TC}} = \text{fund.}, \tag{2}$$

while $(\Delta S)^{(\mathrm{TC})}_{\mathrm{pert.}} = 4/(3\pi)$ for the one-family TC model. Higher $\mathcal{R}_{\mathrm{TC}}$ are discussed below. Another advantage of the model of Eq. (1) is that (for general N_{TC}) all of the three Nambu–Goldstone bosons (NGBs) that arise due to the formation of technicondensates are absorbed to make the $W^\pm$ and Z massive so that there are no problems with unwanted (pseudo) NGBs.

An important property of modern technicolor theories is a TC gauge coupling that runs slowly ("walks") over a certain energy interval extending from Λ_{TC} to a higher ETC scale, Λ_w [19,20]. Walking technicolor (WTC) occurs naturally if the TC gauge coupling has an approximate infrared-stable fixed point (zero of the beta function) $\alpha_{\mathrm{TC,IR}}$ which is slightly larger than the critical value $\alpha_{\mathrm{TC},c}$ for technifermion condensate formation. In such a theory, as the energy scale μ decreases from large values, α_{TC} increases, but its rate of increase, given by $-\beta$, decreases as α_{TC} approaches the zero at $\alpha_{\mathrm{TC,IR}}$. Hence, over an extended energy interval, α_{TC} is O(1) but slowly varying. This is accompanied by a large anomalous dimension $\gamma \simeq 1$ for the bilinear technifermion operator $\bar{F}F$, resulting in the enhancement of SM fermion masses by the factor $\eta = \exp[\int_{\Lambda_{\mathrm{TC}}}^{\Lambda_w} (d\mu/\mu)\gamma(\alpha(\mu))] \simeq \Lambda_w/\Lambda_{\mathrm{TC}}$ and also enhancement of pseudo-Nambu–Goldstone boson masses. In a non-walking scaled-up QCD type of technicolor theory, spectral-function methods yield $(\Delta S)^{(\mathrm{TC})} \simeq 0.1 N_D \simeq 2(\Delta S)^{(\mathrm{TC})}_{\mathrm{pert.}}$ [15]. Nonperturbative estimates of $(\Delta S)^{(\mathrm{TC})}$ in WTC models show that it is reduced relative to nonwalking TC [17], clearly a desirable feature.

An analysis of the beta function of the one-family technicolor model with $N_{\mathrm{TC}} = 2$ and (vectorially coupled) technifermions transforming according to the fundamental representation, i.e., techniisospin $I_{\mathrm{TC}} = 1/2$, suggests that, with its $N_w(N_c + 1) = 8$ technifermions, it can plausibly exhibit walking behavior [19,20] (cf. Eq. (A.2)). The value $N_{\mathrm{TC}} = 2$ has been used for many studies of one-family ETC models [8–13] and also has the advantage that it makes possible a mechanism to obtain light neutrino masses [9]. In contrast, although the technicolor model with the minimal SM-nonsinglet technifermion sector of Eq. (1) with $\mathcal{R}_{\mathrm{TC}} = $ fund. yields a relatively small value of $(\Delta S)^{(\mathrm{TC})}_{\mathrm{pert.}}$, especially for $N_{\mathrm{TC}} = 2$, its two-loop beta function does not have a perturbative IR fixed point or resultant walking behavior. Hence, it may have difficulty producing sufficiently large SM fermion masses, in particular, m_t.

3. Minimal technicolor models with walking

There is thus motivation for constructing technicolor models that have small values of $(\Delta S)^{(TC)}_{\text{pert.}}$ and also have walking behavior to reduce the full (nonperturbatively calculated) $(\Delta S)^{(TC)}$. We proceed to do this. The idea is to use the model of Eq. (1) with the smallest non-Abelian value, $N_{TC}=2$, and the minimal choice, $\mathcal{R}_{TC}=\text{fund.}$, i.e., $I_{TC}=1/2$, together with a SM-singlet, TC-nonsinglet fermion sector that produces the walking. As noted above, this theory has walking behavior for eight $I_{TC}=1/2$ technifermions. Since there are already $N_w=2$ such technifermions from the SM-nonsinglet sector given in Eq. (1), we use six SM-singlet, $I_{TC}=1/2$ technifermions. These should transform nontrivially under a second vectorial gauge symmetry, denoted metacolor, which becomes strongly coupled on a scale $\Lambda_{MC}\simeq\Lambda_{TC}$. The reason for having the SM-singlet technifermions be nonsinglets under metacolor rather than just consisting of the set $\psi^{\tau}_{p,R}$ with $p=1,\dots,12$ (where, without loss of generality, we write SM-singlet fields as right-handed and use the fact that 12 such fermions are equivalent to six Dirac fermions for SU(2)), is that, in the approximation that one neglects SM gauge interactions, which are small at the scale Λ_{TC}, relative to TC gauge interactions, the latter model would have a global chiral symmetry which would be spontaneously broken by the formation of the technicondensates. The subset of the resultant pseudo-Nambu–Goldstone bosons (PNGBs) corresponding to global transformations between the $\psi^{\tau}_{p,R}$ and $F_R^{\pm 1/2\,\tau}$ would be color-singlets with electric charges $\pm 1/2$ and would gain masses of order $e\Lambda_{TC}\sim 100$ GeV due to the explicit breaking of the global chiral invariance by electroweak interactions. These masses are close enough to current experimental limits on new charged leptons, e.g., from LEP, to disfavor such a model.

There are several possibilities for the SM-singlet technifermion representations under metacolor. We shall discuss two in particular. Let us assume that the metacolor gauge group is $SU(2)_{MC}$. Then under $SU(2)_{TC}\times SU(2)_{MC}$ these representations could be

(1) six copies of $(2,2)$, denoted $\zeta^{\tau\alpha}_{p,R}$, $p=1,\dots,6$, or
(2) four copies of $(2,3)$, denoted $\vec{\zeta}^{\,\tau}_{p,R}$, $p=1,\dots,4$,

where τ and α are TC and MC indices, $\vec{\zeta}$ refers to the MC isovector, and p is the copy number. With the strongly coupled metacolor, even neglecting SM gauge interactions, a global transformation of the form $\zeta^{\tau\alpha}_{p,R}\leftrightarrow F_R^{\pm\tau}$ or $\vec{\zeta}^{\,\tau}_{p,R}\leftrightarrow F_R^{\pm\tau}$ is not a symmetry of the model, and hence there are no problematic light electrically charged PNGBs. The masses generated for the charge $q=\pm 1/2$ PNGBs are of order $\Lambda_{MC}\simeq 300$ GeV, since the MC gauge coupling is O(1); these masses should be sufficiently high to agree with experimental limits.

We thus envision the following properties for these models. As the energy scale μ decreases from large values, α_{TC} increases but remains at a large O(1) value throughout a substantial interval because of the walking. As μ approaches the comparable scales $\Lambda_{TC}\simeq\Lambda_{MC}$, the combined attractive TC and MC interactions lead to formation of the condensates

$$\langle \epsilon_{\tau\tau'}\epsilon_{\alpha\alpha'}\zeta^{\tau\alpha T}_{p,R}C\zeta^{\tau'\alpha'}_{p',R}\rangle \tag{3}$$

in model (i) and

$$\langle \epsilon_{\tau\tau'}\vec{\zeta}^{\,\tau T}_{p,R}C\cdot\vec{\zeta}^{\,\tau'}_{p',R}\rangle \tag{4}$$

in model (ii). At the slightly lower scale Λ_{TC} the technifermion condensates $\langle\bar{F}F\rangle$ form. These models yield the appealingly small perturbative estimate (2) together with walking behavior that reduces the full (nonperturbatively calculated) $(\Delta S)^{(TC)}$ relative to its value in a nonwalking theory.

4. Embedding of minimal technicolor model in ETC

We next discuss embedding our $SU(2)_{TC}$ models, presented in the previous section, in an ETC theory. We shall give some formulas for arbitrary N_{TC} to show their general structure. One possible embedding is to use the gauge group $SU(N_{ETC})$ with

$$N_{ETC}=N_{TC}+N_{\text{gen.}}(N_c+1)=N_{TC}+12 \tag{5}$$

(where the number of SM fermion generations $N_{\text{gen.}}=3$) and to assign the left-handed SM-nonsinglet technifermions with weak $I_3=\pm 1/2$ to multiplets containing the SM fermions with the same value of I_3:

$$\begin{aligned}&\left(F^{1/2\,\tau},u^{aj},\nu^{j}\right)_{\chi},\\ &\left(F^{-1/2\,\tau},d^{aj},e^{j}\right)_{\chi},\quad \chi=L,R,\end{aligned} \tag{6}$$

where τ, a, j, and χ denote technicolor, color, generational indices, and chirality, respectively, and we use the compact notation $(u^{a1},u^{a2},u^{a3})\equiv(u^a,c^a,t^a)$, $(d^{a1},d^{a2},d^{a3})\equiv(d^a,s^a,b^a)$, $(e^1,e^2,e^3)\equiv(e,\mu,\tau)$, etc. Here,

$$SU(N_{ETC})\supset SU(N_{TC})\times SU(3)_{\text{gen.}}\times SU(4)_{PS}, \tag{7}$$

where the Pati–Salam $SU(4)_{PS}$ group [21] contains, as a maximal subgroup, $SU(3)_c\times U(1)_{B-L}$, with B and L denoting baryon and lepton number. Hence, $[G_{ETC},G_{SM}]\neq 0$. The left-handed fields form the $SU(2)_L$ doublets $\binom{F^{1/2\,\tau}}{F^{-1/2\,\tau}}_L$, $\binom{u^{aj}}{d^{bj}}_L$, and $\binom{\nu^j}{e^j}_L$. Owing to the "horizontal" structure of the ETC multiplets in Eq. (6), the ETC group does not include $SU(2)_L$ or (if one chooses to gauge this) $SU(2)_R$, and

$$\left[SU(N_{ETC}),SU(2)_{L,R}\right]=0. \tag{8}$$

It follows that $SU(N_{ETC})$ does not contain $U(1)_{em}$ or $U(1)_Y$, as can also be seen since $\mathrm{Tr}(Q)$ and $\mathrm{Tr}(Y)$ are nonzero for the $\chi=R$ multiplets in Eq. (6). We now specialize again to $N_{TC}=2$ so $N_{ETC}=14$.

The requirement that ETC gauge bosons transform SM fermions to the SM-nonsinglet technifermions and back in order to produce SM fermion masses entails the following transitions:

$$\begin{aligned}&u^{aj}_{\chi}\to F^{1/2\,\tau}_{\chi}+V^{aj}_{\tau},\\ &d^{aj}_{\chi}\to F^{-1/2\,\tau}_{\chi}+V^{aj}_{\tau},\\ &\nu^{j}_{\chi}\to F^{1/2\,\tau}_{\chi}+U^{j}_{\tau},\\ &e^{j}_{\chi}\to F^{-1/2\,\tau}_{\chi}+U^{j}_{\tau}.\end{aligned} \tag{9}$$

Under $SU(2)_{TC} \times SU(3)_{gen.} \times SU(3)_c \times U(1)_{B-L}$, the V_τ^{aj} transform as $(2,3,3)_{1/3}$ and the U_τ^j as $(2,3,1)_{-1}$, with corresponding electric charges $Q_V = 1/6$ and $Q_U = -1/2$. To yield the correct generational scales for the SM fermion masses, the ETC vector boson mass eigenstates should have masses of order $\Lambda_3 \simeq$ few TeV for $j = 3$, $\Lambda_2 \simeq 10^2$ TeV for $j = 2$, and $\Lambda_1 \simeq 10^3$ TeV for $j = 1$. There are also TC-singlet ETC gauge bosons (i) X_k^{aj} transforming as $(1,8,3)_{4/3}$ with $Q_X = 2/3$ involved in the transitions $u_\chi^{aj} \to \nu_\chi^k + X_k^{aj}$ and $d_\chi^{aj} \to e^k + X_k^{aj}$; and (ii) G_k^j transforming as $(1,8,1)_0$ occur in the transitions $f_\chi^j \to f_\chi^k + G_k^j$, where $f = u, d, e, \nu$ and j, k are generation indices. The ETC gauge bosons contain a subset corresponding to generators of the Cartan subalgebra of $SU(14)_{ETC}$, which are particle- and flavor-diagonal; these are generically denoted V_{dp}, where d denotes "diagonal" and $p = 1, \ldots, 13$.

With the fermion content in Eq. (6), the ETC model is vectorial (and asymptotically free), so that by itself, as the energy scale decreased from large values, the ETC coupling α_{ETC} would eventually get sufficiently large to form bilinear fermion condensates, but these would be invariant under the $SU(14)_{ETC}$ symmetry, which would thus not self-break. To obtain the sequential dynamical breaking of $SU(14)_{ETC}$ and resultant generational hierarchy of SM fermion masses, one can augment the model with three auxiliary strongly coupled gauge symmetries and an appropriately chosen set of chiral fermions, as in Ref. [22]. For our models we would further augment this with either of the metacolor sectors (1) or (2) discussed above.

The flavor-diagonal ETC gauge bosons V_{dp} produce additional contributions to $(\Delta S)^{(TC)}$ and $(\Delta T)^{(TC)}$ via nondiagonal propagator corrections in which Z goes to a loop of virtual fermions $\bar{f} f$ which then go to V_{dp}. Diagonalizing the vector boson mixing matrix, one finds that the mass of the physical Z is reduced [24]. Since

$$m_Z^2 = (m_Z^2)_{SM} \frac{1-\rho}{1-(m_Z^2)_{SM} G_F S/(2^{3/2}\pi)}, \tag{10}$$

this reduction involves negative and positive contributions to S and T, respectively, which depend on the breaking of $SU(14)_{ETC}$ and resultant values of V_{dp} masses.

The electrically charged ETC gauge bosons couple directly to the Z via the J_{em} part of $J_Z = J_{3L} - \sin^2\theta_W J_{em}$ and hence lead to loop corrections to the ZZ and $Z\gamma$ (and $\gamma\gamma$) 2-point functions. In contrast to fermion loop corrections, these are gauge-dependent and require one also to consider nonoblique box and vertex graphs to the same order (as is the case with analogous W corrections to vector 2-point functions in the SM [25]), so that their effects cannot be subsumed into shifts of the oblique parameters S, T, and U. The most important corrections involve the charged ETC vector bosons with lowest masses, $\sim \Lambda_3$. Because the ETC gauge bosons are $SU(2)_L$ singlets (cf. Eq. (8)), they do not couple directly to W.

The most important ETC corrections to $BR(Z \to b\bar{b})$ arise from graphs in which the Z produces (i) a virtual $b\bar{b}$ pair which exchange a V_{dp} with mass $\sim \Lambda_3$ or (ii) a virtual $F^\pm F^\mp$ pair which exchange a V_τ^{a3} (also with mass Λ_3), yielding the outgoing $b\bar{b}$. These are analogous to the V_{d3} and V_τ^3 exchanges in a one-family ETC model, which were found to tend to cancel each other and hence give acceptably small corrections to this branching ratio [26].

With regard to the SM-singlet, TC-nonsinglet fermion sector, it is interesting to recall that in modern detailed studies of one-family ETC models [8,9,11–13], the SM-singlet, ETC-nonsinglet fermion sectors play a crucial role in the sequential ETC symmetry breaking, and in certain cases (e.g., for the breaking sequence G_b in [9] and $S2$ in [11,13]), they yield SM-singlet sectors of the resultant technicolor field theories that contain more than just a single right-handed technineutrino N_R. These studies thus provide explicit examples of how nonminimal SM-singlet technifermion sectors can arise from ETC breaking.

5. Models having SM-nonsinglet technifermions in rank-2 tensor representations of $SU(N_{TC})$

We next discuss the technicolor model of Eq. (1) with $\mathcal{R}_{TC}$ being the symmetric (S_2) or antisymmetric (A_2) rank-2 tensor representation of $SU(N_{TC})$. Technifermions in higher-dimensional representations of G_{TC} have been of interest [5–7] for several reasons, including walking and the minimization of S (as well as formal connections with supersymmetric non-Abelian gauge theories [27,6,7]). Here they will provide a comparison with our technicolor models presented in section III with respect to predicted S values and embedding in ETC. We first review some of their properties.

We denote the SM-nonsinglet technifermions as $F_\chi^{\pm 1/2\ \tau\tau'}$, where $\chi = L, R$, with $F = S_2, A_2$. The dimensionalities of the rank-n symmetric (antisymmetric) tensor representations of $SU(N)$ are $(1/n!)\Pi_{j=0}^{n-1}(N \pm j)$, respectively, so in the TC models of interest here, there are $d_{S_2,A_2} = (1/2)N_{TC}(N_{TC} \pm 1)$ $SU(2)_L$ doublets comprised of technifermions. In cases where d_{S_2,A_2} is odd, one must add an odd number of other $SU(2)_L$ doublets to avoid a Witten π_4 anomaly in the $SU(2)_L$ theory. Minimally, one would add a single such doublet, and thus the set of new leptons [6] $\binom{\ell^{1/2}}{\ell^{-1/2}}_L$ and $\ell_R^{\pm 1/2}$. The $\ell^{\pm 1/2}$ must get masses that are sufficiently large, $\gtrsim 100$ GeV, to have escaped detection. This addition is necessary, for example, in the case $N_{TC} = 2$, $F = S_2$, where $d_{S_2} = 3$. The models with $F = S_2$ and $N_{TC} = 2$, and possibly also $N_{TC} = 3$, could plausibly exhibit walking [6]. For the $N_{TC} = 2$ case, owing to the necessity of adding the new heavy lepton $SU(2)_L$ doublet, the total new physics contribution to S is comprised of the three technifermion $SU(2)_L$ doublets and the heavy lepton doublet, so that $(\Delta S)_{pert.}^{(NP)} = 2/(3\pi)$. This is larger by a factor of 2 than the value in our models, given in Eq. (2). The full nonperturbative $(\Delta S)^{(NP)}$ values involve walking reductions (relative to the respective $\sim 0.1 N_D$ non-walking estimates). In these models with higher technifermion representations where the walking occurs with the given SM-nonsinglet technifermions, one would not add SM-singlet technifermions. Values of N_{TC} higher than 2 yield larger values of $(\Delta S)^{(TC)}$ and hence are less well motivated.

Regarding the $F = A_2$ case, we first observe that for $N_{TC} = 3$, this antisymmetric rank-2 tensor degenerates to just the $\bar{3}$ (conjugate fundamental) representation, for which $N_{F,c} \simeq 12$ (cf. Eq. (A.2). Since Eq. (1) corresponds to the substantially smaller value, $N_F = 2$, this case would not be expected to exhibit walking, so that $(\Delta S)^{(TC)}_{pert.} = 1/(2\pi)$ without a walking reduction. Higher values of $N_{TC} = 4$ yield larger values of $(\Delta S)^{(TC)}_{pert.}$.

6. Embedding of TC models with $\mathcal{R}_{TC} = S_2, A_2$ in ETC

In this section we investigate embeddings of technicolor models with $\mathcal{R}_{TC} = S_2, A_2$ in an ETC theory. For generality, we will usually take N_{TC} to be arbitrary.

6.1. One-doublet TC

We consider first the case where the technifermions are described by Eq. (1), so that they have the explicit form

$$\begin{pmatrix} F^{1/2\ \tau\tau'} \\ F^{-1/2\ \tau\tau'} \end{pmatrix}_L, \qquad F_R^{\pm 1/2\ \tau\tau'}. \tag{11}$$

In constructing the high-energy ETC-symmetric theory, one treats all of the ETC indices on an equal footing, so the natural embedding of the technifermions in Eq. (11) would be a rank-2 symmetric or antisymmetric representation of $SU(N_{TC} + 12)$. But this is excluded since, among other things, it would lead to various light leptoquark fermions with SM quantum numbers given by $\underline{3}$ of $SU(3)_c$, with lepton number $L = 1$ (and generational indices jk), which are not observed experimentally. (For $F = S_2$, it would also imply fermions transforming as $\underline{6}$'s of $SU(3)_c$ ("quixes"); for $F = A_2$, it would imply $SU(2)_L$ doublets of $\bar{3}$'s of $SU(3)_c$, etc.)

In view of this negative result, one is motivated to investigate whether a higher-dimensional representation of the technicolor group $SU(N_{TC})$ could occur in a fundamental representation of the extended technicolor group $SU(N_{ETC})$ which contains $SU(N_{TC})$. This does not occur for regular embeddings of $SU(N_{TC}) \subset SU(N_{ETC})$. (Here, a regular embedding of a subgroup H in a Lie group G is one in which the generators of the Lie algebra of H can be written as a restriction of, or subset of, the generators of the Lie algebra of G.) In contrast, for embeddings of subgroups $H \subset G$ which are not of this type (and are called "special" embeddings [28]), it is possible for the fundamental representation of a Lie group G to decompose, with respect to a subgroup H in such a manner as to yield a higher-dimensional representation of H. For example, with a special embedding of SU(2) in SU(3), the decomposition of the $\underline{3}$ of SU(3) yields a $\underline{3}$ of SU(2) [28]. However, we have not found any cases that appear promising for semirealistic (E)TC models with $F = S_2, A_2$. It thus remains a challenge to construct acceptable ETC models that yield TC sectors with higher-dimensional technifermion representations.

6.2. One-family TC

Among models with higher TC representations, the minimization of $(\Delta S)^{(TC)}$ motivates one to focus on the one-doublet case. Nevertheless, it is of some interest to consider how one would try to embed a one-family TC model with $F = S_2$ or $F = A_2$ in ETC. This provides a different perspective on how generations might arise, although, as we shall show, it fails to yield an acceptable TC theory.

First, recall, as background, how this embedding is carried out for the simpler case of a one-family $SU(N_{TC})$ model with $\mathcal{R}_{TC} = \text{fund}$. In both cases, $[G_{ETC}, G_{SM}] = 0$ and $SU(N_{TC}) \subset SU(N_{ETC})$. For general N_{TC}, the technifermions are $\binom{U^{a\tau}}{D^{a\tau}}_L$, $U_R^{a\tau}$, $D_R^{a\tau}$, $\binom{N^{\tau}}{E^{\tau}}_L$, N_R^{τ}, and E_R^{τ}. One forms the ETC multiplets with these SM transformation properties by gauging the generation index and combining it with the technicolor index, so that $N_{ETC} = N_{gen.} + N_{TC} = 3 + N_{TC}$. Thus, with the minimal value $N_{TC} = 2$, the ETC group would be $SU(5)_{ETC}$ and, for example, the ETC multiplet transforming as $(5, 3, 1)_{4/3,R}$ under $SU(5)_{ETC} \times SU(3)_c \times SU(2)_L \times U(1)_Y$ would be $(u, c, t, U^4, U^5)_R$. (For $N_{TC} = 2$, one can also construct ETC models with some fermions being assigned to conjugate fundamental representations [11,13].)

For the cases $F = S_2, A_2$ we first determine N_{ETC} for a given N_{TC}. Let

$$N_{ETC} = m + N_{TC} \tag{12}$$

so that

$$SU(N_{ETC}) \supset SU(m) \times SU(N_{TC}). \tag{13}$$

We next extend the S_2 and A_2 representations of $SU(N_{TC})$ to corresponding representations of $SU(N_{ETC})$, denoted with the same symbols. With respect to the direct product subgroup (13) these transform as follows:

$$S_2\colon \left(\frac{m(m+1)}{2}, 1\right) + (m, N_{TC}) + \left(1, \frac{N_{TC}(N_{TC}+1)}{2}\right), \tag{14}$$

$$A_2\colon \left(\frac{m(m-1)}{2}, 1\right) + (m, N_{TC}) + \left(1, \frac{N_{TC}(N_{TC}-1)}{2}\right). \tag{15}$$

Thus, for an S_2 or A_2 ETC fermion multiplet transforming according to a given representation of G_{SM}, the number of technisinglet components, which should be equal to the number of generations, is

$$(N_{gen.})_{S_2,A_2} = \frac{m(m \pm 1)}{2} + \delta_{A_2;N_{TC}=2}, \tag{16}$$

where the second term is a Kronecker delta function which is equal to one if $F = A_2$ and $N_{TC} = 2$ and zero otherwise. This second term is present because in Eq. (15), if $N_{TC} = 2$, the third representation is (1, 1), a technisinglet. The first few sets of pairs for $F = S_2$ are $(m, N_{gen.})$ are (1, 1), (2, 3), and (3, 6), so that in order to reproduce the physical value, $N_{gen.} = 3$, one would take $m = 2$, whence

$$N_{ETC} = 2 + N_{TC} \quad \text{for } F = S_2. \tag{17}$$

The technisinglet components are then $(\psi_\chi^{11}, \psi_\chi^{12}, \psi_\chi^{22})$, $\chi = L, R$. Parenthetically, we note that with the $F = S_2$ assignment, among toy-model values of $N_{\text{gen.}}$, one could accommodate $N_{\text{gen.}} = 1$ (for $m = 1$), but not $N_{\text{gen.}} = 2$.

For the case $F = A_2$, the first few $(m, N_{\text{gen.}})$ values are (i) $(1,1)$, $(2,2)$, $(3,4)$, etc. for $N_{\text{TC}} = 2$; (ii) $(1,0)$, $(2,1)$, $(3,3)$, etc. for $N_{\text{TC}} \geqslant 3$. Evidently, the model with $F = A_2$ and $N_{\text{TC}} = 2$ is not able to accommodate three SM fermion generations, while for $N_{\text{TC}} \geqslant 3$, this is possible with $m = 3$, so

$$N_{\text{ETC}} = 3 + N_{\text{TC}} \quad \text{for } F = A_2,\ N_{\text{TC}} \geqslant 3. \tag{18}$$

In this case, the three generations of a (technisinglet) fermion field with a given set of SM quantum numbers can be written as $(\psi_\chi^{23}, \psi_\chi^{31}, \psi_\chi^{12})$, $\chi = L, R$, where the order is a convention. For this $F = A_2$ case one would preferentially choose the minimal possible value, $N_{\text{TC}} = 3$ to minimize technicolor contributions to the electroweak S parameter.

The fact that there are restrictions on the possible values of $N_{\text{gen.}}$ in these TC models with either S_2 or A_2 fermions is quite different from the situation in ETC models in which the SM-nonsinglet fermions transform according to the fundamental representation of $SU(N_{\text{ETC}})$ and where one can accommodate an arbitrary number of SM fermion generations, subject to the constraints of asymptotic freedom of the color and technicolor groups.

We next calculate the leading-order term in the technicolor beta function. Using the values of d_{S_2} and d_{A_2} and the fact that there are $N_w(N_c + 1) = 8$ Dirac fermion components for each TC gauge index, we have, for the one-family model,

$$b_0^{(\text{TC})} = -\frac{1}{3}\left[5N_{\text{TC}} + 16(m \pm 2) + 2\sum_{\text{SMS}\,f} T(\mathcal{R}_{\text{TC},f})\right], \tag{19}$$

where the $+$ and $-$ signs apply for $F = S_2, A_2$, respectively, and the last term is the contribution from possible SM-singlet (SMS) fermions. For both of the relevant cases (i) $F = S_2$ and hence $m = 2$; and (ii) $F = A_2$ and hence $m = 3$, $N_{\text{TC}} \geqslant 3$, $b^{(\text{TC})} < 0$, i.e., the technicolor theory is not asymptotically free. Since asymptotic freedom is a necessary property of the technicolor theory, being responsible for the confinement and formation of the technifermion condensates that break the electroweak gauge symmetry, this lack of asymptotic freedom rules out these models.

7. Conclusions

In this Letter we have studied fermion representations in technicolor theories with the goal of minimizing technicolor corrections to precision electroweak quantities, in particular, the S parameter. We have constructed $SU(2)_{\text{TC}}$ models with standard-model nonsinglet technifermion sectors of the form of Eq. (1) with $I_{\text{TC}} = 1/2$ technifermions which also plausibly have the desirable property of walking behavior, owing to SM-singlet technifermion sectors. As a consequence, these models yield the rather small estimate $(\Delta S)^{(\text{TC})}_{\text{pert.}}$ in Eq. (2), and a full (nonperturbatively calculated) $(\Delta S)^{(\text{TC})}$ which is reduced by the walking. We have contrasted our results with some models that obtain walking via technifermions in higher-dimensional representations of the technicolor group. For both of these types of models, we have analyzed embeddings in extended technicolor theories. The attractively small value of $(\Delta S)^{(\text{TC})}$ in the one-doublet walking technicolor models with $\mathcal{R}_{\text{TC}} = \text{fund.}$ that we have discussed motivates further study of their embeddings in ETC and the resultant phenomenological predictions, in particular, the differences with respect to one-family ETC models.

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Appendix A

We include here some relevant results used in the text. The beta function for a given gauge interaction G_j is

$$\beta_j = \frac{d\alpha_j}{dt} = -\frac{\alpha_j^2}{2\pi}\left(b_0^{(j)} + \frac{b_1^{(j)}}{4\pi}\alpha_j + O\left(\alpha_j^2\right)\right), \tag{A.1}$$

where $\alpha_j = g_j^2/(4\pi)$, $t = \ln\mu$, and the first two terms $b_0^{(j)}$ and $b_1^{(j)}$ are scheme-independent. Provided that $b_0^{(j)} > 0$, i.e., the theory is asymptotically free, there is an infrared-stable fixed point of the renormalization group equation if $b_1^{(j)} < 0$, at $\alpha_{j,\text{IR}} = -4\pi b_0^{(j)}/b_1^{(j)}$.

Now let $G_j = G_{\text{TC}}$. For a technifermion transforming according to a representation $\mathcal{R}_{\text{TC}}$ of G_{TC}, the critical value of α_{TC} for which a bilinear technifermion condensate forms is denoted $\alpha_{\text{TC},c}$. An analysis of the Schwinger–Dyson gap equation yields the estimate [19,29] $\alpha_{\text{TC},c} \simeq \pi/(3C_2(\mathcal{R}_{\text{TC}}))$, where $C_2(\mathcal{R})$ is defined by $\sum_{a=1}^{\text{order}(G)}\sum_{j=1}^{\dim(\mathcal{R})}(T_a)_{ij}(T_a)_{jk} = C_2(\mathcal{R})\delta_{ik}$. (We also define $T(\mathcal{R})$ via $\sum_{i,j=1}^{\dim(\mathcal{R})}(T_a)^i_j(T_b)^j_i = T(\mathcal{R})\delta_{ab}$.) This estimate of $\alpha_{\text{TC},c}$ involves some theoretical uncertainty because of the strong coupling involved. A vectorial $SU(N_{\text{TC}})$ theory with N_f technifermions in the fundamental representation is expected to exist in a confining phase with SχSB if $N_f < N_{f,c}$, where [20]

$$N_{f,c} \simeq \frac{2N_{\text{TC}}(50N_{\text{TC}}^2 - 33)}{5(5N_{\text{TC}}^2 - 3)} \tag{A.2}$$

and in a non-Abelian Coulomb phase if $N_{f,c} < N_f < 11N_{\text{TC}}/2$. For $N_{\text{TC}} = 2$ and $N_{\text{TC}} = 3$ we have $N_{f,c} \simeq 8$ and $N_{f,c} \simeq 12$, respectively.

In the part of the text dealing with higher-representation technifermions, the motivation was their effect on walking. Here we comment parenthetically on a different application of higher-representation technifermions, namely the idea of using these higher representations to produce the generational mass hierarchy for the SM fermions. Thus, consider, for example, a (vectorial) technicolor theory with a set of SM-nonsinglet technifermions given by Eq. (1) with three different $\mathcal{R}_{\text{TC},j}$'s such that $\dim(\mathcal{R}_{TC,j})$ is an increasing function of the generation index j. Provided that the technicolor theory is asymptotically free and in the phase with spontaneous chiral symmetry breaking (instead of a possible non-Abelian Coulomb phase), as the energy scale decreases through the TeV region, there is a hierarchy of scales at which the technifermions of different representations condense, and hence a hierarchy of dynamical technifermion masses $\Sigma_{TC,j}$. If one could arrange the ETC dynamics so that, to leading order, the SM fermions of generation j communicate with the technifermions with TC representation $\mathcal{R}_{TC,j}$, then the resultant masses of the SM fermions of generation j, namely $m_{f_j} \propto \eta_j \Sigma^3_{TC,j}/\Lambda^2_{ETC,j}$ (where η_j is a possible walking factor), could exhibit a hierarchy due to a combination of the hierarchies in $\Sigma_{TC,j}$ and $\Lambda_{ETC,j}$, in contrast to the situation in usual ETC models with only one Σ_{TC} scale. For the technifermions in Eq. (1) with $\mathcal{R}_{TC,j}$, one has $(\Delta S)^{(TC)}_{pert.} = (6\pi)^{-1} \sum_{j=1}^{3} \dim(\mathcal{R}_{TC,j})$. Examination of specific models shows that the resultant values of $(\Delta S)^{(TC)}$ are excessively large. It also appears difficult to construct models with the appropriate ETC dynamics.

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