Abstract

We present the results of a recent simultaneous study of the muon capture reactions $^2\text{H}(\mu^-,\nu_\mu)nn$ and $^3\text{He}(\mu^-,\nu_\mu)^3\text{H}$. The initial and final $A = 2$ and 3 nuclear wave functions are obtained from the Argonne $v_{18}$ or chiral N$^3$LO two-nucleon potential, in combination with, respectively, the Urbana IX or chiral N$^2$LO three-nucleon potential in the case of $A = 3$. The weak current consists of polar- and axial-vector components. The former are related to the isovector piece of the electromagnetic current via the conserved-vector-current hypothesis. These and the axial currents are derived either in a meson-exchange or in a chiral effective field theory ($\chi$EFT) framework. In the first case, the only parameter is in the axial-vector current and is fixed by reproducing the experimental Gamow-Teller matrix element in tritium $\beta$-decay (GTEXP). In the second case, the low-energy constants, two in the polar and one in the axial-vector current, are fixed, respectively, by reproducing the $A = 3$ magnetic moments and GTEXP. The total rates are found to be $392.0 \pm 2.3 \text{ s}^{-1}$ for $A = 2$, and $1484 \pm 13 \text{ s}^{-1}$ for $A = 3$, where the spread accounts for the model dependence relative to the adopted interactions and currents and to cutoff sensitivity in the $\chi$EFT currents. These values are compared with available experimental data and results of previous calculations.

Keywords: Negative muon capture, Deuteron, $^3\text{He}$

PACS: 23.40.-s, 21.45.-v, 27.10.+h

1. Introduction

There is a significant body of experimental and theoretical work on muon captures on light nuclei, motivated by the fact that these processes provide a testing ground for wave functions and, indirectly, the interactions from which these are obtained, and for models of the nuclear weak current. This is particularly important for processes, such as the astrophysically relevant weak captures on proton and $^3\text{He}$ (the $pp$ and $hep$ reactions), whose rates cannot be measured experimentally, and for which one has to rely exclusively on theory. Thus, it becomes crucial to study within the same theoretical framework related electroweak transitions, whose rates are known experimentally [1]. Muon captures are among such reactions.

In the present work, we focus our attention on muon capture on deuteron and $^3\text{He}$, i.e., on the reactions

$$\mu^- + d \rightarrow n + n + \nu_\mu,$$

$$\mu^- + ^3\text{He} \rightarrow ^3\text{H} + \nu_\mu.$$  

These reactions have been studied extensively through the years, experimentally and theoretically. The observables of interest are the doublet capture rate $\Gamma^D$ for reaction (1), i.e., the rate obtained when the stopped muons are captured from the doublet hyperfine state, and the total capture rate $\Gamma_0$ for reaction (2). The experimental situation for
reaction (2) is quite clear: a very precise determination yielded $\Gamma_0 = 1496(4) \text{ s}^{-1}$ [2], a value consistent with those of the earlier measurements, although these were affected by considerably larger uncertainties. On the other hand, $\Gamma^D$ is poorly known: the available experimental data are $365(96) \text{ s}^{-1}$ [3], $445(60) \text{ s}^{-1}$ [4], $470(29) \text{ s}^{-1}$ [5] and $409(40) \text{ s}^{-1}$ [6]. These measurements, while consistent with each other, are not very precise, with errors in the 6–10 % range. However, there is hope to have this situation clarified by the MuSun Collaboration [7], which is performing at present an experiment at the Paul Scherrer Institut, with the goal of measuring $\Gamma^D$ with a precision of 1 %.

Theoretical work on reactions (1) and (2) is just as extensive, and a list of publications, updated to the late nineties, is given in Table 4.1 of Ref. [8], and in Ref. [9]. Here we comment only on the studies of Ando et al. [10] and Ricci et al. [11] for reaction (1) and Congleton and Fearing [12], Congleton and Truhlik [13], and Gazit [14] for reaction (2). We also will comment on the study of Ref. [15].

Much along the lines of the study of the $pp$ and $hep$ reactions of Ref. [16], the authors of Ref. [10] calculated $\Gamma^D$ for reaction (1) within a hybrid chiral effective field theory ($\chi$EFT) approach, in which matrix elements of weak operators derived in $\chi$EFT were evaluated between wave functions obtained from a realistic potential, specifically the Argonne $v_{18}$ (AV18) [17]. The $\chi$EFT axial current contains a low-energy constant which was fixed by reproducing the experimental Gamow-Teller matrix element (GTEXP) in tritium $\beta$-decay. The calculation, however, retained only the $S$-wave contribution in the final scattering state (the $^1S_0$ state), and higher partial-wave contributions were estimated based on Ref. [18]. This approach yielded a value for $\Gamma^D$ of 386 $\text{ s}^{-1}$, with $\Gamma^D(^1S_0)=245(1) \text{ s}^{-1}$, the theoretical error being related to the experimental uncertainty in GTEXP.

The calculation of Ref. [11] has been performed within the “Standard Nuclear Physics Approach” (SNPA), i.e., using a realistic potential model to obtain the nuclear wave functions, and including in the nuclear weak current operator both one-body (impulse approximation) and two-body contributions. In particular, in Ref. [11], the nuclear wave functions have been obtained with the Nijmegen I and Nijmegen 93 [19] Hamiltonian models, and two-body meson-exchange currents have been derived from the hard pion chiral Lagrangians of the $N\Delta\pi\rho\omega a_1$ system. The final results for $\Gamma^D$ are in the range of 416–430 $\text{ s}^{-1}$ (see Table 1 of Ref. [11]), depending on the potential used, with $\Gamma^D(^1S_0)=254–268 \text{ s}^{-1}$. It should be noticed that the model for the axial current of Ref. [11] is not constrained by data, resulting in the relatively large spread in $\Gamma^D$ values. This is not the case of Ref. [10], as well as the studies of the $pp$ and $hep$ reactions of Refs. [20, 21, 16].

Theoretical studies for reaction (2) within the SNPA have been performed in the early nineties by Congleton and Fearing [12] and Congleton and Truhlik [13]. In this later work, the nuclear wave functions were obtained from a realistic Hamiltonian based on the Argonne $v_{14}$ (AV14) two-nucleon [22] and the Tucson-Melbourne (TM) three-nucleon [23] interactions. The nuclear weak current retained contributions similar to those of Ref. [11]. The value obtained for the total capture rate $G_0$ was 1502(32) $\text{ s}^{-1}$, the uncertainty due to poor knowledge of some of the coupling constants and cutoff parameters entering the axial current.

A first attempt to study muon capture on $^3\text{He}$ in a way that was consistent with the approach adopted for the weak proton capture reactions in Refs. [20, 21], was performed in Ref. [15]. The nuclear wave functions were obtained with the hyperspherical-harmonics (HH) method (see Ref. [24] for a recent review), from a realistic Hamiltonian based on the AV18 two-nucleon and Urbana IX [25] (UIX) three-nucleon interactions. The model for the nuclear weak current was taken from Refs. [20, 21]. However, two additional contributions were included: the single-nucleon pseudoscalar charge operator and the pseudoscalar two-body term in the $N$-to-$\Delta$ transition axial current. Both contributions are of order $O(q^2/m^2)$, where $q$ is the momentum transfer in the process and $m$ is the nucleon mass, and are obviously neglected in the $pp$ and $hep$ captures studies, for which $q \ll m$. The axial coupling constant for the $N$-to-$\Delta$ transition was constrained to reproduce GTEXP. The total capture rate $G_0$ was found to be $1484(8) \text{ s}^{-1}$, where the uncertainty resulted from the adopted fitting procedure and experimental error on GTEXP. A calculation based on the older AV14/TM Hamiltonian model yielded a $G_0$ of $1486(8) \text{ s}^{-1}$, suggesting a weak model-dependence.

A study of reaction (2) within the hybrid $\chi$EFT approach has been performed by Gazit in Ref. [14]. The nuclear wave functions have been obtained with the Effective Interaction HH method [26], and the $\chi$EFT weak current is that of Ref. [16]. It has yielded a value for $G_0$ of $1499(16) \text{ s}^{-1}$, where the error has two main sources: the experimental uncertainty on the triton half-life, and the calculation of radiative corrections.

Recently, a simultaneous study of both reactions (1) and (2) in a consistent framework has been performed in Ref. [27]. Both SNPA and $\chi$EFT models for the weak current operators have been used. The $A=2$ nuclear wave functions have been derived from realistic Hamiltonian models, the AV18 or the chiral N$^3$LO (N3LO) [28] potentials. The $A=3$ nuclear wave functions have been derived with the HH method using the AV18/UIX or chiral N3LO
together with the N^2LO (N2LO) three-nucleon interaction [29]. Here we review this work and discuss its main results.

2. Theoretical formalism

We briefly review the formalism used in the calculation for the muon capture processes, discussed at length in Refs. [27, 15]. The muon capture on deuteron and \(^3\)He is induced by the weak interaction Hamiltonian [30],

\[ H_W = \frac{G_V}{\sqrt{2}} \int dx \, l_\nu(x) j^\nu(x), \]

where \(G_V\) is the Fermi coupling constant, \(G_V = 1.14939 \times 10^{-5}\) GeV\(^{-2}\) [31], and \(l_\nu\) and \(j^\nu\) are the leptonic and hadronic current densities, respectively. The transition amplitude can be written as

\[ T_W(f, f_\nu; s_1, s_2, h_\nu) \equiv \langle nn, s_1, s_2; \nu, h_\nu | H_W | (\mu, d); f, f_\nu \rangle \]

\[ \approx \frac{G_V}{\sqrt{2}} \psi_{AV}^{(1S)} \sum_{s_\mu s_d} \left( \frac{1}{2} s_\mu, \frac{1}{2} s_d \right) l_\nu(h_\nu, s_\mu) \langle \Psi_{p, s_1 s_2}(nn) | j^\nu(q) | \Psi_d(s_d) \rangle, \]

for muon capture on deuteron, \(p\) being the \(nn\) relative momentum, and [15]

\[ T_W(f, f_\nu; s'_3, h_\nu) \equiv \langle \mu, \mu^*; s'_3, h_\nu | H_W | (\mu^*, \mu^*); f, f_\nu \rangle \]

\[ \approx \frac{G_V}{\sqrt{2}} \psi_{AV}^{(1S)} \sum_{s_\mu s_3} \left( \frac{1}{2} s_\mu, \frac{1}{2} s_3 \right) l_\nu(h_\nu, s_\mu) \langle \Psi_{H_3}(s'_3) | j^\nu(q) | \Psi_{He}(s_3) \rangle, \]

for muon capture on \(^3\)He. In order to account for the hyperfine structure in the initial system, the muon and deuteron or \(^3\)He spins are coupled to states with total spin \(f = 1/2\) or \(3/2\) in the deuteron case, and \(f = 0\) or \(1\) in the \(^3\)He case. In Eqs. (4) and (5) we have defined with \(s_\mu\) (\(h_\nu\)) the \(z\)-component of the muon spin (muon neutrino helicity), and with \(s_1, s_2, s_d, s_3, s'_3\) the analogous \(z\)-components of the two neutron, deuteron, \(^3\)He and \(^3\)H spins. The Fourier transform of the nuclear weak current has been introduced as

\[ j^\nu(q) = \int dx \, e^{iq \cdot x} j^\nu(x) \equiv (\rho(q), j(q)), \]

with the leptonic momentum transfer \(q\) defined as \(q = k_\mu - k_\nu \sim -k_\nu, k_\mu\) and \(k_\nu\) being the muon and muon neutrino momenta. The function \(\psi_{AV}^{(1S)}\) has been introduced to take into account the initial bound state of the muon in the atom and the charge distribution of the nucleus. It is typically approximated as [30] \(|\psi_{AV}^{(1S)}|^2 = \frac{\alpha_{\mu\nu}}{\pi}\) for muon capture on deuteron, and [15] \(|\psi_{AV}^{(1S)}|^2 = \mathcal{R} \frac{2 \alpha_{\mu\nu}}{\pi r_{\mu\nu}^2 \mu_{\mu\nu}}\) for muon capture on \(^3\)He, where \(\alpha\) is the fine structure constant (\(\alpha = 1/137\)), \(\mu_{\mu\nu}\) and \(\mu_{\mu\nu}^\mu\) are the reduced masses of the \((\mu, d)\) and \((\mu, \mu^*)\) systems, and the factor \(\mathcal{R}\) approximately accounts for the finite extent of the nuclear charge distribution [30]. This factor has been explicitly calculated in Ref. [27] using the AV18/UIX and N3LO/N2LO Hamiltonian models, and has been found within a percent of 0.98, the value commonly found in the literature [30].

In the case of muon capture on deuteron, the final state wave function \(\Psi_{p, s_1 s_2}(mn)\) is expanded in partial waves, and the calculation is restricted to total angular momentum \(J \leq 2\) and orbital angular momentum \(L \leq 3\), i.e., in a spectroscopic notation, to \(^1S_0, ^3P_0, ^3P_1, ^3P_2\) and \(^1D_2\). Standard techniques [21, 30] are then used to carry out the multipole expansion of the weak charge, \(\rho(q)\), and current, \(j(q)\), operators. Details of the calculation can be found in Ref. [27]. Here we only note that all the contributing multipole operators selected by parity and angular momentum selection rules are included.

The total capture rate for the two reactions under consideration is then defined as

\[ d\Gamma = 2\pi \delta(\Delta E) |T_W|^2 \times \text{(phase space)}, \]
where $\delta(\Delta E)$ is the energy-conserving $\delta$-function, and the phase space is $dp\,dk_x/(2\pi)^6$ for muon capture on deuteron and just $dk_x/(2\pi)^3$ for muon capture on $^3$He. The following notation has been introduced: (i) for muon capture on the deuteron

$$|T_W|^2 = \frac{1}{2f+1} \sum_{s_1s_2} \sum_{s_1} |T_W(f, f_z; s_1, s_2, h_r)|^2,$$

and the initial hyperfine state has been fixed to be $f = 1/2$; (ii) for muon capture on $^3$He

$$|T_W|^2 = \frac{1}{4} \sum_{s_1} \sum_{f_\sigma} |T_W(f, f_z; s_1, h_r)|^2,$$

and the factor 1/4 follows from assigning the same probability to all different hyperfine states.

After carrying out the spin sums, the differential rate for muon capture on deuteron ($d\Gamma^D/dp$) and the total rate for muon capture on $^3$He ($\Gamma^D_0$) are easily obtained [27]. In order to calculate the total rate $\Gamma^D$ for muon capture on deuteron, $d\Gamma^D/dp$ is plotted versus $p$ and numerically integrated.

The initial and final $A = 2$ and 3 nuclear wave functions entering in Eqs. (4) and (5) have been obtained from the AV18 [17] or the N3LO [28] two-nucleon potential, in combination with, respectively, the UIX [25] or chiral N2LO [29] three-nucleon potentials in the case of $A = 3$. The HH expansion method has been used to solve the $A$-body bound and scattering problem. This method, as implemented in the case of $A = 3$ systems, has been reviewed in considerable detail in a series of recent publications [24, 32, 33]. We have used the same method in the context of $A = 2$ systems, for which of course wave functions could have been obtained by direct solution of the Schrödinger equation. A detailed discussion for the $A = 2$ wave functions is given in Ref. [27].

The weak current consists of polar- and axial-vector components, derived within two different frameworks, SNPA and $\chi$EFT. The first one goes beyond the impulse approximation, by including meson-exchange current contributions and terms arising from the excitation of $\Delta$-isobar degrees of freedom. The second approach includes two-body contributions derived in heavy-baryon chiral perturbation theory within a systematic expansion, up to N$^3$LO [16, 34]. It should be noticed that, since the transition operator matrix elements are calculated using phenomenological wave functions, it should be viewed as a hybrid $\chi$EFT approach. Both SNPA and hybrid $\chi$EFT frameworks have been used in studies of weak $pp$ and $hep$ capture reactions in the energy regime relevant to astrophysics [16, 20, 21]. A detailed discussion of the weak current models is given in Ref. [27]. Here we briefly review their main characteristics.

The polar weak current operator is related to the isovector piece of the electromagnetic current via the conserved-vector-current (CVC) hypothesis. In SNPA, no free parameters are present in the model for the electromagnetic current, which is able to reproduce the trinucleon magnetic moments to better than 1% [27], as well as a large variety of electromagnetic observables [35, 36, 37]. In the case of hybrid $\chi$EFT, the vector current can be decomposed into four terms [34]: the soft one-pion-exchange ($1\pi$) term, vertex corrections to the one-pion exchange ($1\pi\rho$), the two-pion exchange ($2\pi$), and a contact-term contribution. Their explicit expressions can be found in Ref. [34]. All the $1\pi$, $1\pi\rho$ and $2\pi$ contributions contain low-energy constants (LECs) estimated using resonance saturation arguments, and Yukawa functions obtained by performing the Fourier transform from momentum- to coordinate-space with a Gaussian regulator characterized by a cutoff $\Lambda$. This cutoff determines the momentum scale below which these $\chi$EFT currents are expected to be valid, i.e., $\Lambda = 500–800$ MeV [16]. The contact-term electromagnetic contribution is given as sum of two terms, isoscalar and isovector, each one with a LEC in front, fixed to reproduce the experimental values of $A = 3$ magnetic moments. The resulting LECs are given in Table V of Ref. [27].

The two-body axial current operators in SNPA can be divided in two classes: the operators of the first class are derived from $\pi$- and $\rho$-meson exchanges and the $\rho\pi$-transition mechanism. These mesonic operators give rather small contributions [27]. The operators in the second class are those that give the largest two-body contributions, and are due to $\Delta$-isobar excitation [20, 21]. In particular, in the dominant $N$-$\Delta$-transition axial current, the $N$-$\Delta$ axial coupling constant is retained as a parameter and is determined by fitting $G^{\text{Exp}}$. It is important to note that the value of this parameter depends on how the $\Delta$-isobar degrees of freedom are treated. Here, the two-body $\Delta$-excitation axial operator is derived in the static $\Delta$ approximation, using first-order perturbation theory (PT). This approach is considerably simpler than that adopted in Ref. [21], where the $\Delta$ degrees of freedom were treated non-perturbatively, within the so-called transition-correlation operator (TCO) approach, by retaining them explicitly in the nuclear wave functions [38]. The results for the $N$-$\Delta$ coupling constant obtained within the two schemes differ by more than
a factor of 2 [21], but the results for the observables calculated consistently within the two different approaches are typically within 1% of each other.

The two-body axial current operator in $\chi$EFT consists of two contributions: a one-pion exchange term and a (non-derivative) two-nucleon contact-term. The explicit expressions for these terms can be found in Ref. [16]. While the coupling constants which appear in the one-pion exchange term are fixed by $\pi N$ data, the LEC which determines the strength of the contact-term has been fixed by reproducing GT$^{\text{EXP}}$. The values of this LEC for $\Lambda=500$–800 MeV are given in Table V of Ref. [27].

3. Results

The results for the doublet capture rate $\Gamma^D$ of reaction (1) and the total capture rate $\Gamma_0$ for reaction (2) are listed in Table 1 and 2, respectively. Both models for the nuclear weak transition operator presented in the previous section have been used, labeled SNPA and $\chi$EFT, respectively. The nuclear wave functions have been calculated with the AV18 [17] or the N3LO [28] two-nucleon interactions in Table 1, and with the AV18/UIX [25] and N3LO/N2LO [29] two- and three-nucleon interactions in Table 2. In the SNPA calculation, two different values for the single-nucleon axial coupling constant $g_A$ are considered, $g_A=1.2654(42)$, taken from Ref. [39] and widely used in studies of weak processes [21, 16, 15, 40], and $g_A=1.2695(29)$, the latest determination quoted by the Particle Data Group (PDG) [41]. This allows for an estimation of the theoretical uncertainty arising from this source. In the $\chi$EFT calculation, for consistency with the work of Refs. [16, 10], the older value for $g_A$ has been used. Within each Hamiltonian model, the parameters present in the SNPA and $\chi$EFT axial current models have been fitted to reproduce GT$^{\text{EXP}}$. Furthermore, the LECs in the $\chi$EFT weak vector current have been fitted to reproduce the $A=3$ magnetic moments.

Inspection of Table 1 shows that the $1S_0$ contribution is the leading one, but $L \geq 1$ contributions are significant and account for $\sim 37\%$ of the total rate. By comparison between the first and second row of the table, we conclude that there is no difference in the results, within uncertainties, when the older value for $g_A$, $g_A=1.2654(42)$, or the most recent one, $g_A=1.2695(29)$, is used. This reflects the fact that the $N$-to-$\Delta$ axial coupling constant has been constrained by GT$^{\text{EXP}}$ in both cases. The cutoff dependence of the $\chi$EFT results is very weak: at the three representative values for $\Lambda$, they agree with each other within the theoretical uncertainties. Also the model dependence due to the interaction model is very weak, as can be seen comparing the $\chi$EFT(AV18) and $\chi$EFT(N3LO) results for $\Lambda=600$ MeV. In conclusion, a total capture rate in the range

$$\Gamma^D = (389.7 - 394.3) \text{s}^{-1},$$

(10)
can be conservatively ascribed to reaction (1). This result is in agreement with the measurements of Refs. [3, 4, 6], but not with that of Ref. [5]. The difference with the theoretical results of Ref. [10] is also very small, and has been traced back [27] to the inclusion in the weak vector current of the $1\pi C$, $2\pi$ and contact-term contributions, not present in Ref. [10]. On the other hand, the results of Ref. [11] are significantly larger than those listed here, presumably because these authors have not constrained their weak current to reproduce GT$^{\text{EXP}}$ and the isovector magnetic moment of the trinucleons. This comparison between the experimental data of Refs. [3, 4, 5, 6] and the theoretical results of Refs. [10, 11, 27] is summarized in Fig. 1.

Similar conclusions can be drawn by inspection of Table 2. In particular, the $\chi$EFT results obtained with the AV18/UIX Hamiltonian model show a very weak $\Lambda$-dependence, and are in excellent agreement with those reported in SNPA. The $\chi$EFT result in the last row is obtained with the N3LO/N2LO Hamiltonian model. The $\chi$EFT (AV18/UIX) and $\chi$EFT (N3LO/N2LO) results differ by 8 s$^{-1}$, or less than 1%. In view of this, we quote conservatively a total capture rate for reaction (2) in the range

$$\Gamma_0 = (1471 - 1497) \text{s}^{-1},$$

(11)
by keeping the lowest and upper bounds in the values of Table 2. The 1% spread due to model dependence is a consequence of the procedure adopted to constrain the weak current. These results are also in very good agreement with the experimental datum of Ref. [2], as well as with the previous theoretical calculations of Refs. [13, 15, 14].
Table 1: Total rate for muon capture on deuteron, in the doublet initial hyperfine state, in s\(^{-1}\). The different partial wave contributions are indicated. The numbers among parentheses indicate the theoretical uncertainty arising from the adopted fitting procedures. Such uncertainty is not indicated when less than 0.1 s\(^{-1}\). The AV18 and N3LO interactions have been used to calculate the deuteron and \(nn\) wave functions. The label “SNPA” and “\(\chi\)EFT” refer to the model used for the weak transition operator. Two different values for the single-nucleon axial coupling constant \(g_A\) in SNPA(AV18) and three values for the cutoff \(\Lambda\) in \(\chi\)EFT(AV18) are used, as explained in the text.

<table>
<thead>
<tr>
<th></th>
<th>(S_0)</th>
<th>(P_0)</th>
<th>(P_1)</th>
<th>(P_2)</th>
<th>(D_2)</th>
<th>(F_2)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNPA (AV18)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(g_A=1.2654(42))</td>
<td>246.6(7)</td>
<td>20.1</td>
<td>46.7</td>
<td>71.6</td>
<td>4.5</td>
<td>0.9</td>
<td>390.4(7)</td>
</tr>
<tr>
<td>(g_A=1.2695(29))</td>
<td>246.8(5)</td>
<td>20.1</td>
<td>46.8</td>
<td>71.8</td>
<td>4.5</td>
<td>0.9</td>
<td>390.9(7)</td>
</tr>
<tr>
<td>(\chi)EFT (AV18)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Lambda=500) MeV</td>
<td>250.0(8)</td>
<td>19.9</td>
<td>46.2</td>
<td>71.2</td>
<td>4.5</td>
<td>0.9</td>
<td>392.7(8)</td>
</tr>
<tr>
<td>(\Lambda=600) MeV</td>
<td>250.0(8)</td>
<td>19.8</td>
<td>46.3</td>
<td>71.1</td>
<td>4.5</td>
<td>0.9</td>
<td>392.6(8)</td>
</tr>
<tr>
<td>(\Lambda=800) MeV</td>
<td>249.7(7)</td>
<td>19.8</td>
<td>46.4</td>
<td>71.1</td>
<td>4.5</td>
<td>0.9</td>
<td>392.4(7)</td>
</tr>
<tr>
<td>(\chi)EFT (N3LO)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Lambda=600) MeV</td>
<td>250.5(7)</td>
<td>19.9</td>
<td>46.4</td>
<td>71.5</td>
<td>4.4</td>
<td>0.9</td>
<td>393.6(7)</td>
</tr>
</tbody>
</table>

Table 2: Total rate for muon capture on \(^3\)He, in s\(^{-1}\). The numbers in parentheses indicate the theoretical uncertainties due to the adopted fitting procedure. The triton and \(^3\)He wave functions are obtained from the AV18/UIX and N3LO/N2LO Hamiltonians. The label “SNPA” and “\(\chi\)EFT” refer to the model used for the weak transition operator. Two different values for the single-nucleon axial coupling constant \(g_A\) in SNPA(AV18/UIX) and three values for the cutoff \(\Lambda\) in \(\chi\)EFT(AV18/UIX) are used, as explained in the text.

<table>
<thead>
<tr>
<th></th>
<th>(S_0)</th>
<th>(P_0)</th>
<th>(P_1)</th>
<th>(P_2)</th>
<th>(D_2)</th>
<th>(F_2)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNPA (AV18/UIX)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(g_A=1.2654(42))</td>
<td>1486(8)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(g_A=1.2695(29))</td>
<td>1486(5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\chi)EFT (AV18/UIX)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Lambda=500) MeV</td>
<td>1487(8)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Lambda=600) MeV</td>
<td>1488(9)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Lambda=800) MeV</td>
<td>1488(8)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\chi)EFT (N3LO/N2LO)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Lambda=600) MeV</td>
<td>1480(9)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Conclusions

We have reviewed the recent results of Ref. [27], where total rates for muon capture on deuteron and \(^3\)He have been calculated within a consistent approach, based on realistic (conventional and chiral) interactions and weak currents consisting of vector and axial-vector components with one- and many-body terms. Two different approaches have been used to derive these operators: the first one goes beyond the impulse approximation, by including meson-exchange current contributions and terms arising from the excitation of \(\Delta\)-isobar degrees of freedom. This approach, labeled SNPA, has been widely and successfully used in studies of electroweak processes (see for instance Refs. [20, 21, 15, 36, 33]). The second approach includes two-body contributions, beyond the impulse approximation, derived within a systematic \(\chi\)EFT expansion, up to N3LO. It is a hybrid \(\chi\)EFT approach, since matrix elements of weak operators derived in \(\chi\)EFT have been evaluated between wave functions obtained from realistic potentials. It should be noticed, however, that when the \(A=2\) and \(3\) wave functions are calculated using chiral potentials, the approach is in principle a full-fledged \(\chi\)EFT calculation, except that these potentials and currents have not (yet) been derived consistently at the same order in the low-momentum scale.

The only parameter in the SNPA nuclear weak current model is present in the axial current and is determined by fitting the experimental value for the triton half-life. In the case of the \(\chi\)EFT approach, three LECs appear, one in the axial-vector and two in the vector component. Of these, one LEC appearing in the isoscalar electromagnetic contact-term does not contribute here. The three LECs are fixed to reproduce, respectively, triton half-life and the \(A=3\) magnetic moments.

The results for the rate of the considered muon capture reactions are summarized in Eqs. (10) and (11). The very accurate experimental datum of Ref. [2] for the total rate in muon capture on \(^3\)He is very well reproduced. For the muon capture on deuteron, a precise measurement, which will become available in the near future [7], will
Figure 1: Experimental data and theoretical results for total rate of muon capture on deuteron, in the doublet initial hyperfine state. The experimental data are shown as function of the publication year, and are taken from Refs. [3, 4, 5, 6]. The theoretical results are from Refs. [10, 11, 27].

discriminate between the theoretical results of Ref. [11] on one side, and those of Ref. [10] and the present one, on the other side. Finally, we remark that the dependence of the results presented here on the input Hamiltonian model, or on the model for the nuclear transition operator, is weak, at less than 1 % level. This weak model dependence is a consequence of the procedure adopted to constrain the weak current.

References

[27] L.E. Marcucci et al., arXiv:1008.1172