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Journal of Algebra 320 (2008) 1255–1260

JOURNAL OF Algebra

www.elsevier.com/locate/jalgebra

Overgroups of the elementary unitary group in linear group over commutative rings

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Received 15 September 2007

Available online 8 May 2008

Communicated by Leonard L. Scott, Jr.

Abstract

For a commutative ring with identity, we give a complete description of all overgroups of the elementary unitary group $EU_{2n}R$ ($n \geqslant 5$) in linear group $GL_{2n}R$.

Keywords: Overgroup; Elementary unitary group; Linear group; Commutative ring

1. Introduction

Let R be a ring. Given an additive homomorphism $f : R \to R$ such that $f[f(a)] = a$ and $f(ab) = f(b)f(a)$ for all $a, b \in R$, then f is called an involution on R. For instance, the usual complex conjugation map gives an involution on the complex field. If R is commutative, the identity map gives an involution on R. In this article, we assume that R is a commutative ring with the identity 1. For simplicity we write an involution f as $f(\cdot) = (\cdot)^*$. An involution $*$ also determines an involution $\star$ of the matrix ring M_nR of all n by n matrices by $(a_{ij})^\star = (a_{ji}^*)$, for $a_{ij} \in R$. An example is that the transpose map T on M_nR determined by the identity map on R gives an involution on M_nR. For a given $\varepsilon \in R$ such that $\varepsilon^*\varepsilon = 1$, let $R^\varepsilon = \{x \mid x = -\varepsilon^* x, x \in R\}$.

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1 Partially supported by NSF of China (Grant No. 10571033) and Science Research Foundation in Harbin Institute of Technology (Grant No. HITC200708).

0021-8693/$ – see front matter
doi:10.1016/j.jalgebra.2008.04.013

The unitary group is defined by

$$U_{2n}R = \{\theta \mid \theta^{\star}\varphi_n\theta = \varphi_n,\ \theta \in GL_{2n}R\},$$

where $\varphi_n = E_{12}^{(2)} \otimes I_n + \varepsilon E_{21}^{(2)} \otimes I_n$, I_k denotes the identity matrix of $M_k R$, $E_{ij}^{(k)}$ the $k \times k$ matrix with 1 at the position (i, j) and zeros elsewhere and $\otimes$ Kronecker product. Clearly $\varphi_n^{-1} = \varepsilon^* E_{12}^{(2)} \otimes I_n + E_{21}^{(2)} \otimes I_n$, $\varphi_n^{\star} = \varphi_n^{-1}$ and $\varepsilon\varphi_n^{\star} = \varphi_n$.

Let $\hat{R}$ be the set of all invertible elements in R. The generalized unitary group is defined by

$$GU_{2n}R = \{\theta \mid \theta^{\star}\varphi_n\theta = x\varphi_n,\ \theta \in GL_{2n}R,\ x \in \hat{R}\}.$$

If $\theta^{\star}\varphi_n\theta = x\varphi_n$, then $x\varphi_n = \varepsilon\theta^{\star}\varphi_n^{\star}\theta = \varepsilon(\theta^{\star}\varphi_n\theta)^{\star} = \varepsilon x^*\varphi_n^{\star} = x^*\varphi_n$. So $x = x^*$. Clearly $U_{2n}R$ is a subgroup of $GU_{2n}R$.

The elementary matrices are defined by $\alpha_{ij}(a) = I_{2n} + aE_{ij}^{(2n)}$ $(i \neq j)$. An ideal J of R is said to be dual, if $J^* = J$. For any ideal J of R, let $E_{2n}J$ denote the subgroup of $GL_{2n}R$ generated by all matrices $\alpha_{ij}(a)$ with $a \in J$, $i \neq j$. For $k = 1, \ldots, n$, let $m_k = k + n$ and $m_{k+n} = k$. For $a \in R$ and $1 \leqslant i \neq j \leqslant 2n$ we define the elementary unitary matrices $\beta_{i,m_i}(a)$ and $\beta_{ij}(a)$ with $j \neq m_i$ as follows: $\beta_{i,m_i}(a) = \alpha_{i,m_i}(a)$ with $a \in R^{\varepsilon}$ when $n + 1 \leqslant i$ and with $a^* \in R^{\varepsilon}$ when $i \leqslant n$; $\beta_{ij}(a) = \beta_{m_j,m_i}(-a') = \alpha_{ij}(a) - a'E_{m_j,m_i}^{(2n)} \in U_{2n}R$ with $a' = a^*$ when $i, j \leqslant n$ or $n + 1 \leqslant i, j$; $a' = \varepsilon^* a^*$ when $i \leqslant n < j$; $a' = \varepsilon a^*$ when $j \leqslant n < i$. The subgroup of $U_{2n}R$ generated by all elementary unitary matrices is denoted by $EU_{2n}R$ and called the elementary unitary group. Let $UE_{2n}(R, J)$ denote the subgroup of $GL_{2n}R$, generated by all elements of the form $\beta_{ij}(r)\alpha_{kl}(a)\beta_{ij}(-r)$ with $a \in J$ and $r \in R$. It is obvious that $UE_{2n}(R, J)$ is a normal subgroup of $GL_{2n}R$. So

$$EUU_{2n}J = EU_{2n}R \cdot UE_{2n}(R, J)$$

is a subgroup of $GL_{2n}R$. For any ideal J, let ϕ be the canonical ring homomorphism: $R \to R/J$. Then ϕ induces the group homomorphism $\phi_J : GL_{2n}R \to GL_{2n}(R/J)$ and $EU_{2n}R \to EU_{2n}(R/J)$. It is clear that

$$GUG_{2n}J = \{\theta \mid \phi_J(\theta) \in GU_{2n}(R/J),\ \theta \in GL_{2n}R\}$$

is a subgroup of $GL_{2n}R$. The main result of this paper is stated as follows.

Theorem 1.1. *Let R be a commutative ring with 1 on which an involution $*$ is defined. Assume that $n \geqslant 5$ and that when $*$ is identical on R and $\varepsilon = 1$, then 2 is torsion-free in R. Let X be an overgroup of $EU_{2n}R$ in $GL_{2n}R$. Then there is a unique dual ideal J of R such that*

$$EUU_{2n}J \subseteq X \subseteq GUG_{2n}J.$$

In [1] Kantor determined all overgroup of $GL_n(F_{q^r})$ in $GL_{nr}(F_q)$. King in [2,3] and Li in [5,7] described all overgroups of $SU(n, K, f)$ and $\Omega(n, K, Q)$ in $GL(n, K)$ where K is a division ring respectively. Some results on overgroups of $G(K)$ in $GL_n K$, where $G(K)$ is a certain classical group over K, can be found in [6]. Li in [4] determined the structure of symplectic group over arbitrary commutative rings. You and Zheng in [20] obtained the overgroups X of symplectic group $Sp_{2n}R$ in $GL_{2n}R$ where R is a local ring. Vaserstein in [10–13] obtained some results on the general linear groups. The results on the overgroups of symplectic and orthogonal groups (with hyperbolic form) over commutative rings were given in [15–17]. Petrov in [9] investigated the overgroups of unitary groups (with hyperbolic form) under a local stable rank condition with form parameter. You in [18,19] classified overgroups of classical groups in linear group over Banach algebras and over commutative rings respectively.

2. Preliminaries

Lemma 2.1. *(See [14].) The following statements hold for* $1 \leqslant i \neq j \leqslant 2n$:

(1) $\alpha_{ij}^{-1}(a) = \alpha_{ij}(-a)$;
(2) $\beta_{ij}^{-1}(a) = \beta_{ij}(-a)$;
(3) $\beta_{ij}(a+b) = \beta_{ij}(a)\beta_{ij}(b)$;
(4) $\beta_{ij}(a) \diamond \beta_{jk}(b) = \beta_{ik}(ab)$ *when* i, j, k, m_i, m_j *and* m_k *are all distinct, here* $\alpha \diamond \beta$ *denotes* $\alpha\beta\alpha^{-1}\beta^{-1}$;
(5) $\beta_{ij}(a) \diamond \beta_{j,m_i}(b) = \beta_{i,m_i}(ab - c)$ *when* $j \neq m_i$, *where* $c = \varepsilon a^* b^*$ *when* $n+1 \leqslant i$ *and* $c = \varepsilon^* b^* a^*$ *when* $i \leqslant n$;
(6) $\beta_{ij}(a) \diamond \beta_{j,m_i}(b) = \beta_{i,m_i}(ab)\beta_{i,m_i}(c)$ *when* $j \neq m_i$, *where* $b^* \in R^{\varepsilon}$ *and* $c = ab^*a^*$ $(i, j \leqslant n)$, $b^* \in R^{\varepsilon}$ *and* $c = \varepsilon aba^*$ $(j \leqslant n < i)$, $b \in R^{\varepsilon}$ *and* $c = -ab^*a^*$ $(i \leqslant n < j)$ *and* $b \in R^{\varepsilon}$ *and* $c = -\varepsilon ab^*a^*$ $(n+1 \leqslant i, j)$;
(7) $\alpha_{ij}(a) \diamond \beta_{jk}(b) = \alpha_{ik}(ab)$ *when* i, j *and* k *are all distinct and* $j \neq m_i$, *where* $b \in R^{\varepsilon}$ *or* $b^* \in R^{\varepsilon}$ *if* $k = m_j$;
(8) $\alpha_{ij}(a) \diamond \beta_{k,m_j}(b) = \alpha_{i,m_k}(c)$ *when* i, j *and* m_k *are all distinct, where* $c = -\varepsilon a^* b^*$ $(j, k \leqslant n)$, $c = -\varepsilon ab^*$ $(n+1 \leqslant j, k)$ *and* $c = -ab^*$ $(j \leqslant n < k$ *or* $k \leqslant n < j)$;
(9) *For* $1 \leqslant k \leqslant n$, $\gamma_{k,m_k} = I_{2n} - E_{kk}^{(2n)} - E_{m_k,m_k}^{(2n)} + E_{k,m_k}^{(2n)} + \varepsilon E_{m_k,k}^{(2n)} \in EU_{2n}R$;
(10) *For* $i \neq j \leqslant n$, $\varpi_{ij} = E_{11}^{(2)} \otimes P_{ij} + E_{21}^{(2)} \otimes (P_{ij}^{\star})^{-1} \in EU_{2n}R$, *where* P_{ij} *is an* $n \times n$ (i, j)-*permutation matrix.*

Lemma 2.2. $\theta \in GU_{2n}R$ *if and only if there is an* $x \in \hat{R}$ *such that* $xu_i = \varepsilon^* v_{m_i}^{\star}\varphi_n$ *when* $1 \leqslant i \leqslant n$ *and* $u_i = v_{m_i}^{\star}\varphi_n$ *when* $n+1 \leqslant i \leqslant 2n$, *where* u_i *is the* i*th row of* θ^{-1} *and* v_i *is the* i*th column of* θ.

Proof. From $\theta \in GU_{2n}R$ we have $\theta^{\star}\varphi_n\theta = x\varphi_n,\ x \in \hat{R}$. Then $x\theta^{-1} = \varphi_n^{-1}\theta^{\star}\varphi_n$. So

$$x\begin{bmatrix} u_1 \\ \vdots \\ u_n \\ u_{n+1} \\ \vdots \\ u_{2n} \end{bmatrix} = \varepsilon^*\varphi_n^{\star}\begin{bmatrix} v_1^{\star} \\ \vdots \\ v_n^{\star} \\ v_{n+1}^{\star} \\ \vdots \\ v_{2n}^{\star} \end{bmatrix}\varphi_n = \begin{bmatrix} \varepsilon^* v_{n+1}^{\star} \\ \vdots \\ \varepsilon^* v_{2n}^{\star} \\ v_1^{\star} \\ \vdots \\ v_n^{\star} \end{bmatrix}\varphi_n.$$

It is clear that the converse holds also. □

Note. $\theta \in U_{2n}R$ if and only if there is an $x \in \hat{R}$ such that $u_i = \varepsilon^* v_{m_i}^{\star}\varphi_n$ when $1 \leqslant i \leqslant n$ and $u_i = v_{m_i}^{\star}\varphi_n$ when $n+1 \leqslant i \leqslant 2n$, where u_i is the ith row of θ^{-1} and v_i is the ith column of θ.

Lemma 2.3. *For any overgroup X of $EU_{2n}R$ in $GL_{2n}R$ which is not in $GU_{2n}R$, there is an element $\theta \in X$ such that at least one column v_i of θ satisfies $xu_i \neq \varepsilon^* v^{\star}_{m_i}\varphi_n$ when $1 \leqslant i \leqslant n$ or $u_i \neq v^{\star}_{m_i}\varphi_n$ when $n+1 \leqslant i \leqslant 2n$, where u_i is the ith row of θ^{-1} and $x \in \hat{R}$.*

Proof. From Lemma 2.2, we get the result. □

For a commutative ring R, the Jacobson radical $\operatorname{rad} R$ of R is defined by the intersection of all the maximal ideals of R. A commutative ring R is said to be semi-local if R has finitely many maximal ideals. For a commutative semi-local ring R, let $M_1, \ldots, M_k$ denote all the maximal ideals of R, then $R/\operatorname{rad} R = \bigoplus_{i=1}^{k} F_i$, where $F_i = R/M_i$, $i = 1, \ldots, k$, are fields. For $x \in R$, if $x \pmod{M_i} \neq 0$, $i = 1, \ldots, k$, then $x \in \hat{R}$. If $x \pmod{M_i} \neq 0$, $i = 1, \ldots, k$, then $x \in \operatorname{rad} R$. A vector $u = [r_1, \ldots, r_k]^T \in R^k$ is said to be unimodular if there exists a $v \in R^k$ such that $u^T v = 1$. From Theorem 8.1 in [8] and Proposition 2.3 in [14] a lemma follows.

Lemma 2.4. *Let $u = [r_1, \ldots, r_{2n}]^T \in R^{2n}$ be a unimodular vector over a semi-local ring R and $n \geqslant 3$. Then there is a $\theta \in EU_{2n}R$ such that $\theta u = [1, 0, \ldots, 0, r'_{n+1}, 0, \ldots, 0]^T$. If u is also a column of a unitary matrix in $U_{2n}R$, then $r'_{n+1} = 0$.*

A unimodular vector u is said to be a unitary vector if u is a column of a unitary matrix.

Lemma 2.5. *For a semi-local ring R with at most two maximal ideals and $n \geqslant 4$, let $\xi = [u_1, \ldots, u_{2n}] \in GL_{2n}R$ with $u_1 = [1, 0, \ldots, 0, r_{n+1,1}, 0, \ldots, 0]^T$. Then there exist $\theta_1, \theta_2 \in EU_{2n}R$ such that $\theta_1\xi\theta_2 = [u_1, u'_2, \ldots, u'_{2n}]^T$ with u_1 is not changed and $u'_{n+2} = [0, 1, 0, \ldots, 0, r'_{n+1,n+2}, r'_{n+2,n+2}, 0, \ldots, 0]^T$.*

Proof. Discuss just for a semi-local ring R with two maximal ideals. Let $\eta = \prod_{i=n+2}^{2n} \beta_{1i}(-r_{1i})$ where r_{1i} is the first element of u_i. Then $\xi\eta = [u_1, u'_2, \ldots, u'_{2n}]^T$ with unimodular columns $u'_i = [0, r_{2,i}, \ldots, r_{n+1,i}, r'_{n+2,i}, r_{n+1,i}, \ldots, r_{2n,i}]^T$, $i = n+2, \ldots, 2n$. Let $v_i = [r_{2,i}, \ldots, r_{n+1,i} r_{n+1,i}, \ldots, r_{2n,i}]^T$, $i = n+1, \ldots, 2n$. Since R only has two maximal ideals M_1 and M_2, and $R/\operatorname{rad} R = F_1 \oplus F_2$ where F_i $(i = 1, 2)$ are fields, applying the linear independency of $u'_{n+2} \pmod{M_i}, \ldots, u'_{2n} \pmod{M_i}$, $i = 1, 2$, we have that one of $v_{n+2}, \ldots, v_{2n}$ is unimodular. Since ϖ_{ij} $(2 \leqslant i < j \leqslant n)$ is in $EU_{2n}R$, we may assume that v_{n+2} is unimodular. By Lemma 2.4 we can find a matrix $\theta_1 = I_2 \oplus \zeta$, $\zeta \in EU_{2n-2}R$ such that $\theta_1\xi\eta$ or $\theta_1\xi\eta\varpi_{ij}$ has the required form. □

Note that in Lemma 2.5 if $r_{n+1,1} = 0$, then $r'_{n+1,n+2} = 0$.

Lemma 2.6. *Under the conditions in Lemma 2.5 and $n \geqslant 5$, let $\xi = [u_1, \ldots, u_{2n}] \in GL_{2n}R$ with $u_1 = [1, 0, \ldots, 0, r_{n+1,1}, 0, \ldots, 0]^T$. Then there exist $\theta_1, \theta_2 \in EU_{2n}R$ such that $\theta_1\xi\theta_2 = [u_1, u'_2, \ldots, u'_{2n}]^T$ with $u'_{n+2} = [0, 1, 0, \ldots, 0, r'_{n+1,n+2}, r'_{n+2,n+2}, 0, \ldots, 0]^T$ and $u'_{n+3} = [0, 0, 1, 0, \ldots, 0, r'_{n+1,n+3}, r'_{n+2,n+3}, r'_{n+3,n+3}, 0, \ldots, 0]^T$.*

Proof. Repeat the process of proving Lemma 2.5. □

Lemma 2.7. *(See [19].) Let X be an overgroup of $EU_{2n}R$ in $GL_{2n}R$ and $n \geqslant 3$. If X contains an elementary matrix $\alpha_{i,m_i}(a)$ with $a \notin R$ or $a^* \notin R^\varepsilon$, then X contains an $\alpha_{kl}(c)$ with $l \neq m_k$ and $c \in R$ except for the case that $*$ is identical on R, $\varepsilon = 1$, and 2 is a torsion element in R.*

Let R_* denote the subring of R generated by all rr^* with $r \in R$. Obviously, $1 \in R_*$ and $R^\varepsilon R_* \subseteq R^\varepsilon$.

Lemma 2.8. *Let R be a commutative ring with 1 on which an involution $*$ is defined. Then the ring $S^{-1}R$ is a semi-local ring which has at most two maximal ideals for every maximal ideal M of R_*, where $S = R_* \backslash M$.*

Proof. Similar to the process of proving Lemma 1.4 in [14]. □

For a maximal ideal M of R_*, the localization: $R \to S^{-1}R(S = R_* \backslash M)$ induces the group homomorphism $\psi_M : GL_{2n}R \to GL_{2n}(S^{-1}R)$ and $EU_{2n}R \to EU_{2n}(S^{-1}R)$.

Lemma 2.9. *(See [19].) Let X be an overgroup of $EU_{2n}R$ in $GL_{2n}R$.*

(1) *If $X \nsubseteq GU_{2n}R$, then there exists a maximal ideal M of R_* such that $\psi_M(X) \nsubseteq \psi_M(GU_{2n}R)$;*
(2) *for a given $\theta \in EU_{2n}(S^{-1}R)$, where $S = R_* \backslash M$, then for $n \geqslant 3$, there exists an s in S such that $\theta\psi_M(EU_{2n}(sR))\theta^{-1} \subseteq \psi_M(EU_{2n}(R))$;*
(3) *if $X \nsubseteq GU_{2n}R$, then there exists a θ in $EU_{2n}(S^{-1}R)$ such that $\theta\psi_M(X)\theta^{-1}$ contains an elementary matrix $\alpha_{ij}(a)$ with $j \neq m_i$ ($a \in S^{-1}R$) or $j = m_i$ ($a \notin S^{-1}R^\varepsilon$ or $a^* \notin S^{-1}R^\varepsilon$);*
(4) *if $X \nsubseteq GU_{2n}R$, then there exists an elementary matrix $\alpha_{ij}(a) \in X$ with $j \neq m_i$.*

3. A proof of the main result

Proof of Theorem 1.1. If $X \subseteq GU_{2n}R$, then $EU_{2n}R \cdot UE_{2n}(R, 0) = EU_{2n}R \subseteq X$. If $X \nsubseteq GU_{2n}R$, by Lemma 2.9 there exists an element $a \in R$ such that $\alpha_{ij}(a) \in X$ with $j \neq m_i$. By Lemmas 2.1 and 2.7, we have that X contains all $\alpha_{ij}(ab)$ for $k + 1 \leqslant i \neq j \leqslant n + k$ ($k = 0, n$). Therefore $\alpha_{i,m_i}(ab) = \alpha_{ij}(a) \diamond \beta_{j,m_i}(b) \in X$ for $1 \leqslant i \leqslant 2n$ ($j \neq m_i$) and $\alpha_{ij}(ab) = \alpha_{i,m_j}(a) \diamond \beta_{m_j,j}(b) \in X$ for $1 \leqslant i \leqslant n, n + 1 \leqslant i \leqslant 2n$ and $1 \leqslant j \leqslant n, n + 1 \leqslant i \leqslant 2n$ ($j \neq m_i$). So there exists an $a \in R$ such that $E_{2n}(aR) \subseteq X$. From $\alpha_{m_i,m_i}(c) = \beta_{ij}(a)\alpha_{ij}(-a)$, where $c = a^*$ or $c = \varepsilon a^*$ or $c = \varepsilon^* a^*$, we have $E_{2n}(a^*R) \subseteq X$ for the same $a \in R$ satisfying $E_{2n}(aR) \subseteq X$. Let $J = \{a \mid E_{2n}(aR) \subseteq X,\ a \in R\}$. Clearly $ab \in J$ for any $a, b \in J$. Since $\alpha_{ij}(a + b) = \alpha_{ij}(a)\alpha_{ij}(b)$, we have $a + b \in J$ for any $a, b \in J$. So J is a subring of R. For any $r \in R$ and $a \in J$, we easily have $ar \in J$, that is, J is an ideal of R. From $J^* = \{a^* \mid E_{2n}(aR) \subseteq X,\ a \in R\} = \{a \mid E_{2n}(a^*R) \subseteq X,\ a \in R\}$, we have $J^* = J$. Therefore $EU_{2n}R \cdot UE_{2n}(R, J) \subseteq X$. From $EU_{2n}R \subseteq X$ we have $\phi_J(EU_{2n}R) \subseteq \phi_J(X)$. Since ϕ_J is surjective, then $EU_{2n}(R/J) \subseteq \phi_J(X)$. If $\phi_J(X) \nsubseteq GU_{2n}(R/J)$, there is $a \notin J$ such that $\alpha_{ij}(a + J) \in \phi_J(X)$ with $j \neq m_i$. So there is a $\theta \in X$ satisfying $\phi_J(\theta) = \phi_J(\alpha_{ij}(a))$ and $\theta \neq \alpha_{ij}(a)$. Take $\xi = \alpha_{ij}(-a)\theta \in \ker\phi_J$. By Theorem 3 in [12], $\beta_{k,m_j}(a) \diamond \xi \in UE_{2n}(R, J) \subseteq X$. Since $\alpha_{ij}(a)(\beta_{k,m_j}(a) \diamond \xi)\alpha_{ij}(-a) \in EU_{2n}(R, J)$, then

$$\alpha_{i,m_k}(c) = \alpha_{ij}(a) \diamond \beta_{k,m_j}(1) = \alpha_{ij}(a)\big(\beta_{k,m_j}(1) \diamond \xi\big)\alpha_{ij}(-a)\big(\theta \diamond \beta_{k,m_j}(1)\big) \in X,$$

where $c = -\varepsilon^* a$ ($j, k \leqslant n$), $c = -\varepsilon a$ ($n + 1 \leqslant j, k$), $c = -a$ ($j \leqslant n < k$ or $k \leqslant n < j$). This is a contradiction to $a \notin J$. So $\phi_J(X) \subseteq GU_{2n}(R/J)$, where J is the unique maximal ideal such that $EU_{2n}R \cdot UE_{2n}(R, J) \subseteq X$. Furthermore $X \subseteq \phi_J^{-1}(\phi_J(X)) \subseteq \phi_J^{-1}(GU_{2n}(R/J))$. Therefore $X \subseteq \phi_J^{-1}(GU_{2n}(R/J)) \cap GU_{2n}(R) = GUG_{2n}J$. $\square$

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