Revisiting the perforation of ductile plates by sharp-nosed rigid projectiles

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A B S T R A C T

The process of ductile plate perforation by sharp-nosed rigid projectiles is further examined in this work through 2D numerical simulations. We highlight various features concerning the effective resisting stress \( \sigma_r \) which a finite thickness plate, with a flow stress of \( \sigma_f \), exerts on the projectile during perforation. In particular, we show that the normalized resisting stress \( \sigma_r/\sigma_f \) can be represented as a unique function of the normalized thickness of the plate \( H/D \), where \( H \) is plate thickness and \( D \) is projectile diameter), for a large range of normalized thicknesses. Our simulations for very thin target plates show that the penetration process is achieved through the well-known dishing mechanism, where the target material is pushed forward by the projectile’s nose. An important observation, which emerges from our simulations, is that the transition between the dishing and the hole enlargement mechanisms takes place at a normalized thickness of about \( H/D = 1/3 \). We also find that the normalized resistive stress for intermediate plate thicknesses, \( 1/3 < H/D < 1.0 \), is relatively constant at a value of \( \sigma_r/\sigma_f \approx 2.0 \). This range of thicknesses conforms to a state of quasi plane stress in the plates. For thicker plates \( H/D > 1 \) the \( \sigma_r/\sigma_f \) ratio increases monotonically to values which represent the resistance to penetration of semi-infinite targets, where the stress state is characterized by plane strain conditions. Using a simple model, which is based on energy conservation, we can predict the values of the ballistic limit velocities for many projectile/target combinations, provided the perforation is done through the ductile hole enlargement mechanism. Good agreement is demonstrated between predictions from our model and experimental data from different sources, strongly enhancing the confidence in both the validity and usefulness of our model.

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1. Introduction

In a recent paper (Rosenberg and Dekel, 2009a) we revisited the deep penetration of rigid rods in semi-infinite metallic targets, using 2D numerical simulations. In particular, we followed the deceleration of rigid rods, with different nose shapes, as they penetrate various targets. We found that for impact velocities \( V_o \), which are smaller than some threshold value \( V_c \), the deceleration of the rod is constant during the penetration process. Moreover, this deceleration does not depend on impact velocity, as long as it is lower than the threshold velocity, which marks the onset of cavitation in the penetration channel. We also found that the constant deceleration is achieved at penetration depths beyond the entrance phase, which is typically of the order of six rod diameters. From these constant decelerations, we obtained the resistive stress \( R_e \) which the target exerts on the rigid rod. The values of \( R_e \) and the threshold velocities \( V_c \) were found to depend on both the strength of the target and the nose shape of the rod. Using the appropriate values of these constant decelerations we were able to construct a simple penetration model for rigid rods, impacting semi-infinite metallic targets, both at low \( V_o < V_c \) and high \( V_o > V_c \) velocities.

In a subsequent paper (Rosenberg and Dekel, 2009b) we investigated the process of ductile plate perforation, by sharp-nosed rigid projectiles. We showed that due to the free surfaces (the impact face and the back surface) the deceleration of these projectiles is far from constant during perforation. This fact means that a simple perforation model, which is based on the actual resisting stresses, must be very difficult (if not impossible) to construct. Thus, we used a different approach, based on energy considerations, in order to define an effective resisting stress \( \sigma_r \) which the plate exerts on the projectile during perforation. With this effective stress we were able construct a simple perforation model which accounts for both our simulation results and for much of the available data in the literature. Our most important result in Rosenberg and Dekel (2009b) concerns the relation between the values for the effective stress, normalized by the flow stress of the target material \( \sigma_f/\sigma_f \) as a function of the ratio \( H/D \), where \( H \) is the plate thickness and \( D \) is the projectile’s diameter. The purpose of the present paper is to further investigate the mechanics of ductile plate perforation by rigid projectiles with additional simulations and comparisons with recently published data. In particular, we investigate the perforation of thin plates \( H/D \leq 1 \) in order

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to follow the transition from the dishing mechanism, in thin plates, to the ductile hole enlargement process in thick plates. Using our simulation results we present a simple analytical model for the ballistic limit velocity \( (V_0) \), for a large range of plate thicknesses.

2. Numerical simulations

The simulations were performed with the AUTODYN-2D code, using the Lagrange processor for both the projectile and the target. In order to assure that the projectiles do not deform, during the impact and perforation processes, their strength (von-Mises) was kept at 50 GPa in all simulations. Thus, in all the simulations performed here the projectile is within its elastic range. The meshing was the same as described in Rosenberg and Dekel (2009a) and Rosenberg and Dekel (2009b), namely, eleven cells on the projectile’s radius and a similar meshing around the symmetry axis of the targets. In the simulations for very thin plates (0.1–0.2 mm) we used shell elements for the target. The idea here is to treat the plate as a thin shell for which the strength and the bulk modulus are defined. We checked the validity of this configuration by comparing the results with those from the usual meshing, for plates of intermediate thickness (0.5–1.0 mm), and found that the residual velocities were the same in both simulations. The lateral dimensions of the target were large enough to avoid any influence from their lateral boundaries. The constitutive relation for the target materials was the simple elasto-plastic von-Mises relation without any strain rate or strain hardening terms, in order to simplify the analysis. The main purpose of this simplification is to explore the relations between the relevant ballistic parameters, such as the residual velocities of the projectiles, and the strength of the target plate. Many materials show appreciable strain hardening and thermal softening during their deformation and in order to account for experimental results with such materials, one has to implement these variations in their constitutive relations. However, these additions often mask the relations which we are seeking in the present study. On the other hand, there are many materials, especially strong alloys, which do not strain harden appreciably and their sensitivity to strain rate is relatively small. For these materials one can choose an average value for their strength from a dynamic compression test, e.g. the Kolsky bar system, at a strain rate of \( 10^4 \text{s}^{-1} \), which is the typical rate in a ballistic test.

The code library was used for all the necessary data in the relevant equation of states of the materials, as described in Rosenberg and Dekel (2009a) and Rosenberg and Dekel (2009b). We used the shock equations of state simply because they are given for many materials in the code. Other equations can be also used in these simulations since the pressures in the projectile and target are not very high in these impact events. Moreover, since we are dealing only with the ductile enlargement of holes, we did not have to consider failure mechanisms, like spalling or scabbing, which take place under the impact of blunt projectiles. This is a very important simplification of the perforation process by sharp-nosed projectiles. Thus, we did not specify any failure model for the target materials and set the erosion criterion at \( e_i = 2.0 \) which is high enough to avoid any problems with mass erosion in the target. The erosion strain has to be defined in such simulations in order to ensure that cells which are heavily deformed will not stop the computation process. It does not specify a true property of the material, but with higher values of \( e_i \), the material behaves in a more ductile manner. In order to check the influence of this parameter on the simulation results, we performed several simulations with lower values of the erosion threshold, \( e_i = 0.5 \) and 1.0. The resulting residual velocities were higher, as expected, but the differences were limited to about 10%, as compared with results from the simulations with the erosion strain of 2.0. Low values of these strains can be considered as representing early failure of the target material. As we deal with ductile hole enlargement in the present work, our targets have to behave in a ductile manner. However we shall point out where failure mechanisms are important, especially when reviewing experimental data which include failure modes such as petalling, spalling and thermal softening. It is well known that such failure modes play an important role in the impact of blunt projectiles on thin targets, but this issue is outside the scope of the study presented here.

We shall first present our simulation results for 3CRH ogive-nosed steel projectiles perforating aluminum plates of different strengths and thicknesses. These simulations follow the experimental results from two recent works, (Forrestal et al., 2009 and Borvik et al., 2009), where 0.3”AP projectiles were shot at aluminum plates made of different alloys. The results from these simulations enhance our model as far as the dependence of \( \sigma_r/Y \) on \( H/D \), for the range of \( H/D > 1 \), is concerned. These results complement those which we obtained in Rosenberg and Dekel (2009b), with conical and 1.5CRH ogive nosed projectiles made of tungsten, steel and aluminum. Our main aim in the present work is to investigate the universality of this curve, \( \sigma_r/Y \) vs. \( H/D \), for all the relevant nose shapes and for different target materials. We then deal with the question whether a single plate of a given thickness has the same ballistic properties as an equal thickness target made of several thinner plates which are stacked together. This issue has a practical importance as many workers are using such stacks of thin plates in their ballistic tests (as in Forrestal et al., 2009; Borvik et al., 2009, for example). Finally, we investigate the perforation processes of thin and intermediate plates \( (H/D < 1) \) in order to follow the transition from the dishing to the hole enlargement mechanisms.

3. The resistive stress for thick plates \((H/D > 1)\)

In all the simulations described here we used short rigid projectiles with a diameter of \( D = 6 \text{ mm} \) and effective lengths of about 24 mm, depending on their nose shapes. These included 3CRH ogive nosed and several types of conical-nosed projectiles. The first set of simulations, for \( H/D > 1 \), will be described in this chapter in order to add more results for the numerical data which we presented in Rosenberg and Dekel (2009b). Our aim is to supply a numerically-based curve for the normalized resisting stress \( (\sigma_r/Y) \) as a function of the normalized thickness of the plates \( (H/D) \).

3.1. Deriving the effective stresses

The impact velocities \( (V_0) \) and the resulting residual velocities \( (V_r) \) from each simulation are used to determine the inferred ballistic limit velocities \( (V_0) \) through the model of Recht and Ipson (1963), as described in Rosenberg and Dekel (2009b):

\[
V_{bl} = \left( V_0^2 - V_r^2 \right)^{0.5}
\]

or, in its normalized version:

\[
V_r/V_{bl} = \left[ \left( V_0/V_{bl} \right)^2 - 1 \right]^{0.5}.
\]

This simple relation, Eq. (1b), means that all the data for the normalized residual velocities \( (V_r/V_{bl}) \), as a function of \( V_0/V_{bl} \), should fall on a single curve. This was clearly demonstrated in Rosenberg and Dekel (2009b) for several sets of data concerning the perforation of different plates by rigid projectiles and long rods. Thus, the only physical parameter which has to be determined accurately, either experimentally or analytically, is the ballistic limit velocity for each projectile/plate combination.
Our next step is to use this inferred value of \( V_b \) in order to calculate the effective resistance of the target (\( \sigma_r \)) through the energy-based equation, which was derived in Rosenber and Dekel (2009b):

\[
\sigma_r = \frac{\rho_p \cdot L_{eff} \cdot V_{bi}^2}{2H},
\]

(2a)

where \( \rho_p \) is the density of the projectile and \( L_{eff} \) is its effective length. This effective stress depends on target thickness, as we have shown in Rosenber and Dekel (2009b), and we are going to further investigate this issue here. In fact, many workers are using a simple relation like Eq. (2a) with a constant resisting stress. Some workers use the dynamic cavity expansion analysis in order to determine the value of \( \sigma_r \), which is independent on target thickness (see Forrestal et al., 2009; Borvik et al., 2009, for example). In contrast, our approach, which is a “numerically empirical” model, shows that the inherent resistance of a plate to the perforation process is strongly dependent on plate thickness. The simulations which we performed enabled us to construct the relation between \( \sigma_r / Y_t \) and \( H/D \) for a large range of values for \( H/D \). Once this relation is determined, we can predict the value of \( V_b \) for every projectile/target combination by rewriting Eq. (2a) as follows:

\[
V_{bl} = (2H \cdot \sigma_r / \rho_p \cdot L_{eff})^{0.5}.
\]

(2b)

The dependence of \( \sigma_r \) on \( H \) means that the relation between \( V_{bi} \) and \( (H/D)^{0.5} \) is not simply linear, as one may infer from Eq. (2b). Moreover, by keeping the mass of the projectile constant while changing its diameter (and its effective length), one may be led to the conclusion that \( V_{bi} \) is linearly dependent on \( D \). However, the strong dependence of \( \sigma_r \) on the ratio \( H/D \) results in a more complicated relation between \( V_{bi} \) and \( D \). This issue will be further discussed in Section 4.4.

3.2. Comparing our model with experiments for aluminum plates

Before we describe the simulations which were performed in this study, we wish to demonstrate the usefulness of our model (from Rosenber and Dekel, 2009b) by comparing its predictions with experimental results from two recent studies (Forrestal et al., 2009; Borvik et al., 2009). A series of experiments with 0.3” APM2 projectiles, perforating 20 and 40 mm thick 7075-T651 aluminum plates, is described in Forrestal et al. (2009) and a similar set of experiments, for 20, 40 and 60 mm thick 5083-H116 aluminum plates, are described in Borvik et al. (2009). In fact, the 40 and 60 mm targets included two and three plates, each 20 mm thick, which were held together. These laminate targets are, obviously, not identical with monolithic plates and we shall discuss the effects of lamination later on. For the present purpose we consider only the single, 20 mm plates, in these studies. Impact and residual velocities were measured for both the jacketed projectiles and for their hard steel cores which were launched with a specially designed sabot. We shall consider only the steel core experiments since they do not include the complicating effects of the soft jackets surrounding them. These 3CRH ogive nosed hard steel cores have a diameter of 6.17 mm and a mass of 5.25 g, resulting in an effective length of \( L_{eff} = 22.4 \) mm. Thus, the normalized thickness of the plates in these experiments was: \( H/D = 3.24 \). Using the simulation results of our earlier work (Rosenberg and Dekel, 2009b) we find that the normalized effective stress, for this value of \( H/D \), should be \( \sigma_r / Y_t = 2.8 \). The compressive stress–strain curve of the 7075-T651 alloy, as given in Forrestal et al. (2009), shows a clear yield point at 0.52 GPa followed by a strain hardening curve at high strains. The flow stresses increase from 0.65 GPa, at a strain of 0.1, to about 0.68 GPa at a strain of 0.8. Thus, a value of \( Y_t = 0.65 \) GPa seems adequate to represent the average flow stress of this material. The stress strain curve for the 5083 alloy, as shown in Borvik et al. (2009), has a yield point at 0.24 GPa and a rather constant strain hardening between 0.39 GPa at a strain of 0.2, and 0.47 GPa at a strain of 0.8. The average flow stress of the 5083-H116 alloy, as given in Borvik et al. (2009), is around \( Y_t = 0.43 \) GPa. Thus, the corresponding values for the effective resisting stresses are: \( \sigma_r = 1.82 \) GPa, for the 7075 alloy, and \( \sigma_r = 1.2 \) GPa, for the 5083 alloy.

We have now all the information needed in order to predict the values for \( V_b \), using Eq. (2b). These values are given in Table 1 together with the experimental values, and one can clearly see the excellent agreement between model and data which strongly supports the validity and usefulness of our model. All velocities are given in m/s.

As described above, for a given ballistic limit velocity, the energy-based model of Recht and Ipson (1963) provides a simple relation for the residual velocity as a function of the impact velocity, as given by Eq. (1a). We can check the agreement between predicted and measured values of \( V_r \) for the two sets of experiments in Forrestal et al. (2009) and Borvik et al. (2009). Fig. 1(a) shows this agreement for \( V_r(V_b) \), using Eq. (1a) and the values which we predicted for \( V_b \). One can clearly see that the predicted curve falls very near the data points, further enhancing the model by Recht and Ipson (1963).

Another set of experiments, with a much larger projectile, is described in Borvik et al. (2009). These were 3CRH ogive nosed projectiles and (b) the 197 g projectile.

### Table 1

<table>
<thead>
<tr>
<th></th>
<th>( V_b ) (measured)</th>
<th>( V_b ) (predicted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7075-T651</td>
<td>633</td>
<td>643.5</td>
</tr>
<tr>
<td>5083-H116</td>
<td>513</td>
<td>523.5</td>
</tr>
</tbody>
</table>

Fig. 1. Our model’s predictions and the experimental results in Forrestal et al. (2009) and Borvik et al. (2009), for 20 mm aluminum plates perforated by (a) the 0.3” APM2 hard steel core, (b) the 197 g projectile.
projectiles with a mass of 197 g and a diameter of \( D = 20 \) mm, and an effective length of \( L_{\text{eff}} = 80 \) mm. These projectiles were shot at the 20 mm 5083-H116 aluminum plates (\( H/D = 1 \)), and the resulting ballistic limit velocity was \( V_{0} = 244 \) m/s. From our results in Rosenberg and Dekel (2009b) we know that for \( H/D = 1 \) the ratio \( \sigma_{r}/\sigma_{r} \) is equal to 2.0. Thus, for this projectile-plate combination the effective resisting stress is \( \sigma_{e} = 0.86 \) GPa. Inserting this value, together with the appropriate values of \( L_{\text{eff}} \) and \( H \), in Eq. (2b), results in a value of \( V_{0} = 234 \) m/s, which is lower than the measured value by about 4\%. Fig. 1b shows the predicted \( V(V_{0}) \) curve and the data from Borvik et al. (2009) for these 197 g ogive nosed projectiles. Again, the agreement is quite good. The main conclusion which we can draw from this agreement is that the energy based Eq. (1a), which was first suggested by Recht and Ipson (1963), is able to account for the data if one has a good estimate for \( V_{0} \). We stress this point here because several workers account for their data through a different equation which is essentially empirical by its nature. This is the Lambert–Jonas equation which was also used in Forrestal et al. (2009). The relation between \( V, V_{0} \) and \( V_{bl} \) according to this equation is

\[
V_{f} = \left( V_{0}^{2} - V_{b}^{2} \right)^{1/2},
\]

where \( p \) is an empirical constant which is chosen to best fit the data with the least square method. It turns out that the values for \( p \) have a relatively large range, \( p = 1.85 - 2.6 \), as shown in Forrestal et al. (2009) and Borvik et al. (2009) for the two aluminum alloys tested there. Moreover, there is no consistent relation between these values and the thicknesses of the plates or any other property of the plate material. Thus, we prefer to use the value of \( p = 2 \) which identifies Eq. (3) with the physically based Eq. (1a) from the work of Recht and Ipson (1963). Our understanding is that physically based model is a better choice than any empirical formula even if the latter does a better job with fitting to the data.

### 3.3. The effect of lamination

In order to investigate the influence of lamination on the ballistic limit velocities we performed several sets of simulations with 40 and 60 mm aluminum targets, with strength of 0.41 GPa, which were either single-plated or laminated. The ogive nosed projectiles in these simulations had a length of \( L_{\text{eff}} = 23.8 \) mm. The results of these simulations, in terms of inferred ballistic limit velocities from Eq. (1a), and the corresponding effective stresses from Eq. (2a), are given in Table 2. We also give the simulation results for the 20 mm plate, for completeness.

The effect of lamination can be clearly seen by the values we obtained for \( V_{bl} \) of the laminated 40 and 60 mm plates in these simulations. Due to the additional surfaces the projectile is expected to lose less energy in the laminated target, a trend which should increases with the number of plates. This is what we actually see in Table 2 for the monolithic and laminated 60 mm targets. Clearly, the addition of laminations lowers the values of \( V_{bl} \), but the effect itself is not very large. For the three layered target \( V_{bl} \) is lower by only 2.5\% than that of the monolithic 60 mm target. One should note that the space between the plates in these simulations was about 0.017 mm, which would be difficult to achieve with real plates in the experiments. Thus, we can expect a somewhat larger difference between monolithic and stacked targets (of the same thickness) in actual experiments. In summary, lamination of closely stacked plates reduces their ballistic resistance, against sharp-nosed projectiles, by a few percent.

One can also predict that the changes in \( V_{bl} \) will be more pronounced for widely separated plates, due to the new free surfaces which are introduced in this arrangement. In order to have some appreciation for the effect of separation we repeated the simulations of the 60 mm target, which consists of three 20 mm plates. The plates were separated by 30 mm from each other (more than the projectile’s length). The impact velocity in this simulation was 1250 m/s and the residual velocity, after emerging from the three plates, was 846.6 m/s. These values result in an inferred value of \( V_{bl} = 920 \) m/s. This value is lower by 9\% than the value of \( V_{bl} = 1010 \) m/s, for the closely stacked plates impacted at 1250 m/s (see Table 2). Thus, the resistance to penetration of the closely stacked target is, indeed, higher than that of a widely separated set of plates, as expected. This trend was already observed by many workers, as in Dey et al. (2007), for example. In that work the ballistic limit velocities of Weldox 700E steel plates, with several thickness combinations, were determined. The hard steel projectiles were 3CRH ogive nosed, with \( D = 20 \) and \( L_{\text{eff}} = 80 \) mm and the thicknesses of the plates were 6 and 12 mm. Targets made of two closely stacked 6 mm plates, and with an air gap of 24 mm, were tested and compared with the monolithic 12 mm plate. The results of these experiments, in terms of the \( V_{bl} \) values for each target configuration, are given in Table 3 below.

As with the simulations described above, these experiments also show that the values of \( V_{bl} \) for laminated targets are lower than those for the monolithic ones and that an air gap between the plates lowers this value even more, as expected. Dey et al. (2007) attribute the lower \( V_{bl} \) of the laminated target, as compared with the monolithic one, to the fact that shear and tensile stresses cannot be transmitted between the layered plates, thus reducing their shear resistance. One should note that the 6 mm plate in this study experienced a considerable bending around the perforation hole, while the 12 mm plate deformed much less. This difference may explain the non-linear relation between the values of \( V_{bl} \) for these plates, as their thickness ratio is 2.0 while their corresponding ballistic limits are related by a factor of 1.6. The extra energy spent by the projectile in deforming the 6 mm plate is manifested by the relatively high value of its ballistic limit velocity (198 m/s) as compared with the value of 318 m/s for the 12 mm plate.

We can have an estimate for the reduction in the value of \( V_{bl} \) for the laminated target with large air gaps between the plates. Consider two plates of thickness \( H \) separated by large enough distance such that the projectile’s nose is out of the first plate when it impacts the second one. A simple calculation, using Eq. (1a) for this combination, shows that the ballistic limit velocity for this double plate target is given by:

\[
V_{bl}(Hx2) = 2 \cdot V_{bl}(H).
\]

Applying this argument to the experimental results of Dey et al. (2007), we expect

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**Table 2**

Simulation results for the single and laminated aluminum plates.

<table>
<thead>
<tr>
<th>( H ) (mm)</th>
<th>( V_{c} ) (m/s)</th>
<th>( V_{b} ) (m/s)</th>
<th>( V_{bl} ) (m/s)</th>
<th>( \sigma_{r} ) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>700</td>
<td>332</td>
<td>500</td>
<td>1.17</td>
</tr>
<tr>
<td>40</td>
<td>900</td>
<td>486</td>
<td>503</td>
<td>1.47</td>
</tr>
<tr>
<td>20X2</td>
<td>900</td>
<td>426</td>
<td>793</td>
<td>1.47</td>
</tr>
<tr>
<td>60</td>
<td>1000</td>
<td>604</td>
<td>797</td>
<td>1.47</td>
</tr>
<tr>
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<td>1000</td>
<td>444</td>
<td>783</td>
<td>1.43</td>
</tr>
<tr>
<td>30X2</td>
<td>1150</td>
<td>615.7</td>
<td>788</td>
<td>1.43</td>
</tr>
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<td></td>
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<tr>
<td>1250</td>
<td>720</td>
<td>1021.8</td>
<td>1.56</td>
<td></td>
</tr>
</tbody>
</table>

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**Table 3**

Results from the experiments in Dey et al. (2007).

<table>
<thead>
<tr>
<th>( H ) (mm)</th>
<th>6</th>
<th>12</th>
<th>6 × 2 (closely packed)</th>
<th>6 × 2 (air 24 mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{bl} ) (m/s)</td>
<td>198</td>
<td>318</td>
<td>288.3</td>
<td>280</td>
</tr>
</tbody>
</table>

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that the ratio of the $V_{bl}$ value for the laminated $6 \times 2$ target with the air gap, to that of the single 6 mm plate, will be equal to $\sqrt{2} = 1.414$. This is exactly what we get from the results of Dey et al. (2007), as given in Table 3: $V_{bl}(6 \times 2)/V_{bl}(6) = 280/198 = 1.414$.

In fact, for $N$ widely separated identical plates of thickness $H$, one can easily show that:

$$V_{bl}^2(H \times N) = NV_{bl}^2(H).$$

(4a)

Using the same analysis it is easy to show that when the target consists of $N$ separated plates, of different thicknesses ($H_i$) and strengths ($Y_i$), its ballistic limit velocity is given by

$$V_{bl}^2 = V_{bl}^2(H_1) + V_{bl}^2(H_2) + \cdots + V_{bl}^2(H_N) = \left(\frac{2}{\rho V_f L_{off}}\right) \sum \sigma_{ri} - H_i,$$

(4b)

where $\sigma_{ri}$ is the effective resisting stress of the $i^{th}$ plate and the sum is over all the plates. It is clear from this simple equation that $V_{bl}$ does not depend on the ordering of the plates. This was also shown in other works, such as the analytical study of Ben-Dor et al. (1998) who cite some experimental data to support their analysis. The same conclusions hold for laminations with different distances between the plates, as long as these do not touch each other during the perforation process. We should stress that all of these conclusions hold for separated ductile plates perforated by sharp nosed rigid projectiles. The situation can be totally different when a failure mechanism takes place, either in the target or the projectile. In such a case the ordering of the different plates and the space between them, may be important.

Another set of experiments with laminated plates is given in Gupta and Madhu (1997). In Table 3 of Gupta and Madhu (1997) we find the results of impact and residual velocities of 0.3” AP projectiles penetrating layers of aluminum and steel plates. Consider the data for 6 mm mild steel plates which include stacks of up to six plates. Each stack was impacted by a single shot. From the measured values of $V_0$ and $V_r$, we determine the inferred ballistic limit velocity ($V_{bl}$) through Eq. (1a). The basic result for our analysis is the inferred value for $V_{bl}$ of the single 6 mm plate, which is $V_{bl} = 350.6$ m/s. With this value we can predict $V_{bl}$ values for each stack according to Eq. (4a). These predicted values are compared with the inferred values using Eq. (1a). The relevant data from these experiments, for each stack of plates, are shown in Table 4. We can also use the inferred values of $V_{bl}$ in order to predict the residual velocities for each stack, through Eq. (1b) and compare it to the experimental value of $V_r$. These values are given in the last column of Table 4. All the velocities are given in m/s.

We can see that the predicted values, for both $V_{bl}$ and $V_r$, are very close to the experimental values, strongly enhancing our simple analysis for laminated targets. In conclusion, the analysis presented here, as well as the experimental data cited above, suggest that the ballistic limit of a monolithic plate should be always higher than that of an equal thickness target composed of laminated plates. In addition a closely stacked laminated should have a higher ballistic resistance than a configuration with separated plates. This is the expected behavior for ductile plates which are penetrated by sharp-nosed projectiles.

Finally, we wish to point out an important issue which emerges from the simulations for a given target struck at different impact velocities. As seen in Table 2, with higher impact velocities we obtain a somewhat higher value for the inferred ballistic limit. This is the result of the fact that the energy which the projectile loses by penetrating the target is not strictly constant but increases slightly with increasing impact velocity. This increase is manifested by a somewhat larger volume of target material which is deformed plastically. This is inferred to as global deformation in several articles. Thus, the true ballistic limit should be inferred by the lowest impact velocities, both experimentally and through simulations. On the other hand, the differences in the inferred values of $V_{bl}$ are not very large, as long as the impact velocity is not too high. In order to have an estimate of this effect we compare the inferred ballistic limits from the simulations for the 20 mm plates in Table 2 adding the simulation for the three widely spaced 20 mm plates which we described above. Note that in this simulation the impact velocity was 1250 m/s and the residual velocities for each plate were, in effect, the impact velocities for the subsequent plate. These velocities are given in Table 5 below, together with inferred ballistic limits for each case. All velocities are given in m/s.

One can clearly see the monotonic increase in $V_{bl}$ with impact velocity, which was also noted in the simulations we presented in Rosenberg and Dekel (2009b). However, this increase is not large and it can be ignored as long as the impact velocity is not much higher than the ballistic limit. This was the reason we recommended in Rosenberg and Dekel (2009b) that $V_{bl}$ values be determined by impact velocities which are not higher than about 1.5 $V_{bl}$. From Table 5 we learn that increasing $V_0$ from 600 to 1250 m/s resulted in an increase of only 8% in the inferred values for $V_{bl}$, from 500 to 540 m/s, respectively. Moreover, for the triple plate target with $V_0 = 1250$ m/s and $V_{bl} = 500$ m/s, we get by Eq. (1b): $V_r = 1145.6, 1030.7$ and 901.3 m/s for the three plates. These values are higher by 1.6%, 3.7% and 6.5% than the values we obtained from the simulations for the first, second and third plate in Table 5. Thus, one may use the lowest value of $V_{bl}$ and get predicted values for $V_r$, which are higher by several percent only than those obtained with simulations at high velocities.

3.4. Summary for thick targets ($H/D \geq 1$) and comparison with experiments

Fig. 2 summarizes all our simulation results, including those from Rosenberg and Dekel (2009b), for the normalized effective stresses as a function of $H/D$ of thick targets (with $H/D \geq 1$). These simulations include ogive and conical nosed projectiles, made of steel and tungsten, impacting steel and aluminum targets with strengths of 0.4 and 0.8 GPa. As is clearly seen, the results, in terms of $\sigma_1/Y_1$ vs. $H/D$, are scattered around a curve which starts at the point $\sigma_1/Y_1 = 2$ for $H/D = 1$ and increases asymptotically to values which characterize semi-infinite targets, as explained in Rosenberg et al. (2007), as given in Table 3: $V_{bl}(6 \times 2)/V_{bl}(6) = 280/198 = 1.414$.

In Table 4 we use the experimental values of $V_{bl}$ determined by impact velocities which are not higher than about 1.5 $V_{bl}$. From Table 5 we learn that increasing $V_0$ from 600 to 1250 m/s resulted in an increase of only 8% in the inferred values for $V_{bl}$, from 500 to 540 m/s, respectively. Moreover, for the triple plate target with $V_0 = 1250$ m/s and $V_{bl} = 500$ m/s, we get by Eq. (1b): $V_r = 1145.6, 1030.7$ and 901.3 m/s for the three plates. These values are higher by 1.6%, 3.7% and 6.5% than the values we obtained from the simulations for the first, second and third plate in Table 5. Thus, one may use the lowest value of $V_{bl}$ and get predicted values for $V_r$, which are higher by several percent only than those obtained with simulations at high velocities.

<table>
<thead>
<tr>
<th>Target</th>
<th>$V_0$</th>
<th>$V_r$</th>
<th>$V_{bl}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single plate</td>
<td>600</td>
<td>332</td>
<td>500</td>
</tr>
<tr>
<td>Triple plates</td>
<td>1250</td>
<td>1127.4</td>
<td>540</td>
</tr>
<tr>
<td></td>
<td>1127.4</td>
<td>994.6</td>
<td>532</td>
</tr>
<tr>
<td></td>
<td>994.6</td>
<td>846.6</td>
<td>527</td>
</tr>
</tbody>
</table>
and Dekel (2009b). Since all these points are quite close to each other we did not look for further refinements in the different sets of simulations. Instead, we outlined a curve through these points which accounts for all of them to within 5% in the resisting stresses ($\sigma_r$). Since the ballistic limit velocity is dependent on the square root of $\sigma_r$, as given by Eq. (2b), we expect differences of about 2.5% between experimental values for $V_{bl}$ and those calculated from our best fit curve, which is given by

$$\frac{\sigma_r}{Y_t} = 2.0 + 0.8 \ln(H/D) .$$

(5)

One can check the validity of this relation by analyzing sets of experiments where the targets have different thicknesses. Several examples were given in Rosenberg and Dekel (2009b) and the agreement between the data and the model was good. As another example we bring the data from Gupta and Madhu (1992) for the conical on steel 0.40 GPa and Ogive 1.5CHN on Al 0.40 GPa. The inferred flow stresses for the targets in Gupta and Madhu (1992). The inferred flow stresses for the targets in Gupta and Madhu (1992).

The fact that we obtained an increasing value of the resistive stress with plate thickness enhances the validity of our model, since the stress state in the plate should change gradually from plane stress to plane strain for thicker plates. These effective stresses should approach the resistance to penetration of semi-infinite targets ($R_i$). We have shown in Rosenberg and Dekel (2009a) and Rosenberg and Dekel (2009b) that the values of $R_i$ are typically 4–6 times the strength of the target material and we expect the resistive stresses to approach these values as the thickness of the plate increases.

The table below lists the inferred flow stresses ($\sigma_r$) and the corresponding values of $Y_t$ for each plate and these are inserted in Eq. (2b), with the appropriate values for $L_{ef}$, in order to get our predictions of $V_{bl}$ for each projectile/plate combination. Table 8 shows the excellent agreement between our predictions for $V_{bl}$ and the experimental results in Cheeseman et al. (2008), as given by the $V_{bl}$ values for the two projectiles. This agreement enhances the validity of our simulations and demonstrates the usefulness of our model, as it is presented by Eqs. (5) and (2b).

Finally, we compare our model’s predictions with the experimental data of Cheeseman et al. (2008) who list the values of $V_{bl}$ for aluminum 2139-T8 plates, of different thicknesses, impacted by 0.3” and 0.5” APM2 projectiles. These projectiles are very similar in shape, having hard steel cores with soft jackets, which do not participate in the whole penetration process. Thus, we shall consider only their hard steel cores in our analysis. The 5.25 g 0.3”APM2 core has a mass of 25 g, diameter of $D = 6.2$ mm and an effective length of $L_{ef} = 22.4$ mm. The hard steel core of the 0.5” projectile has a mass of 29 g, diameter of $D = 10.8$ mm and an effective length of $L_{ef} = 34.8$ mm. The aluminum alloy tested in Cheeseman et al. (2008) had an ultimate tensile strength of 0.5 GPa (see Table II in Cheeseman et al. (2008)) and we use this value for $Y_t$ in our analysis. The thickness of these plates ranged between 25–40 mm for the 0.3” projectiles, and between 39–64 mm for the 0.5” projectiles. Thus, the values of $H/D$ in these tests ranged between 3.8 and 6.6, and with Eq. (5) we find the corresponding values of $\sigma_r/Y_t$ for each projectile/plate combination. With $Y_t = 0.5$ GPa we find $\sigma_r$ for each plate and these are inserted in Eq. (2b), with the appropriate values for $L_{ef}$, in order to get our predictions of $V_{bl}$ for each projectile/plate combination. Table 8 shows the excellent agreement between our predictions for $V_{bl}$ and the experimental results in Cheeseman et al. (2008), as given by the $V_{bl}$ values for the two projectiles. This agreement enhances the validity of our simulations and demonstrates the usefulness of our model, as it is presented by Eqs. (5) and (2b).

Table 6

<table>
<thead>
<tr>
<th>$H$ (mm)</th>
<th>$H/D$</th>
<th>$V_r$ (experimental)</th>
<th>$V_r$ (inferred)</th>
<th>$\sigma_r$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.6</td>
<td>827.5</td>
<td>702.2</td>
<td>1.65</td>
</tr>
<tr>
<td>12</td>
<td>1.94</td>
<td>818</td>
<td>661.5</td>
<td>1.66</td>
</tr>
<tr>
<td>16</td>
<td>2.58</td>
<td>819.7</td>
<td>562</td>
<td>1.92</td>
</tr>
<tr>
<td>20</td>
<td>3.2</td>
<td>820.6</td>
<td>404.8</td>
<td>2.2</td>
</tr>
<tr>
<td>25</td>
<td>4.03</td>
<td>842.3</td>
<td>107.6</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Table 7

<table>
<thead>
<tr>
<th>$H$ (mm)</th>
<th>$\sigma_r$ (GPa)</th>
<th>$Y_t$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.65</td>
<td>0.69</td>
</tr>
<tr>
<td>12</td>
<td>1.66</td>
<td>0.66</td>
</tr>
<tr>
<td>16</td>
<td>1.92</td>
<td>0.7</td>
</tr>
<tr>
<td>20</td>
<td>2.2</td>
<td>0.75</td>
</tr>
<tr>
<td>25</td>
<td>2.4</td>
<td>0.71</td>
</tr>
</tbody>
</table>
and length of the projectile, results in our predicted value for $V_{bl}$. These are given in Table 9 together with the experimental data for the two velocities $V_1$ and $V_2$ which bound the value of $V_{bl}$ (all in units of m/s).

We see that the predicted values are in good agreement with the experiments. Even for the 76.2 mm plate, where the predicted value for $V_{bl}$ seems to be outside the permitted range, it is higher by only a few percent from the average value of $V_1$ and $V_2$.

A similar series of experiments were performed on plates of 6061-T6 aluminum which were perforated by long steel rods. These works, by Piekutowski et al. (1996), Forrestal et al. (1987) and Rosenberg and Forrestal (1988), were described and analyzed in Rosenberg and Dekel (2009b). Table 10 lists the relevant data from these works and our predicted values for $V_{bl}$ which were derived in the way explained above. The flow stress of this aluminum alloy was taken as $Y_t = 0.4$ GPa.

These three sets of data show the good agreement between our model predictions for $V_{bl}$ and experimental data. Concluding this section we can state that, as far as thick plates ($H/D > 1$) are concerned, our numerically-based model is able to predict ballistic limit velocities to within a few percent.

4. The resistive stress for thin plates ($H/D \leq 1$)

4.1. The transition from the dishing to the ductile hole enlargement mechanism

As discussed in Rosenberg and Dekel (2009b), the fact that we obtained a value of $\sigma_r/Y_t = 2$ for the case $H/D = 1$ is in excellent agreement with the analytical model of Bethe (1941). He calculated the work needed to open a hole of diameter $D$ in a ductile plate with a similar thickness. A short account of Bethe's model, which is based on a plane stress situation in the plate, is given by Woodward (1978). An important result of Bethe's model is that the effective resisting stress, which has to be applied on the hole's wall, is twice the flow stress of the target material. This effective stress is manifested through the expression for the work needed to open a hole of radius $r$ in a plate of thickness $H$ and a flow stress $Y_t$. From the plane stress analysis of Bethe, as described by Woodward (1978), this work is

$$W = \pi r^2 \cdot H \cdot 2Y_t,$$

which means that the effective resisting stress, when the hole enlargement process is the dominant one, is equal to $2 Y_t$. We should note that Bethe's analysis was done for plates with thickness of the same order as the projectile's diameter.

On the other hand the perforation of very thin plates is achieved through a dishing mechanism, where the material around the hole is pushed in the direction of the projectile's trajectory. The analysis of Thomson (1955), for the dishing mechanism in very thin plates, results in the following expression for the work needed to perforate the plate:

$$W = \pi r^2 \cdot H \cdot Y_t/2,$$

resulting in a value of $Y_t/2$ for the effective resisting stress which controls the dishing of thin plates.

Thus, according to these models the effective resisting stresses ($\sigma_r$) which a plate exerts on a sharp projectile during its perforation, increases from $Y_t/2$ for very thin plates, to $2 Y_t$ for moderate thicknesses, of the order of $H/D = 1$. For thicker plates $\sigma_r$ increases appreciably, as we have shown above, approaching the corresponding values of $R_t$ (about $5 Y_t$) for very thick plates. The transition from very thin to moderate plates is accompanied by a change in mechanism, from dishing to ductile hole enlargement, as discussed by Woodward (1978). In fact, Woodward (1978) tried to account for the transition from dishing to ductile hole enlargements by using the models of Bethe (mentioned above) and Thomson (1955), for very thin plates. Woodward added the energy needed to bend the plate material around the hole in order to obtain a criterion for the transition from bending and dishing of the thin plates, to the ductile hole enlargement of intermediate and thick plates. Using these two models Woodward found that the transition should occur at about $H/D = 1$. However, he points out the difficulties with these idealized models and, particularly, the fact that on moving from very thin plates to intermediate thicknesses the stress state in the plates changes from plane stress to plane strain. Thus, the situation with intermediate plate thicknesses must be very complex and its analysis far from simple. This is where 2D simulations can help since they include the actual stress states in the targets inherently. In addition they result in the shape of the plate around the hole, with which we can determine the transition point from dishing to the ductile hole enlargement processes, as we show below.
In order to follow these complicated processes we performed a large number of simulations for thin plates, both of steel and aluminum, with strengths of 0.41 and 0.82 GPa. These were impacted by \( D = 6 \) mm steel projectiles with the 3CRH ogive and several conical noses as well. The resulting \( V_r \) values from each simulation were used to obtain the ballistic limit velocities through Eq. (1a) and these were inserted in Eq. (2a) to derive the corresponding value of the effective resisting stress (\( \sigma_r \)). These stresses, normalized to the target’s strength (\( Y_t \)), were then plotted as a function of \( H/D \) for the range of \( H/D = 0 - 1.0 \), as shown in Fig. 3 below.

One can clearly see that the simulation results fall on two very distinctive curves; a fast rising one for the very thin targets, up to a value of about \( H/D = 1/3 \), and a nearly horizontal line with \( \sigma_r/Y_t = 2 \) for the range of \( H/D = 1/3 - 1.0 \). This behavior is a clear indication for a change of mechanism in the perforation process.

In order to demonstrate the fact that we are dealing with the transition from dishing to ductile hole enlargement we show in Fig. 4 simulation results for the shape of the target around the hole for three thicknesses: 1.5, 1.8 and 3.0 mm of the 0.82 GPa steel plates perforated by the ogive nosed \( D = 6 \) mm projectiles. It is clearly seen that the thicker plate (\( H = 3 \) mm, \( H/D = 0.5 \)) is defeated by the hole enlargement process, while the thinnest plate (\( H = 1.5 \) mm, \( H/D = 0.25 \)) is defeated by dishing. The intermediate plate with \( H/D = 0.3 \) seems to be on the threshold from dishing to ductile hole enlargement.

The simulation results in Fig. 3 do not, in fact, lie on a definite curve and there is a small scatter which is the result of the differences between the nose shapes of the projectiles and the strengths of the targets. Also, the points seem to approach the value of \( \sigma_r/Y_t = 0.5 \), as \( H \) approaches zero, in agreement with Thomson’s model in Thomson (1955). However, it is much more practical to draw a straight line through all our points, in the range of \( H/D = 0–1/3 \), and obtain a simple relation which will best represent them, for all practical values of \( H \), according to

\[
\sigma_r/Y_t = 2/3 + 4 \cdot (H/D).
\]

With this relation we can determine the values of \( \sigma_r \) for very thin plates, in the range of \( H/D < 1/3 \). For the range of \( H/D = 1/3 - 1.0 \) we shall use the value of \( \sigma_r/Y_t = 2.0 \), as explained above.

It is interesting to follow the improved version of the analytical model by Woodward (1978) for the energy spent by a projectile in dishing and bending while it is perforating a thin plate. According to this model the work done by a projectile with diameter \( D \), in perforating a thin plate of thickness \( H \) and flow stress \( Y \), through the dishing mechanism, is given by the sum of three terms:

\[
W_{dish} = \pi D^2 \cdot H \cdot Y/8 + \pi^2 D^2 \cdot H^2/8 + 0.43 \pi \cdot H^3 \cdot Y.
\]  

(9)

The first term represents the work done in radial stretching of the dished material (Thomson’s model Thomson, 1955), the second term is due to the bending forward of the plate and the third is due to the indentation by the sharp-nosed projectile, before the plate begins to dish. Note that our result for the work needed to perforate a plate of intermediate thickness (\( 1/3 < H/D \leq 1 \)), agrees with Bethe’s analysis as given by Eq. (6), which can be rewritten as

\[
W = \pi D^3 H \cdot Y/2.
\]  

(10)

In order to find the transition point from dishing to hole enlargement mechanisms we equate the two expressions for the corresponding energies, Eqs. (9) and (10), resulting in a quadratic equation in the parameter \( H/D \):

\[
3.464(H/D)^2 + \pi(H/D) - 3 = 0,
\]  

(11)

for which the positive solution is \( H/D = 0.56 \). Thus, the analytical approach of Woodward (1978) results in a transition value for \( H/D \) which is not very far from our simulation results, of \( H/D = 1/3 \). The post mortem examination of the targets in the work of Woodward (1978) showed that for the range of \( H/D = 0.67 - 2.67 \) the penetration was through the hole enlargement process, while for \( H/D = 0.33 \) it was through the dishing process, in agreement with what we found here for the transition between the two mechanisms.

We would like to point out that for \( H/D > 1 \) we obtained \( \sigma_r/Y_t \) values which are increasing with \( H/D \), while in the range of \( H/D = 1/3 - 1.0 \) the normalized stresses are constant, \( \sigma_r/Y_t = 2.0 \). This means that the normalized thickness \( H/D = 1 \) is the onset of another change in material behavior, which can be attributed to the transition from a state of quasi plane stress to that of plane strain, at very large thicknesses. Obviously, this transition is not an abrupt one and it occurs more gradually.

4.2. Comparison with experimental data

We wish to compare our model’s prediction, with the experimental results for the ballistic limit velocities from several studies starting with Woodward’s work (Woodward, 1978). He used two different conical-nosed steel projectiles perforating various steel...
and aluminum plates, ranging in thickness between 1.6 and 12.7 mm. These projectiles are termed AP (D = 4.76 mm, \( L_{\text{eff}} = 20.3 \) mm) and BP (D = 6.35 mm, \( L_{\text{eff}} = 12.2 \) mm) in Woodward (1978). Except for the aluminum plates all the other plates had \( H/D \) values in the range of 0.33–1.0. Thus, we can use the value of \( \sigma /Y_t = 2.0 \) for these plates, as our model predicts. For both the 5083 alloy and the “commercial aluminum”, with \( H/D > 1 \), we use Eq. (5) in order to use the proper \( \sigma /Y_t \) ratios for these cases. Woodward also lists the flow stresses of the various target materials, and we use the same values for the corresponding \( Y_t \) in our model. Finally, we have all the necessary information to insert in Eq. (2b) and derive the values of \( V_{bl} \) for each projectile/target combination in Woodward (1978). These are compared with the experimental results in Table 11, where all velocities are given in m/s.

We see that except for the weaker mild steel plates (with \( Y_t = 0.563 \) GPa), all the predicted \( V_{bl} \) values are within 15% of the experimental ones. In fact, most predictions are within 10% of the experimental results, which is a very good agreement, considering the variety of materials which were tested here.

One should also note that both our simulations and Woodward’s model in Woodward (1978) ignore failure mechanisms, which accompany perforation events in less ductile materials, such as: petaling, spalling, adiabatic shear and thermal softening. An excellent example for the complexity of the perforation process, when several failure mechanisms are at work, is given by Leppin and Woodward (1986). They shot conical-nosed hard steel projectile at plates made of the titanium alloy Ti-6Al-4V, which is well known for its propensity to adiabatic shearing. They identified four different modes of failure, depending on the \( H/D \) ratios, as well as on impact velocities. These modes resulted in the ejection of small and large shear plugs, as well as thin rings of target material around the projectiles. Thus, even a sharp-nosed projectile, like the one used by Leppin and Woodward (1986), may result in failure modes which are very difficult to simulate or to account for in an analytical model.

In order to enhance the validity of our simulation results for very thin targets, we compare our model’s predictions with experimental data from the works of Radin and Goldsmith (1988) and from Gupta et al. (2007), where the \( H/D \) ratios were particularly small. In Radin and Goldsmith (1988), conical-nosed steel projectiles with \( D = 12.7 \) mm and \( M = 29 g \) (\( L_{\text{eff}} = 29.2 \) mm), were shot at thin 2024–0 aluminum plates. The thickness of the plates ranged between 1.65–6.35 mm, which correspond to \( H/D = 0.13 - 0.5 \), covering the range of values around the transition point at \( H/D = 1/3 \). The flow stress of this material, as given by the static compression curve in Radin and Goldsmith (1988), is \( Y_t = 0.24 \) GPa, and this is the value we use to derive the effective resisting stresses (\( \sigma_r \)) through Eq. (8). As before, once we have the value of \( \sigma_r \), we insert it in Eq. (2b) and calculate the predicted value for \( V_{bl} \). Table 12 lists the relevant data from these experiments (plate thicknesses and ballistic limit velocities) together with our predictions for \( V_{bl} \) and we see that the agreement between model predictions and data is very good. The velocities in the table are given in m/s.

Another set of experiments, with polycarbonate targets (Lexan), was also performed by Radin and Goldsmith (1988), with the same projectiles. The stress–strain curve for this material shows that its flow stress is about \( Y_t = 0.08 \) GPa. Table 13 lists the relevant data from these experiments and shows the good agreement between our model’s predictions and the data for the values of \( V_{bl} \), except for the thickest plate, where the difference amounts to about 20%. Velocities are given in m/s.

Gupta et al. (2007) performed a large number of perforation experiments using hollowed steel projectiles, 19 mm in diameter with different nose shapes, which impacted very thin aluminum plates (0.5–3.0 mm). These experiments are characterized by very low values of \( H/D = 0.026 - 0.16 \), which are within the dishing range of perforation (\( H/D < 1/3 \)). This is an excellent opportunity to check our model in this range of thicknesses. The projectiles in this study had an outer diameter of 19 mm, a total length of 50.8 mm and mass of 52.5 g. We consider only the ogive nosed (1.5SCRH) projectiles in Gupta et al. (2007), whose effective length as calculated from their mass and outer diameter, was 23.6 mm. The aluminum alloy 1100-H12 which was used for the targets had a stress–strain curve which shows a strain hardening feature from \( Y = 0.25 \) GPa, at a strain of 0.05, to \( Y = 0.32 \) GPa at a strain of 0.5. Thus, a reasonable value for the average flow stress of this alloy would be \( Y_t = 0.28 \) GPa.

Table 14 lists the relevant data for all the plates in this study, in terms of their \( H/D \) ratios and the two impact velocities which bound the ballistic limit \( V_1 < V_{bl} < V_2 \) (see Table 2 in Gupta et al. (2007)). Also listed in Table 14 are the corresponding values for

### Table 11
Comparison between the data of Piekutowski et al. (1996) and our model’s predictions.

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Target</th>
<th>( H/D )</th>
<th>( Y_t ) (GPa)</th>
<th>( V_{bl} ) (experimental)</th>
<th>( V_{bl} ) (predicted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AP</td>
<td>4130 steel</td>
<td>0.67</td>
<td>1.18</td>
<td>330</td>
<td>308</td>
</tr>
<tr>
<td>AP</td>
<td>Mild steel</td>
<td>1.0</td>
<td>0.976</td>
<td>375</td>
<td>342</td>
</tr>
<tr>
<td>AP</td>
<td>Commercial Al</td>
<td>1.33</td>
<td>0.229</td>
<td>225</td>
<td>201</td>
</tr>
<tr>
<td>AP</td>
<td>5083 Al</td>
<td>2.67</td>
<td>0.452</td>
<td>454</td>
<td>466</td>
</tr>
<tr>
<td>BP</td>
<td>Mild steel</td>
<td>0.75</td>
<td>0.976</td>
<td>442</td>
<td>440</td>
</tr>
<tr>
<td>BP</td>
<td>Commercial Al</td>
<td>1.0</td>
<td>0.229</td>
<td>293</td>
<td>246</td>
</tr>
<tr>
<td>BP</td>
<td>Hadfield steel</td>
<td>1.0</td>
<td>1.69</td>
<td>603</td>
<td>669</td>
</tr>
<tr>
<td>BP</td>
<td>5083 Al</td>
<td>2.0</td>
<td>0.452</td>
<td>536</td>
<td>553</td>
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<tr>
<td>AP</td>
<td>Mild steel</td>
<td>0.33</td>
<td>0.563</td>
<td>202</td>
<td>150</td>
</tr>
<tr>
<td>AP</td>
<td>4130 steel</td>
<td>0.33</td>
<td>1.21</td>
<td>243</td>
<td>220</td>
</tr>
<tr>
<td>AP</td>
<td>4130 steel</td>
<td>0.33</td>
<td>1.62</td>
<td>240</td>
<td>247</td>
</tr>
</tbody>
</table>

### Table 12
The data from Radin and Goldsmith (1988) and our predicted results for the aluminum plates.

<table>
<thead>
<tr>
<th>( H ) (mm)</th>
<th>( H/D )</th>
<th>( \sigma /Y_t )</th>
<th>( \sigma_r ) (GPa)</th>
<th>( V_{bl} ) (model)</th>
<th>( V_{bl} ) (experimental)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>0.126</td>
<td>1.17</td>
<td>0.28</td>
<td>62.6</td>
<td>52.8</td>
</tr>
<tr>
<td>3.2</td>
<td>0.25</td>
<td>1.667</td>
<td>0.40</td>
<td>105.2</td>
<td>95.2</td>
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<tr>
<td>4.83</td>
<td>0.38</td>
<td>2</td>
<td>0.48</td>
<td>142.4</td>
<td>144</td>
</tr>
<tr>
<td>6.35</td>
<td>0.5</td>
<td>2</td>
<td>0.48</td>
<td>164</td>
<td>184.4</td>
</tr>
</tbody>
</table>

### Table 13
The data from Radin and Goldsmith (1988) and our predicted results for Lexan plates.

<table>
<thead>
<tr>
<th>( H ) (mm)</th>
<th>( H/D )</th>
<th>( \sigma /Y_t )</th>
<th>( \sigma_r ) (GPa)</th>
<th>( V_{bl} ) (model)</th>
<th>( V_{bl} ) (experimental)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2</td>
<td>0.25</td>
<td>1.667</td>
<td>0.133</td>
<td>60.9</td>
<td>62.8</td>
</tr>
<tr>
<td>4.8</td>
<td>0.38</td>
<td>2</td>
<td>0.16</td>
<td>81.9</td>
<td>82.4</td>
</tr>
<tr>
<td>5.7</td>
<td>0.45</td>
<td>2</td>
<td>0.16</td>
<td>89.2</td>
<td>93.9</td>
</tr>
<tr>
<td>11.7</td>
<td>0.92</td>
<td>2</td>
<td>0.16</td>
<td>127.8</td>
<td>161.6</td>
</tr>
</tbody>
</table>
\( \sigma_r/Y_t \) from Eq. (8) and the resulting values of \( \sigma_p \) using \( Y_t = 0.28 \) GPa. Thus, we have all the parameters which we need to calculate the predicted \( V_{bl} \) values with Eq. (2b), and these are listed in Table 14. We see that for most of these cases the predicted \( V_{bl} \) values fall in between the two velocities which bound the experimental value of \( V_{le} \). This agreement strongly enhances the confidence in our model for very low \( H/D \) ratios.

Finally we wish to compare our model’s prediction with an experimental result by Rusinek et al. (2009), who shot a 154 g spherical nosed steel projectile, at a very thin (0.8 mm) mild steel plate. The perforation process was clearly by the dishing mechanism including petalling of the plate. The projectile’s dimensions were \( D = 22 \) mm and \( L_{eff} = 51.6 \) mm, thus \( H/D = 0.036 \). Inserting this value in Eq. (8) results in a value of \( \sigma_r/Y_t = 0.812 \) for this projectile/plate combination. The compressive strength of the mild steel at the highest strain rate tested by Rusinek et al. (2009), was \( V_{bl} = 40 \) m/s. For this projectile/plate combination. This value is in excellent agreement with the experimental value of \( V_{le} \), which simplify the analysis and Eq. (2b) can be written in the following way:

\[
V_{bl} = (4HY_t/\rho_p L_{eff})^{0.5} \quad \text{for} \quad 1/3 \leq H/D \leq 1.0. \tag{12}
\]

Moreover, using the definition of \( L_{eff} \) through the mass of the projectile, \( M = \rho_p L_{eff} D^2/4 \), we can write for \( V_{bl} \):

\[
V_{bl} = \left( \pi D^2 HY_t/M \right)^{0.5} = D(\pi HY_t/M)^{0.5} \quad \text{for} \quad 1/3 \leq H/D \leq 1.0. \tag{13}
\]

This equation shows that for the limited range of \( H/D \) discussed here, a linear relation between \( V_{bl} \) and \( D \) is obtained, for a constant mass projectile. Thus, if one is changing the diameter of the projectile, while keeping its mass constant, such a linear relation between \( V_{bl} \) and \( D \) is expected for normalized \( H/D \) values between 1/3 and 1.0. This was exactly the subject of a recent work by Rusinek et al. (2008) who performed a large number of simulations for conical-nosed steel projectiles with the same mass, perforating a 12 mm steel plate. The constitutive relation for the steel plate in these simulations was that of Weldox 460E which has yield strength of about 0.5 GPa and a strain hardening behavior up to its failure at a stress of about 1.0 GPa. Considering the data given in Rusinek et al. (2008) for the rate and temperature dependence of this steel we can choose a value of \( Y_t = 0.85 \) GPa for the flow stress of this steel at the high strains and strain rates which characterize the ballistic event. The projectiles in these simulations had the same cone angle and the same mass of \( M = 200 \) g. Their diameters were: \( D = 10, 15, 20 \) and 25 mm, corresponding to normalized thickness of: \( H/D = 1.2, 0.8, 0.6 \) and 0.48. Thus, except for the \( D = 10 \) mm projectile, these values fall within the plateau range where \( \sigma_r/Y_t = 2.0 \), so that we can use Eq. (13) for the 15, 20 and 25 mm plates. For the 10 mm plate, with \( H/D = 1.2 \), we use Eq. (5) in order to calculate the value of \( V_{bl} = 2.146 \) (instead of the value of 2.0 for the other plates). This is a small difference but we shall account for it in order to be consistent with our approach. In fact, this correction means that we have to multiply the result for the 10 mm plate, as obtained by Eq. (13), by a factor of \( (2.146/2)^{0.5} = 1.036 \). Finally, the simulation results for \( V_{le} \), as given in Fig. 22 of Rusinek et al. (2008), are listed in Table 15 together with the predicted values from our model, using Eq. (13), with the corresponding values of \( Y_t, H \) and \( M \). The velocities are given in m/s.

One can clearly see the excellent agreement between our model and the simulation results of Rusinek et al. (2008). However, as we stated above, the nearly linear relation between \( V_{bl} \) and \( D \) which we see here, is valid only because the corresponding \( H/D \) ratios fall within the plateau range of \( \sigma_r/Y_t \). We stress this point here because we do not expect to find this linear relation for other \( H/D \) ratios.

### 4.4. The effect of strain hardening

One of the most common properties of many metals and alloys is their propensity to strain harden at large strains. Thus, it is quite difficult to determine their average flow stress (\( Y_t \)) which can be used for our model and, consequently, the analysis we presented in this paper cannot be applied for these materials. We wish to find out whether the general trends which we found for the effective resisting stress (\( \sigma_r \)), in strain hardening materials, are the same as those which we found for the simple elasto-plastic materials, as shown in Figs. 2 and 3. In particular, we are looking at the crossover from dishing to hole enlargement mechanism at \( H/D = 1/3 \), the constant value of \( \sigma_r \) for the range \( H/D = 1/3 – 1.0 \) and the increasing values of \( \sigma_r \) for \( H/D > 1.0 \). In order to investigate these

<table>
<thead>
<tr>
<th>( H ) (mm)</th>
<th>( H/D )</th>
<th>( \sigma_r/Y_t )</th>
<th>( \sigma_r ) (GPa)</th>
<th>( V_{bl} ) (m/s)</th>
<th>( V_{le} ) (m/s)</th>
<th>Calculated ( V_{le} ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>0.158</td>
<td>1.3</td>
<td>0.363</td>
<td>90.4</td>
<td>96.9</td>
<td>108.4</td>
</tr>
<tr>
<td>2.5</td>
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<td>0.334</td>
<td>79.4</td>
<td>96.7</td>
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<tr>
<td>2.0</td>
<td>0.105</td>
<td>1.087</td>
<td>0.304</td>
<td>67.2</td>
<td>83.7</td>
<td>81.0</td>
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<tr>
<td>1.5</td>
<td>0.079</td>
<td>0.983</td>
<td>0.276</td>
<td>54.3</td>
<td>62.9</td>
<td>66.4</td>
</tr>
<tr>
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<td>0.877</td>
<td>0.246</td>
<td>45.3</td>
<td>51.3</td>
<td>51.5</td>
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<tr>
<td>0.71</td>
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<td>0.815</td>
<td>0.228</td>
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<td>0.216</td>
<td>33.7</td>
<td>40.7</td>
<td>34.1</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>( D ) (mm)</th>
<th>( V_{bl} ) (simulations)</th>
<th>( V_{bl} ) (model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>135</td>
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</tr>
<tr>
<td>15</td>
<td>185</td>
<td>190</td>
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<td>20</td>
<td>255</td>
<td>253</td>
</tr>
<tr>
<td>25</td>
<td>310</td>
<td>316</td>
</tr>
</tbody>
</table>
issues we performed a series of simulations for a conical-nosed steel projectiles \( (D = 6 \text{ mm}, L_{\text{eff}} = 24 \text{ mm}) \) impacting 304 stainless steel plates of thicknesses ranging between 1 and 30 mm \( (H/D = 0.167 - 5.0) \). This alloy is well known for its high tendency for strain hardening and we used the Steinberg model with its constitutive properties which are given in the AUTODYN library. In particular, the strength of this material is increasing between 0.34 GPa at low strains to 2.5 GPa at very high strains. The constitutive relations for this material also include strain rate sensitivity but this has a minor effect on the simulation results since the range of rates involved in these perforations is quite limited. The procedure for determining \( \sigma_r \) for each plate thickness is the same as described above for the other materials. We infer the value of \( V_{\text{ref}} \) for each plate thickness from the simulation results for \( V_r \), through Eq. (1a), and then insert this value in Eq. (2a) to obtain the value of \( \sigma_r \) for the corresponding value of \( H/D \). Obviously, these effective stresses cannot be normalized by a definite value for the flow stress of this alloy, due to its strong hardening, and we shall analyze the changes in \( \sigma_r \) with \( H/D \), instead of the changes in \( \sigma_r/Y_t \).

Fig. 5 shows the values of \( \sigma_r \) for the 304SS alloy, as a function of \( H/D \), for a large range of plate thicknesses. One can clearly see here all the features we found earlier for the simple elasto-plastic materials, as shown in Figs. 2 and 3. These features are: (1) a steep rise in \( \sigma_r \) with \( H/D \) for the very thin plates, which are perforated by dishing, (2) a constant level of \( \sigma_r \) starting at about \( H/D = 0.4 \) up to \( H/D = 1.0 \) and, (3) an asymptotic increase for the thicker plates, with \( H/D > 1 \). The fact that the crossover point between dishing to hole enlargement is near \( H/D = 0.45 \), rather than at \( H/D = 1/3 \), can be attributed to the fact that the strength of these plates is not constant and is strongly dependent on the plastic strain. Still, a value of 0.45 is not very far from the \( H/D = 1/3 \) value which we found for the simple elasto-plastic materials. As with the other simulations we see the evidence for the dishing to hole enlargement transition by observing the shape of the plate around the perforation hole. It is interesting to note that the values we obtained for \( \sigma_r \), at the plateau in Fig. 5(b), are around 1.64 GPa. Assuming that the ratio of \( \sigma_r/Y_t \) should be equal to 2.0 at the plateau, for 304SS also, we get a value of \( Y_t = 0.82 \text{ GPa} \) for its effective flow stress in this range of plate thicknesses. It is quite difficult, if not impossible, to relate this value with the strength of 304SS in Steinberg’s model, which increases from 0.34 GPa at yield, to 2.5 GPa at the highest strains. However, a value of 0.82 GPa seems a reasonable average for the effective flow stress at the large strains which the target material, around the hole, experiences. The main point to note here is, that the results shown in Fig. 5 have all the features we highlighted in this paper, even for a material with such a high strain hardening tendency as this steel.

4.5. The effect of the back surface

One of the issues which we dealt with in Rosenberg and Dekel (2009a) is related to the effect of the entrance phase on the deceleration of rigid rods. In particular, our simulations showed that the free impact surface influences the deceleration of the rod until it penetrates to about six diameters. This issue is even more important for projectiles with small aspect ratios, which do not penetrate more than several diameters at impact velocities in the ordnance range. As we demonstrated in Rosenberg and Dekel (2009b) the situation is even more complicated for thin plates where the back free surface is also playing an important role in diminishing the forces on the projectile. In order to have a qualitative measure for the effect of the back surface we followed the deceleration–time histories in several simulations for thick targets. In these simulations one can exactly determine the time when the back surface starts to exert its influence \( (t_f) \). Fig. 6 shows the deceleration–time history for a conical-nosed tungsten projectile, with a diameter of 8.3 mm, as it penetrates a 100 mm aluminum plate with strength of 0.4 GPa. The impact velocity in this simulation was 800 m/s and one can clearly see the entrance phase effect in the early part of this history and the starting time \( (t_f) \) of the back surface effect. This time can be used in order to determine the position of the projectile at the moment the back surface starts affecting the process. Using several simulations we found that the distance between the nose of the projectile and the back surface of the target, at \( t_f \), is in the range of 2.5–3.5 diameters. This is about half the value we found for effect of the impact face (about six diameters). Thus, to a first approximation, we can state that a target has to be thicker than about ten projectile diameters in order to exert its maximal stress on the projectile, at least for part of the perforation time.
5. Conclusions

This paper presents the results of a large number of numerical simulations for the perforation of ductile plates by sharp-nosed rigid projectiles, in order to follow the various processes which take place during such perforations. In particular, we were able to distinguish between the dishing and the hole enlargement processes which are the main perforation mechanisms for thin and thick plates, respectively. Our main result is that the normalized resisting stresses, which are exerted by the plates, can be simply related to the normalized thickness of the plates, through:

\[
\sigma_r/Y_t = \begin{cases} 
\frac{2}{3} + 4(H/D) & \text{for } 0 < H/D \leq 1/3 \\
2.0 & \text{for } 1/3 \leq H/D \leq 1.0 \\
2.0 + 0.8 \cdot \ln(H/D) & \text{for } H/D \geq 1.0.
\end{cases}
\]

The values of \(\sigma_r\) should be inserted in Eq. (2b) in order to obtain the values of \(V_{bl}\) for the projectile/target combination. Comparisons of these predictions for \(V_{bl}\) with experimental data from several sources were shown to be good, for a large range of plate thicknesses. Clearly, a lot of analytical work remains in order to account for these numerically derived relations. Once the value of \(V_{bl}\) is determined by our model, the data for residual velocities as a function of impact velocity can be predicted by the energy-based model of Recht and Ipson (1963), through Eq. (1b). Some additional issues were discussed here, such as the effect of lamination on target performance and the back surface influence on the perforation process.

References


