Multi-attribute decision making based on z-valuation

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Abstract

In this paper we investigate multi-attribute decision making problem, where the attribute values are Z-numbers, and the weight information on attributes are partially reliable. The presented method is based on overall criteria positive ideal and negative ideal solution of alternatives and distance between Z-vectors. Final decision alternative is selected on basis of degree of membership of candidates belonging to the positive ideal solution. A numerical example on multi-attribute decision making for Web Services selection is given to illustrate the solution processes of the suggested method.

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Keywords: Decision making; Z-number; Ideal solution; Distance between Z-vectors; Web services selection.

1. Introduction

The process of multi-attribute criteria decision making (MADM) is to find the best all of the existing alternatives. The use of one or another multi-attribute decision theory depends mainly on decision making situations.

One of the widely used theories to model human decisions is of fuzzy set theory. Some of the most popular theories that emerged for uncertainty modeling were fuzzy sets\textsuperscript{1,2,3,4} and possibility theory\textsuperscript{5,6}, the rough set theory\textsuperscript{7}, Dempster\textsuperscript{8,9} and Shafer’s\textsuperscript{10} evidence theories. It is needed to find the relevant methodology suitable for a particular problem\textsuperscript{11,12}. Some of these methods are the Analytical Hierarchy Process (AHP)\textsuperscript{13}, Analytic Network Process (ANP)\textsuperscript{14}, Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)\textsuperscript{15}, Multi-criteria Optimization and Compromise Solution (VIKOR)\textsuperscript{16}, Simple Additive Weighting Method (SAW)\textsuperscript{17}, Elimination Et Choice Translating EReality (ELECTRE)\textsuperscript{18}, Preference Ranking Organization METHODS for Enrichment Evaluations (PROMETHEE)\textsuperscript{19}, Fuzzy expert systems\textsuperscript{20} etc. Unfortunately, up to day there are scarce research on multi-attribute decision making problems.

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decision making under Z-environment. In this paper, we suggest a new approach to study of multi-attribute decision analysis using Z-number concept.

The rest of this paper is structured as follows. In Section 2 we present some prerequisites material on Z-number. In Section 3 we describe the statement of the problem and the suggested approach to MADM with Z-information. In Section 4 we illustrate an application of the suggested approach to a real-world investment problem. Finally, conclusions are given in Section 5.

2. Preliminaries

Definition 1. A Continuous Z-number. A continuous Z-number is an ordered pair $Z = (A, B)$ where $A$ is a continuous fuzzy number which describes a fuzzy constraint on values that a random variable $X$ may take:

$X$ is $A$

and $B$ is a continuous fuzzy number with a membership function $\mu_B : [0, 1] \rightarrow [0, 1]$, which describes a fuzzy constraint on the probability measure of $A$:

$P(A)$ is $B$.

Definition 2. A discrete Z-number. A discrete Z-number is an ordered pair $Z = (A, B)$ where $A$ is a discrete fuzzy number which describes a fuzzy constraint on values that a random variable $X$ may take:

$X$ is $A$, and $B$ is a discrete fuzzy number with a membership function $\mu_B : \{b_1, \ldots, b_n\} \rightarrow [0, 1]$, $\{b_1, \ldots, b_n\} \subseteq [0, 1]$, which describes a fuzzy constraint on the probability measure of $A$:

$P(A)$ is $B$.

Definition 3. A distance between Z-number-valued vectors. The distance between Z-number valued vectors is $Z_1 = (Z_{11}, Z_{12}, \ldots, Z_{1n})$ and $Z_2 = (Z_{21}, Z_{22}, \ldots, Z_{2n})$ defined as

$$D(Z_1, Z_2) = \max_{i=1,\ldots,n} d(Z_{1i}, Z_{2i}),$$

$$d(Z_{1i}, Z_{2i}) = \left( \frac{1}{n+1} \sum_{i=1}^{n} \left[ |a_{1i}^L - a_{2i}^L| + |a_{1i}^R - a_{2i}^R| \right] + \frac{1}{m+1} \sum_{k=1}^{m} \left[ |b_{1i}^L - b_{2i}^L| + |b_{1i}^R - b_{2i}^R| \right] \right),$$

where $a_{i}^L = \min A_i^a, a_{i}^R = \max A_i^a, b_{i}^L = \min B_i^a, b_{i}^R = \max B_i^a$.

2. Statement of the problem and its solution.

Assume that $A = \{A_1, A_2, \ldots, A_n\}$ is a set of alternatives and $C = \{C_1, C_2, \ldots, C_m\}$ is a set of attributes. Every attribute $C_j, j = 1, m$ is characterised by weight $W_j$ assigned by expert or decision maker. As we deal with Z-information valued decision environment, the characteristic of the alternative $A_i, i = 1, n$ on attribute $C_j (j = 1, m)$ is described by the form

$$A_i = \{Z(A_{i1}, B_{i1}), Z(A_{i2}, B_{i2}), \ldots, Z(A_{in}, B_{in})\}.$$
where \( Z(A_{ij}, B_{ij}) \) is evaluation of an alternative \( A_i \) with respect to a attribute \( C_j \). Value of attributes and weights of attributes are usually derived from decision maker or experts and are vague and characterized with partial reliability. In this case, the weights \( W_j, i = 1, m \) are represented as

\[
W_j = \{Z(A_j^w, B_j^w)\} , \ j = 1, m
\]  

(4)

Where \( A_j^w \) is value of weight of \( j \)-th is attribute, \( B_j^w \) is reliability of this value.

Hence, we can represent decision matrix \( D_{nm} \) as Table 1.

<table>
<thead>
<tr>
<th></th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( \cdots )</th>
<th>( C_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>( Z(A_{1,1}, B_{1,1}) )</td>
<td>( Z(A_{1,2}, B_{1,2}) )</td>
<td>( \cdots )</td>
<td>( Z(A_{1,m}, B_{1,m}) )</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>( Z(A_{2,1}, B_{2,1}) )</td>
<td>( Z(A_{2,2}, B_{2,2}) )</td>
<td>( \cdots )</td>
<td>( Z(A_{2,m}, B_{2,m}) )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( A_n )</td>
<td>( Z(A_{n,1}, B_{n,1}) )</td>
<td>( Z(A_{n,2}, B_{n,2}) )</td>
<td>( \cdots )</td>
<td>( Z(A_{n,m}, B_{n,m}) )</td>
</tr>
</tbody>
</table>

The common approach in the MADM is the use of the utility theories. This approach leads to transformation of a vector-valued alternative to a scalar-valued quantity. This transformation leads to loss of information. It is related to restrictive assumptions on preferences underlying utility models. In human decision it is not needed to use artificial transformation.

In this case we will use the concept of positive and negative ideal point in multi-attribute decision making\(^{15}\). We present an ideal Z-point for attributes as

\[
A_{p}^id = \left( Z\left( A_{p1}^id, B_{p1}^id \right), Z\left( A_{p2}^id, B_{p2}^id \right), \ldots, Z\left( A_{pm}^id, B_{pm}^id \right) \right)
\]  

(5)

A negative attributes ideal point will be described as

\[
A_{n}^id = \left( Z\left( A_{n1}^id, B_{n1}^id \right), Z\left( A_{n2}^id, B_{n2}^id \right), \ldots, Z\left( A_{nm}^id, B_{nm}^id \right) \right)
\]  

(6)

Solution at the stated decision making problem, i.e. choice best alternative among \( A = \{A_1, A_2, \ldots, A_n\} \) consist of the following steps:

1. Weighted distances \( d_p^i \) \( i \)-th alternative and positive ideal solutions (5) is defined by (1).
2. Weighted distances \( d_n^i \) between \( i \)-th alternative and negative ideal solutions (5) is defined by (1).
3. Degree of membership \( r_i, i = 1, n \) of each alternatives belonging to the positive ideal solution is calculated. For this (7) is used\(^{27}\):

\[
r_i = \frac{1}{1 + \left( \frac{d_n^i}{d_p^i} \right)}
\]  

(7)
4. Final decision alternative is selected as \( \max(r_i), i = 1, n \)

3. Practical example

We consider multi-attribute decision making for Web services selection problem\(^{21}\). Today a wide variety of services are offered that can satisfy quality of services for agents. The number of options, i.e. Web services is 8 \( A_1, A_2, A_3 \ldots A_8 \). An agent has to make a decision taking into account 5 attributes \( C_1 \) (cost), \( C_2 \) (time), \( C_3 \) (reliability), \( C_4 \) (availability), \( C_5 \) (repetition). In this case all 8 alternatives are evaluated under 5 attributes by Z-numbers. Components of these Z-numbers are presented by triangle fuzzy number and scaled decision matrix shown in Tables 2, 3.

Table 2. Decision matrix

<table>
<thead>
<tr>
<th></th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>(0.45 0.5 0.55)(0.5 0.6 0.7)</td>
<td>(0.441 0.49 0.539)(0.5 0.6 0.7)</td>
<td>(0.621 0.69 0.759)(0.5 0.6 0.7)</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>(0.126 0.14 0.154)(0.5 0.6 0.7)</td>
<td>(0.531 0.59 0.649)(0.5 0.6 0.7)</td>
<td>(0.423 0.47 0.517)(0.5 0.6 0.7)</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>(0.225 0.25 0.275)(0.5 0.6 0.7)</td>
<td>(0.711 0.79 0.869)(0.7 0.8 0.9)</td>
<td>(0.27 0.3 0.33)(0.5 0.6 0.7)</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>(0.612 0.68 0.748)(0.5 0.6 0.7)</td>
<td>(0.603 0.67 0.737)(0.5 0.6 0.7)</td>
<td>(0.378 0.42 0.462)(0.5 0.6 0.7)</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>(0.333 0.37 0.407)(0.5 0.6 0.7)</td>
<td>(0.225 0.25 0.275)(0.5 0.6 0.7)</td>
<td>(0.522 0.58 0.638)(0.5 0.6 0.7)</td>
</tr>
<tr>
<td>( A_6 )</td>
<td>(0.432 0.48 0.528)(0.5 0.6 0.7)</td>
<td>(0.549 0.61 0.671)(0.5 0.6 0.7)</td>
<td>(0.621 0.69 0.759)(0.5 0.6 0.7)</td>
</tr>
<tr>
<td>( A_7 )</td>
<td>(0.738 0.82 0.902)(0.7 0.8 0.9)</td>
<td>(0.324 0.36 0.396)(0.5 0.6 0.7)</td>
<td>(0.522 0.58 0.638)(0.5 0.6 0.7)</td>
</tr>
<tr>
<td>( A_8 )</td>
<td>(0.531 0.59 0.649)(0.5 0.6 0.7)</td>
<td>(0.378 0.42 0.462)(0.5 0.6 0.7)</td>
<td>(0.648 0.72 0.792)(0.7 0.8 0.9)</td>
</tr>
</tbody>
</table>

For the simplicity the weights vector of the 5 attributes is given as
weight for \( C_1 \) is \( W_1 = 0.3 \), for \( C_2 \) is \( W_2 = 0.2 \), for \( C_3 \) is \( W_3 = 0.12 \), for \( C_4 \) is \( W_4 = 0.18 \) and for \( C_5 \) is \( W_5 = 0.2 \)

The positive ideal alternative is presented as
\[ A^p = ((0.738 0.82 0.902)(0.7 0.8 0.9),(0.711 0.79 0.869)(0.7 0.8 0.9),(0.648 0.72 0.792)(0.7 0.8 0.9),(0.747 0.83 0.913)(0.7 0.8 0.9),(0.828 0.92 1.012)(0.7 0.8 0.9)) \]

The negative ideal alternative is presented as
\[ A^n = ((0.126 0.14 0.154)(0.5 0.6 0.7), (0.225 0.25 0.275)(0.5 0.6 0.7), (0.27 0.3 0.33)(0.5 0.6 0.7), (0.216 0.24 0.264)(0.5 0.6 0.7), (0.126 0.14 0.154)(0.5 0.6 0.7)) \]

According to the (1)-(2) weighted distances between Z-vectors of alternatives and positive ideal solution Z-vector are obtained as
\[ d_{p1} = 0.32 \quad d_{p2} = 0.42 \quad d_{p3} = 0.375 \quad d_{p4} = 0.24 \]

Analogously we have obtained weighted distances between Z-vectors of alternatives and negative ideal solution Z-vector:
The membership degree \( r_i, i = 1,8 \) are calculated according to (7) and have obtained
\[
\begin{align*}
    r_1 &= 0.27, r_2 = 0.37, r_3 = 0.29, r_4 = 0.56, r_5 = 0.12, r_6 = 0.42, r_7 = 0.75, r_8 = 0.4.
\end{align*}
\]

The final decision is determined as
\[
\max(r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8) = 0.75.
\]

The best alternative is \( A_7 \).

### 4. Conclusion

Despite a lot of methods have been developed to deal with interval and fuzzy MADM unfortunately, today research on multi-attribute decision making under Z-information is scarce. The mentioned above dictated to create new approach for MADM under decision situation where decision relevant information are characterized by fuzzy uncertainty and partial reliability. For this purpose we have suggested MADM procedure based on overall criteria positive ideal and negative ideal solutions of alternatives, distance between Z-vectors and Z-information processing. Numerical example on MADM Web services selection problem demonstrates applicability and efficiency of proposed method.

### References