



Short distance non-perturbative effects of large distance modified gravity

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Abstract

In a model of large distance modified gravity we compare the non-perturbative Schwarzschild solution of [G. Gabadadze, A. Iglesias, Phys. Rev. D 72 (2005) 084024, hep-th/0407049] to approximate solutions obtained previously. In the regions where there is a good qualitative agreement between the two, the non-perturbative solution yields effects that could have observational significance. These effects reduce, by a factor of a few, the predictions for the additional precession of the orbits in the Solar system, still rendering them in an observationally interesting range. The very same effects lead to a mild anomalous scaling of the additional scale-invariant precession rate found by Lue and Starkman.

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1. Introduction

The DGP model of large distance modified gravity [1] has one adjustable parameter—the distance scale r_c . Distributions of matter and radiation which are homogeneous and isotropic at scales $\gtrsim r_c$ exhibit in this model the following properties: for distance/time scales $\ll r_c$ the solutions approximate General Relativity (GR) to a high accuracy, while for scales $\gtrsim r_c$ they dramatically differ [1–4]. Postulating that $r_c^{-1} \sim H_0 \sim 10^{-42}$ GeV the deviations from GR could lead to interesting observational consequences in late-time cosmology, see, e.g., [3,5–11].

On the other hand, sources of matter and radiation with typical inhomogeneity scale less than r_c have somewhat different properties. These are easier to discuss for a Schwarzschild source—a spherically-symmetric distribution of matter of the mass M and radius r_0 , such that $r_M < r_0 \ll r_c$ ($r_M \equiv 2G_N M$ is the Schwarzschild radius and G_N the Newton constant). For such a source a new scale, that is a combination of r_c and r_M , emerges (the so-called Vainshtein scale)¹ [4]:

$$r_* \equiv (r_M r_c^2)^{1/3}. \quad (1)$$

Above this scale gravity of a compact object deviates substantially from the GR result. Note that r_* is huge for typical astrophysical objects. An isolated star of a solar mass would have $r_* \sim 100$ pc. However, if we draw a sphere of a 100 pc radius with the Sun in its center there will be many other stars enclosed by that sphere. The matter enclosed by this sphere would have even larger r_* . We could draw a bigger sphere, but it will enclose more matter which would yield yet larger r_* and so on. An isolated object which could be separated from a neighboring one by a distance larger than its own r_* is a cluster of galaxies. For typical clusters, $r_* \sim$ (few Mpc) is just somewhat larger than their size and is smaller than their average separation. The above arguments suggest that interactions of isolated clusters will be different in the DGP model. On the other hand, at scales beneath a few Mpc or so, there will be agreement with the GR results with potentially interesting small deviations. Below we discuss these issues in detail on an example of a single isolated Schwarzschild source. There exist in the literature two different solutions for the Schwarzschild problem in the DGP model. The first one is based on approximate expansions in the $r \ll r_*$ and $r \gg r_*$ regions [1,4,13] (see also [14,15]). We call this set of results the perturbative Schwarzschild (PS) solution. The second one [16] is a solution that interpolates smoothly from $r \ll r_*$ to $r \gg r_c \gg r_*$, and is non-analytic in the either parameters used to obtain the PS solution. We call this the non-perturbative Schwarzschild (NPS) solution. It is important to understand which of these two solutions, if any, is physically

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¹ A similar, but not exactly the same scale was discovered by Vainshtein in massive gravity [12], hence the name.

viable. Since neither of the two can be solved completely without numerical simulations, a first step to discriminate between them would be to look closely at the theoretical differences, as well as predictions that could be tested observationally. This is the goal of the present note.

2. Qualitative discussions

We will study separately two regimes, $r \ll r_*$ and $r \gg r_*$.

(I) $r \ll r_*$. In this regime the standard G_N expansion breaks down [4]. How could one proceed? One way is to perform an expansion in powers of $m_c = r_c^{-1}$ [4]. This expansion breaks down above $r \sim r_*$ but is well suited for the $r \ll r_*$ domain. (Kaloper [17] recently used a different expansion. His proposal could prove to be useful for a broad class of problems.) A Schwarzschild metric in the small m_c expansion was calculated by Gruzinov [13] (see also [14]). It is instructive to compare the result of [13] with the NPS solution of [16].

Let us start with the Newton potential $\phi(r)$. The expansion of the exact result of [16] for $r \ll r_*$ leads:

$$-2\phi = \frac{r_M}{r} - \alpha m_c^2 r^2 \left(\frac{r_*}{r} \right)^{\frac{3}{2} - \beta} + \dots, \quad (2)$$

where $\beta = 3/2 - 2(\sqrt{3} - 1) \simeq 0.04$, and α is a number to be discussed in detail below. The above result, but with $\beta = 0$, is what was first obtained in a small m_c expansion [13]. The NPS solution of [16] gives $\beta \simeq 0.04$, it depends on irrational powers of m_c [16], and it differs by that from the small m_c expansion results.

Is the above difference important? As was demonstrated in Refs. [14] and [18], the modification of the Newton potential in (2), although tiny, could lead to a measurable precession of orbits in the Solar system (see, Ref. [19] for further studies). The above works used the potentials obtained in the small m_c expansion, e.g., used (2) with $\beta = 0$. Although β is tiny, the ratio (r_*/r) is typically huge in the cases of interest, therefore, taking into account the effects of a non-zero β could lead to appreciable differences in the predictions of the PS and NPS solutions. We will study this issue in the next section.

Consequences of the modified potential (2) could be understood as well in terms of invariant curvatures. The Schwarzschild solution in GR has zero scalar curvature. In contrast with this, the solution (2) generates a non-zero Ricci scalar that extends to $r \sim r_*$ in the NPS solution (see, [16] and discussions below). This can be seen by looking at the trace equation in the DGP model:

$$R - 3m_c K = T, \quad (3)$$

where R is the 4D Ricci scalar, K is a trace of an extrinsic curvature and T is a trace of the stress-tensor times $8\pi G_N$ (for the ADM formalism in the DGP model see, e.g., [20,21]). This has to be compared with the trace equation in GR: $R = T$. The second term on the LHS of (3) is not zero outside the source and, therefore, gives rise to non-zero R . This curvature, although tiny, extends to enormous scales of the order of $r \sim r_*$ [16].

The sign of the curvature depends on a choice of the boundary conditions in the bulk, since the latter determines the sign of K . There are two choices for this. The so-called conventional branch corresponds to a negative (AdS like) curvature produced by the Schwarzschild source, while the selfaccelerated branch [2] corresponds to a positive (dS like) R . This is reflected in the sign of the coefficient α in (2) which takes a positive value on the conventional branch and becomes negative on the selfaccelerated branch: $\alpha \simeq \pm 0.84$. Therefore, there is an additional tiny attraction toward the source on the conventional branch and a repulsion of the same magnitude on the selfaccelerated branch. This change of sign was first found by Lue and Starkman [14] in the context of the PS solution.

(II) $r \gg r_*$. In this regime the small m_c expansion breaks down. However, the conventional G_N expansion can be readily used [1,4]. The results are [1]:

(A) For $r \gg r_*$ DGP gravity is a tensor-scalar theory, where the extra scalar couples to matter with the gravitational strength: the vDVZ phenomenon [22,23].

(B) The Newton potential scales as $1/r$ for $r_* \ll r \ll r_c$ which smoothly transitions into the $1/r^2$ potential at $r \gg r_c$.

These properties of the PS solution were reconfirmed in detailed studies of Refs. [13–15,24,25]. Could the PS solution interpolate from $r \ll r_*$ to $r \gg r_c$? The above question is related to the following one: what is a gravitational mass that is felt by an object separated from the source at a distance $r \gg r_*$? The PS solution implies that this is just the bare mass M of the original source. On the other hand, one may expect that the curvature created by the source in the domain $r \ll r_*$ would also contribute to this effective mass (the ADM mass) [16]. If so, unless there is a hidden non-trivial cancellation, a putative observer at $r \gg r_*$ would measure an effective mass different from M . The above property is captured by the NPS solution of Ref. [16]. It has the following features:

(A') For $r \gg r_*$ it is a solution of a tensor-scalar gravity (as in (A) above);

(B') The Newton potential scales as $1/r^2$ for $r \gg r_*$ (different from (B)).

An attractive feature of the NPS solution is that it smoothly interpolates from $r \ll r_*$ to $r \gg r_c$. However, a somewhat unusual fact is that it does not recover the results of the G_N expansion. This will be discussed in the reminder of this section (readers who are not interested in these somewhat technical issues could directly go to the next section without loss of clarity).

Why is that, that the NPS solution [16] does not agree with the results of the perturbative G_N expansion, even in the regime $r \gg r_*$, where the latter approximation is internally self-consistent? There could be a few different reasons for this. Formally, one is solving non-linear partial differential equations and these can have different solutions even with the same boundary conditions. In our two cases, however, the boundary

conditions are somewhat different: the PS solution is supposed to describe the same mass M at short and large distances, while the NPS solution matches M at the short scales but asymptotes to a screened mass at the large scales.² Then either the PS and NPS solutions belong to different sectors and are both stable, or at least one of them should be unstable. In the former case, one should distinguish between them observationally, while in the latter case a relevant point would be that the ADM mass of the NPS solution is smaller [16]. In a very qualitative way, this can be understood as follows. A deviation from the conventional metric at $r \ll r_*$ scales as $m_c \sqrt{r_M r}$ (we ignore small β here). This can give rise to a scaling of the scalar curvature $m_c \sqrt{r_M r}^{-3/2}$. The curvature extends roughly to $r \sim r_*$, and the integrated curvature scales as $m_c \sqrt{r_M r_*}^{3/2} \sim r_M$. Then, the “effective mass” due to this curvature can be estimated as $r_M M_P^2 \sim M$, which is of the order of the mass itself.

On the other hand, it may well be that there is a certain “discontinuity” between the linearized and full non-linear versions of the DGP model in 5D. This could result from a different number of constraints one has to satisfy depending on whether solutions are looked for in the linearized approximation or in the full non-linear theory. For instance, one of the bulk equations can be combined with the junction condition in 4D to yield:

$$3m_c^2 R = R^2 - 3R_{\mu\nu}^2. \quad (4)$$

On a flat background both terms on the RHS of (4) contain at least quadratic terms in the fields. Therefore, according to (4), R has to be zero in the linearized approximation. The latter condition happens to be a consequence of the other linearized equations of the theory as well; therefore, (4) is trivially satisfied as long as those other equations are fulfilled. This changes at the non-linear level: Eq. (4) becomes an additional constraint that one has to satisfy on top of the other equations. Because of this: (i) The solutions of the linearized theory may not be supported by the non-linear equations (the phenomenon known as “linearization instability” in gravity). (ii) New non-perturbative solutions that do not exist in the linearized theory may emerge. One way to decide on the point (i), is to study solutions for other sources and see whether a similar phenomenon takes place. The NPS solution of [16] is an explicit example of the point (ii).

3. Explicit solution

We consider the action of the DGP model [1]:

$$S = M_*^3 \int d^5x \sqrt{-g} R + M_P^2 \int d^4x \sqrt{-\tilde{g}} \tilde{R}. \quad (5)$$

Here, the $(4+1)$ coordinates are $x^M = (x^\mu, y)$, $\mu = 0, \dots, 3$ and g and R are the determinant and curvature of the 5-dimensional metric g_{MN} , while \tilde{g} and \tilde{R} are the determinant and curvatures of the 4-dimensional metric $\tilde{g}_{\mu\nu} = g_{\mu\nu}(x^\mu, y=0)$. The Gibbons–Hawking [26] surface term that guarantees correct equations of motion is implied in the action (5).

M_P denotes the 4D Planck mass and is fixed by the Newton constant. On the other hand, the scale M_* is traded for the parameter $r_c \equiv M_P^2/2M_*^3$ discussed in the previous section.

The NPS solution studied in [16] is found by considering a static metric with spherical symmetry on the brane and with \mathbf{Z}_2 symmetric line element:

$$ds^2 = -e^{-\lambda} dt^2 + e^\lambda dr^2 + r^2 d\Omega^2 + \gamma dr dy + e^\sigma dy^2, \quad (6)$$

where λ, γ, σ are functions of $r = \sqrt{x^\mu g_{\mu\nu} x^\nu}$ and y . The \mathbf{Z}_2 symmetry across the brane implies that γ is an odd function of y while the rest are even. The brane is chosen to be straight in the above coordinate system.³

The exact solution for $y \rightarrow 0+$ is given implicitly as follows:

$$e^{-\lambda} = 1 - \frac{P(r)}{r}, \quad (7)$$

where P is obtained from

$$P(r) = -\frac{3}{2}m_c^2 \int dr r^2 U(r), \quad (8)$$

in which U can have two different behaviors corresponding to the solution of the following two equations (giving rise to a conventional and selfaccelerated branch respectively):

$$(k_1 r)^8 = -\frac{(1+3U+f)}{U^2(3+3U+\sqrt{3}f)^{2\sqrt{3}}(-5-3U+f)}, \quad (9)$$

$$(k_2 r)^8 = -\frac{(-5-3U+f)(-3-3U-\sqrt{3}f)^{2\sqrt{3}}}{(U+2)^2(1+3U+f)}, \quad (10)$$

where $f = \sqrt{1+6U+3U^2}$ and k is an integration constant.

Note that in this parametrization the gravitational potential ϕ in weak field approximation is easily obtained, namely

$$\phi \equiv -\frac{P(r)}{2r}. \quad (11)$$

The off-diagonal metric component, γ , is determined from

$$\frac{2r^2 P_r}{P_{rr}} = \frac{(r^4 \gamma e^{-\lambda})_r}{(r \gamma e^{-\lambda})_r}, \quad (12)$$

and the yy component from

$$e^\sigma = m_c^2 \left[\frac{(r^4 \gamma e^{-\lambda})_r}{2r^2 P_r} \right]^2 + e^{-\lambda} \gamma^2. \quad (13)$$

The profile λ_y for $y \rightarrow 0+$ can be computed as well:

$$\lambda_y = e^{-\lambda} \gamma_r. \quad (14)$$

The two integration constants, k and the one produced in the integration (8), are determined by imposing appropriate boundary conditions at the source (namely, $P(r \rightarrow 0+) \rightarrow r_M$) and at large distances, (namely, $\lambda \sim \tilde{r}_M^2/r^2$ in the conventional branch or $\lambda \sim m_c^2 r^2 + \tilde{r}_M^2/r^2$ in the selfaccelerated branch and no $1/r$ term).

³ One could transform (6) to the coordinate system where the metric is diagonal $ds^2 = -A(r, z) dt^2 + B(r, z) d\rho^2 + C(r, z) d\Omega^2 + dz^2$, and $A \neq B$. In this system our brane will be bent.

² The boundary conditions at the brane are also different, see footnote 3.

3.1. Conventional branch

The conventional branch is obtained from the solution of (9). As shown in [16] the boundary conditions ($P(0) = r_M$, $P(+\infty) = 0$) determine the exact relation between k_1 and r_* , namely

$$2(r_*k)^3 = c, \quad (15)$$

where c is the following integral:

$$c = \int_0^\infty \left[-\frac{(1+3U+f)}{U^2(3+3U+\sqrt{3}f)^{2\sqrt{3}}(-5-3U+f)} \right]^{3/8} dU \approx 0.43. \quad (16)$$

The solution has the following asymptotic behavior. At large distances, $r \gg r_*$ ($U \rightarrow 0^+$), we obtain

$$\frac{P(r)}{r} = \frac{\tilde{r}_{M_1}^2}{r^2} + \dots, \quad (17)$$

where,

$$\tilde{r}_{M_1}^2 = \frac{3\sqrt{2}}{4(3+\sqrt{3})\sqrt{3}} \frac{m_c^2}{k_1^4} \approx 0.56r_M r_*, \quad (18)$$

while at short distances, $r \ll r_*$ ($U \rightarrow +\infty$), we get

$$\frac{P(r)}{r} = \frac{r_M}{r} - \alpha_1 m_c^2 r^2 \left(\frac{r_*}{r} \right)^{2(\sqrt{3}-1)} + \dots, \quad (19)$$

where

$$\alpha_1 = 6^{(\sqrt{3}-1)/2} \frac{1+\sqrt{3}}{4(3\sqrt{3}-1)} \left(\frac{3+\sqrt{3}}{3-\sqrt{3}} \right)^{(\sqrt{3}-1)/4} (k_1 r_*)^{2(1-\sqrt{3})} \approx 0.84. \quad (20)$$

As we see, a short distance observer at $r_M \ll r \ll r_*$ would measure the gravitational mass M with a small corrections to Newton's potential, while the large distance observer at $r \gg r_*$ would measure an effective gravitational mass $\sim M(r_M/r_c)^{1/3}$ [16]. The latter includes the effects of the 4D curvature.

3.2. Selfaccelerated branch

The solution on the selfaccelerated branch is obtained from (10). The relation between k_1 and r_* is obtained, as in the conventional case, by imposing boundary conditions ($P(0) = r_M$, $P(r) - m_c^2 r^3 \rightarrow 0$ for large r). This gives

$$\begin{aligned} 2(r_*k_2)^3 &= - \int_{-\infty}^{-2} (U+2) \frac{d}{dU} \left[-\frac{(1+3U+f)}{U^2(3+3U+\sqrt{3}f)^{2\sqrt{3}}(-5-3U+f)} \right]^{3/8} dU \\ &= 6^{3\sqrt{3}/4} c \approx 4.41. \end{aligned} \quad (21)$$

The second line in (21), that is generated by a change of variables in the integral ($\tilde{U} = -U - 2$) while using (15), also gives a relation between k_1 and k_2 ,

$$k_2 = 6^{\sqrt{3}/4} k_1. \quad (22)$$

The solution has the following asymptotic behavior. At large distances, $r \gg r_*$ ($U \rightarrow -2^-$), we derive

$$\frac{P(r)}{r} = -\frac{\tilde{r}_{M_2}^2}{r^2} + m_c^2 r^2 + \dots, \quad (23)$$

where,

$$\tilde{r}_{M_2}^2 = \frac{3}{(3-\sqrt{3})2\sqrt{3}} \frac{m_c^2}{k_2^4} \approx 0.45r_M r_*, \quad (24)$$

while at short distances, $r \ll r_*$ ($U \rightarrow -\infty$), we get

$$\frac{P(r)}{r} = \frac{r_M}{r} - \alpha_2 m_c^2 r^2 \left(\frac{r_*}{r} \right)^{2(\sqrt{3}-1)} + \dots, \quad (25)$$

where $\alpha_2 = -\alpha_1 \approx -0.84$ is, in absolute value, the same constant appearing in the conventional branch short distance expansion (19). Note, however, that the sign of the correction to the 4D behavior is opposite in the two branches.

At intermediate distances, $r_* \ll r \ll r_c$, the potential contains a 5D gravitational term that is *repulsive*, \tilde{r}_M^2/r^2 . This looks like a 5D negative mass. However, this is not an asymptotic value of the mass since one can only cover the solution in the above coordinate system till $r \sim r_c$ where the dS like horizon is encountered. Moreover, in the intermediate regime $r_* \ll r \ll r_c$, the de Sitter term $m_c^2 r^2$ in the potential always dominates over the \tilde{r}_M^2/r^2 term suggesting that the effects due to the Schwarzschild source are strongly suppressed.

3.3. Perihelion precession

The deviation from 4D gravity (2) gives rise to the additional perihelion precession of circular orbits [14,18] (see also [19] for comprehensive studies of these and related issues). In a simplest approximation this effect is quantified by a fraction of the deviation of the potential from its Newtonian form

$$\epsilon \equiv \frac{\Delta\phi}{\phi}. \quad (26)$$

This can be used to evaluate an additional perihelion precession of orbits in the Solar system [14,18].⁴ As we discussed in Section 1, the ϵ ratio is somewhat different for the non-perturbative solution (NPS solution) as compared to the approximate solution (the PS solution) used in Refs. [14,18]. We can easily calculate this difference:

$$\frac{\epsilon_{\text{NPS}}}{\epsilon_{\text{PS}}} \simeq \frac{|\alpha|}{\sqrt{2}} \left(\frac{r}{r_*} \right)^\beta \simeq 0.59 \left(\frac{r}{r_*} \right)^{0.04}. \quad (27)$$

The perihelion precession per orbit is

$$\Delta\phi = 2\pi + \frac{3\pi r_M}{r} \mp \frac{3\pi|\alpha|}{4} \left(\frac{r}{r_*} \right)^{3/2} \left(\frac{r}{r_*} \right)^{0.04}. \quad (28)$$

⁴ Note that in the leading order of the relativistic expansion the answer is given by the correction to the Newtonian potential, while the correction to the rr component of the metric is not important.

The second term on the RHS is the Einstein precession, and the last term arises due to modification of gravity. For the PS solution this was first calculated in Refs. [14,18]; the solution (28) is written for the NPS solution and is somewhat different.

For the Earth–Moon system $r = 3.84 \times 10^{10}$ cm and $r_*^{\text{Earth}} \simeq 6.59 \times 10^{12}$ cm; as a result the ratio in (27) is approximately 0.48. Therefore, the predictions of the NPS solution for the additional perihelion precession of the Moon is a factor of two smaller than the predictions of the approximate solution. The result of (28) for the additional precession (the last term on the RHS) is $\mp 0.7 \times 10^{-12}$ (the plus sign for the selfaccelerated branch). This is below the current accuracy of 2.4×10^{-11} [27], but could potentially be probed in the near future [28].⁵

A similar calculations can be performed for the anomalous Martian precession [14,18]. For the Sun–Mars system we get:

$$\frac{\epsilon_{\text{NPS}}}{\epsilon_{\text{PS}}} \simeq 0.59 \left(\frac{r_{\text{Sun-Mars}}}{r_*^{\text{Sun}}} \right)^{0.04} \simeq 0.30, \quad (29)$$

where we used $r_{\text{Sun-Mars}} = 2.28 \times 10^{13}$ cm and $r_*^{\text{Sun}} = 4.9 \times 10^{20}$ cm. Therefore, we see that the suppression in the NPS result for the precession of the Martian orbit is stronger. The additional precession of the Mars orbit is $\sim \mp 1.3 \times 10^{-11}$, which should be contrasted with a potential accuracy of the Pathfinder mission $\sim 9 \times 10^{-11}$.

Last but not least, Lue and Starkman (LS) [14], found that the PS solution gives rise to a correction to the precession rate (additional precession per unit time),

$$\Gamma_{\text{LS}} = \mp \frac{3}{8r_c}, \quad (30)$$

that is universal, i.e., is independent of the source. The NPS solution, predicts a weak anomalous violation of the universal Lue–Starkman scaling due to the RHS of (27). The results is

$$\Gamma = \Gamma_{\text{LS}} \times \frac{|\alpha|}{\sqrt{2}} \left(\frac{r}{r_*} \right)^{0.04}. \quad (31)$$

This rate depends mildly on the source mass and a separation from it. The rate is a slowly increasing function of r , as opposed to the rate due to the second term on the RHS of (28), which is decreasing with growing r as $\Gamma_{\text{Einstein}} = \sqrt{9r_M^3/8r^5}$.

4. Outlook

In this note we compared the PS [1,4,13–15] and NPS [16] solutions in the DGP model. We emphasized different, but interesting predictions that these two solutions make in the observationally accessible domain of $r \ll r_*$. These predictions are testable.

As we have also mentioned, there will be important differences in the predictions at $r \gg r_*$. These need further detailed studies, especially in the context of the structure formation. We would expect that both the linear as well as non-linear regimes of the structure formation will be affected. If the NPS solution

is the right one, then even at very large scales non-perturbative techniques should be used. Moreover, the non-linear regime of the structure formation could be sensitive to, and be able to discriminate between, the PS and NPS solutions.

The same issue of non-linear interactions arises in the context of strong coupling behavior in the 5D DGP model [4,30–33]. This is related to the problem of the UV completion of the quantum theory [30,31] for which seemingly two different proposals were put forward in Refs. [32] and [33]. It would be interesting to pursue these studies further. The string theory realizations of brane induced gravity of Refs. [34–36] can be taken as a guideline. It would also be interesting to understand the NPS solution in terms of the approach of Refs. [30, 33].

We have not touched upon the issue whether the small fluctuations on the selfaccelerated branch contain a negative norm state [30,33], or not (see also [37]), and when these fluctuations are relevant. Additional investigations on this issue are being conducted.

It would also be interesting to look at the Schwarzschild solutions in models of large distance modified gravity where non-linear interactions do not exhibit the strong coupling behavior. This is the case [38] in a certain models of brane induced gravity in more than five dimensions [38,39], as well as in the “dielectric regularization” of the 5D DGP model [40]. Finally we would also point out the constrained approach to the 5D DGP model [41–43] in which case strong interactions also seem to be absent. All the above deserves further detailed investigations.

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⁵ An interesting possibility that similar effects could lead to seemingly observable increase of the astronomical unit was recently discussed in [29].

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